CHAPTER 23

Problem

57. A proton moving to the right at 3.8×10^5 m/s enters a region where a 56 kN/C electric field points to the left. (a) How far will the proton get before its speed reaches zero? (b) Describe its subsequent motion.

Solution

(a) Choose the x-axis to the right, in the direction of the proton, so that the electric field is negative to the left. If the Coulomb force on the proton is the only important one, the acceleration is $a_x = e(-E)/m$. Equation 2-11, with $v_{ox} = 3.8 \times 10^5$ m/s and $v_x = 0$, gives a maximum penetration into the field region of $x - x_0 = -v_{ox}^2/2a_x = mv_{ox}^2/2eE =$

$$\frac{(1.67 \times 10^{-27}\ \text{kg})(3.8 \times 10^5\ \text{m/s})^2}{2(1.6 \times 10^{-19}\ \text{C})(56 \times 10^3\ \text{N/C})} = 1.35\ \text{cm}.$$

(b) The proton then moves to the left, with the same constant acceleration in the field region, until it exits with the initial velocity reversed.

Problem

65. A dipole with dipole moment 1.5 nC·m is oriented at 30° to a 4.0-MN/C electric field. (a) What is the magnitude of the torque on the dipole? (b) How much work is required to rotate the dipole until it's antiparallel to the field?

Solution

(a) The torque on an electric dipole in an external electric field is given by Equation 23-11; $\tau = |\mathbf{p} \times \mathbf{E}| = pE\sin\theta = (1.5\ \text{nC}\cdot\text{m})(4.0\ \text{MN/C})\sin 30° = 3.0\ \text{mN}\cdot\text{m}$. (b) The work done against just the electric force is equal to the change in the dipole's potential energy (Equation 23-12); $W = \Delta U = (-\mathbf{p}\cdot\mathbf{E})_f - (-\mathbf{p}\cdot\mathbf{E})_i = pE(\cos 30° - \cos 180°) = (1.5\ \text{nC}\cdot\text{m}) \times$ $(4.0\ \text{MN/C})(1.866) = 11.2\ \text{mJ}$.

Chapter 24

Problem

17. The electric field at the surface of a uniformly charged sphere of radius 5.0 cm is 90 kN/C. What would be the field strength 10 cm from the surface?

Solution

The electric field due to a uniformly charged sphere is like the field of a point charge for points outside the sphere, i.e., $E(r) \propto 1/r^2$ for $r \geq R$. Thus, at 10 cm from the surface, $r = 15$ cm and $E(15\ \text{cm}) = (5/15)^2 E(5\ \text{cm}) = (90\ \text{kN/C})/9 = 10\ \text{kN/C}$.

Problem

18. A solid sphere 25 cm in radius carries 14 μC, distributed uniformly throughout its volume. Find the electric field strength (a) 15 cm, (b) 25 cm, and (c) 50 cm from the sphere's center.

Solution

Example 24-1 shows that (a) at $15\text{ cm} = r < R = 25\text{ cm}, E = kQr/R^3 = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(14\ \mu\text{C})(15\text{ cm})/(25\text{ cm})^3 = 1.21\text{ MN/C},$ (b) at $r = R,\ E = kQ/R^2 = (\frac{5}{3})(1.21\text{ MN/C}) = 2.02\text{ MN/C},$ and (c) at $r = 2R > R,\ E = kQ/(2R)^2 =$ $(\frac{1}{4})(2.02\text{ MN/C}) = 504\text{ kN/C}.$

Problem

20. Positive charge is spread uniformly over the surface of a spherical balloon 70 cm in radius, resulting in an electric field of 26 kN/C at the balloon's surface. Find the field strength (a) 50 cm from the balloon's center and (b) 190 cm from the center. (c) What is the net charge on the balloon?

Solution

(a) Inside a uniformly charged spherical shell, the electric field is zero (see Example 24-2). (b) Outside, the field is like that of a point charge, with total charge at the center, so
$E(190\text{ cm}) = E(70\text{ cm})(70/190)^2 = (0.136)(26\text{ kN/C}) = 3.53\text{ kN/C}.$
(c) Using the given field strength at the surface, we find a net charge
$Q = ER^2/k = (26\text{ kN/C})(0.7\text{ m})^2/(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 1.42\ \mu\text{C}.$

Problem

22. A solid sphere 2.0 cm in radius carries a uniform volume charge density. The electric field 1.0 cm from the sphere's center has magnitude 39 kN/C. (a) At what other distance does the field have this magnitude? (b) What is the net charge on the sphere?

Solution

(a) Referring to Example 24-1, we see that at $r = \frac{1}{2}R,\ E = kQ(\frac{1}{2}R)/R^3 = kQ/2R^2$. This is also the field strength outside the sphere at a distance $r = \sqrt{2}R = \sqrt{2}(2\text{ cm}) = 2.83\text{ cm}.$ (b) Using the given field strength at $r = \frac{1}{2}R,$ we find $Q = 2R^2E/k = 2(2\text{ cm})^2(39\text{ kN/C})/(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 3.47\text{ nC}.$

Problem

23. A point charge $-2Q$ is at the center of a spherical shell of radius R carrying charge Q spread uniformly over its surface. What is the electric field at (a) $r = \frac{1}{2}R$ and (b) $r = 2R$? (c) How would your answers change if the charge on the shell were doubled?

Solution

The situation is like that in Problem 21. (a) At $r = \frac{1}{2}R < R$ (inside shell),
$E = E_{\text{pt}} + E_{\text{shell}} = k(-2Q)/(\frac{1}{2}R)^2 + 0 = -8\,kQ/R^2$ (the minus sign means the direction is radially inward).
(b) At $r = 2R > R$ (outside shell), $E = E_{\text{pt}} + E_{\text{shell}} = k(-2Q + Q)/(2R)^2 = -kQ/4R^2$ (also radially inward). (c) If $Q_{\text{shell}} = 2Q,$ the field inside would be unchanged, but the field outside would be zero (since $q_{\text{shell}} + q_{\text{pt}} = 2Q - 2Q = 0$).

Problem

24. A spherical shell of radius 15 cm carries $4.8\ \mu\text{C},$ distributed uniformly over its surface. At the center of the shell is a point charge. (a) If the electric field at the surface of the sphere is 750 kN/C and points outward, what is the charge of the point charge? (b) What is the field just inside the shell?

Solution

(a) As in the previous solution, the field strength at the surface of the shell $(r = R)$ is $E_{pt} + E_{shell} = k(q_{pt} + q_{shell})/R^2$, hence $q_{pt} = [(15\text{ cm})^2(750\text{ kN/C})/(9\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)] - 4.8\ \mu\text{C} = -2.93\ \mu\text{C}$. (b) Just inside the shell, at $r = 15\text{ cm} - \delta$ (where $\delta \ll 15\text{ cm}$), the field is due to the point charge only: $E = k(-2.93\ \mu\text{C})/(15\text{ cm} + \delta)^2 \approx -1.17\text{ MN/C}$, directed radially inward.

Problem

26. The thick, spherical shell of inner radius a and outer radius b shown in Fig. 24-45 carries a uniform volume charge density ρ. Find an expression for the electric field strength in the region $a < r < b$, and show that your result is consistent with Equation 24-7 when $a = 0$.

Solution

Use the result of Gauss's law applied to a spherically symmetric distribution, $E = q_{enclosed}/4\pi\varepsilon_0 r^2$. For $a < r < b$ in a spherical shell with charge density ρ, $q_{enclosed} = \frac{4}{3}\pi(r^3 - a^3)\rho$, so $E = \rho(r^3 - a^3)/3\varepsilon_0 r^2 = (\rho/3\varepsilon_0)(r - a^3/r^2)$. If $a \to 0$, Equation 24-7 for a uniformly charged spherical volume is recovered.

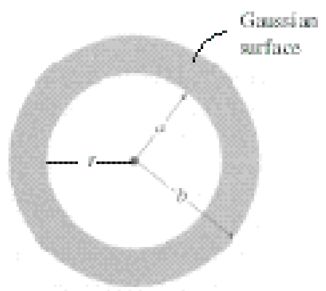


FIGURE 24-45 Problem 26 Solution.

Problem

27. How should the charge density within a solid sphere vary with distance from the center in order that the magnitude of the electric field in the sphere be constant?

Solution

Assume that ρ is spherically symmetric, and divide the volume into thin shells with $dV = 4\pi r^2\,dr$. From Gauss's law and Equation 24-5,

$$E = \frac{1}{4\pi\varepsilon_0 r^2}\int_V \rho\,dV = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r')4\pi r'^2 dr' = \frac{1}{\varepsilon_0 r^2}\int_0^r \rho r'^2 dr'.$$

It can be seen that if $\rho(r') \propto 1/r'$ then E is constant, but we can obtain the same result mathematically, by differentiation. If E is constant, $dE/dr = 0$. This implies

$$0 = \frac{d}{dr}\left(\frac{1}{r^2}\int_0^r \rho r'^2\,dr'\right) = \frac{1}{r^2}\frac{d}{dr}\left(\int_0^r \rho r'^2 dr'\right) + \left(\int_0^r \rho r'^2\,dr'\right)\frac{d}{dr}\left(\frac{1}{r^2}\right) = \frac{1}{r^2}\rho(r)r^2 - \frac{2}{r^3}\int_0^r \rho r'^2 dr',$$

or $$\rho(r) = \frac{2}{r^3}\int_0^r \rho(r')r'^2 dr'.$$

Since $r^{-2}\int_0^r \rho(r')r'^2\,dr' = \varepsilon_0 E$ is a constant, by hypothesis, $\rho(r) = 2\varepsilon_0 E/r \propto 1/r$, as suspected. (Look up how to take the derivative of an integral in any calculus textbook.) Note that constant magnitude does not imply constant direction; $\mathbf{E} = E\hat{\mathbf{r}}$ is spherically symmetric, not uniform.

Problem

29. A long solid rod 4.5 cm in radius carries a uniform volume charge density. If the electric field strength at the surface of the rod (not near either end) is 16 kN/C, what is the volume charge density?

Solution

If the rod is long enough to approximate its field using line symmetry, we can equate the flux through a length ℓ of its surface (Equation 24-8) to the charge enclosed. The latter is the charge density (a constant) times the volume of a length ℓ of rod. Thus, $2\pi R\ell E = q_{\text{enclosed}}/\varepsilon_0 = \rho\pi R^2\ell/\varepsilon_0$, or $\rho = 2\varepsilon_0 E/R = 2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(16\text{ kN/C})/(4.5\text{ cm}) = 6.29\ \mu\text{C/m}^3$. (This is the magnitude of ρ, since the direction of the field at the surface, radially inward or outward, was not specified.)

Problem

31. An infinitely long rod of radius R carries a uniform volume charge density ρ. Show that the electric field strengths outside and inside the rod are given, respectively, by $E = \rho R^2/2\varepsilon_0 r$ and $E = \rho r/2\varepsilon_0$, where r is the distance from the rod axis.

Solution

The charge distribution has line symmetry (as in Problem 29) so the flux through a coaxial cylindrical surface of radius r (Equation 24-8) equals $q_{\text{enclosed}}/\varepsilon_0$, from Gauss's law. For $r > R$ (outside the rod), $q_{\text{enclosed}} = \rho\pi R^2\ell$, hence $E_{\text{out}} = \rho\pi R^2\ell/2\pi r\ell\varepsilon_0 = \rho R^2/2\varepsilon_0 r$. For $r < R$ (inside the rod), $q_{\text{enclosed}} = \rho\pi r^2\ell$, hence $E_{\text{in}} = \rho\pi r^2\ell/2\pi r\ell\varepsilon_0 = \rho r/2\varepsilon_0$. (The field direction is radially away from the symmetry axis if $\rho > 0$, and radially inward if $\rho < 0$.)

Problem

34. A square nonconducting plate measures 4.5 m on a side and carries charge spread uniformly over its surface. The electric field 10 cm from the plate and not near an edge has magnitude 430 N/C and points toward the plate. Find
(a) the surface charge density on the plate and (b) the total charge on the plate. (c) What is the electric field strength 20 cm from the plate.

Solution

We assume that the field due to the surface charge on the plate has plane symmetry (at least for the points considered in this problem), so that $E = \sigma/2\varepsilon_0$ (positive away from the surface and negative toward it). Then (a) $\sigma = 2\varepsilon_0 E = 2(8.85\times10^{-12}\text{ N}\cdot\text{m}^2/\text{C}^2)(-430\text{ N/C}) = -7.61\text{ nC/m}^2$, (b) $q = \sigma A = (-7.61\text{ nC/m}^2)(4.5\text{ m})^2 = -154\text{ nC}$, and (c) $E = -430\text{ N/C}$
(E is independent of distance, as long as the distance is small enough to justify approximate plane symmetry).

Problem

36. A slab of charge extends infinitely in two dimensions and has thickness d in the third dimension, as shown in
Fig. 24-46. The slab carries a uniform volume charge density ρ. Find expressions for the electric field strength
(a) inside and (b) outside the slab, as functions of the distance x from the center plane.

Solution

If the slab were really infinite, the electric field would be everywhere normal to it (the x direction) and symmetrical about the center plane. (b) Gauss's law, applied to the surface superposed on Fig. 24-46, gives, for points outside the slab $(|x| > \frac{1}{2}d)$, $EA + EA = \rho dA/\varepsilon_0$, or $E = \rho d/2\varepsilon_0$ (equivalent to a sheet with

$\sigma = \rho d$). (a) For points inside the slab $(|x| \le \frac{1}{2}d)$, $2EA = \rho 2xA/\epsilon_0$, or $E = \rho x/\epsilon_0$. **E** is directed away from (toward) the central plane for positive (negative) charge density.

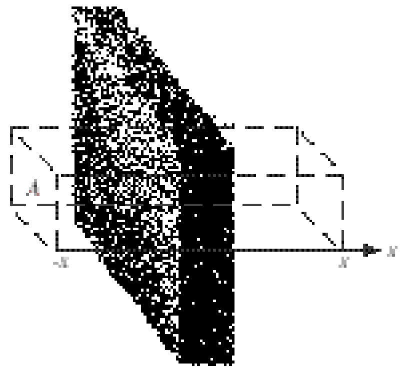


FIGURE 24-46 Problem 36 Solution.

Problem

41. The electric field strength on the axis of a uniformly charged disk is given by $E = 2\pi k\sigma(1 - x/\sqrt{x^2 + a^2})$, with σ the surface charge density, a the disk radius, and x the distance from the disk center. If $a = 20$ cm, (a) for what range of x values does treating the disk as an infinite sheet give an approximation to the field that is good to within 10%? (b) For what range of x values is the point-charge approximation good to 10%?

Solution

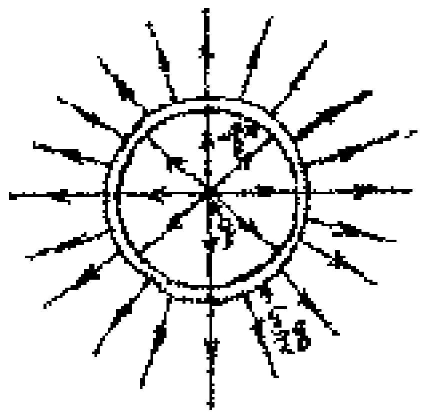
(Note: The expression given, for the field strength on the axis of a uniformly charged disk, holds only for positive values of x.) (a) For small x, using the field strength of an infinite sheet, $E_{sheet} = \sigma/2\epsilon_0 = 2\pi k\sigma$, produces a fractional error less than 10% if $|E_{sheet} - E|/E < 0.1$. Since $E_{sheet} > E$, this implies that $E_{sheet}/E < 1.1$ or $2\pi k\sigma/2\pi k\sigma(1 - x/\sqrt{x^2 + a^2}) < 1.1$. The steps in the solution of this inequality are: $1.1x < 0.1\sqrt{x^2 + a^2}$, $1.21x^2 < 0.01(x^2 + a^2)$, $x < a\sqrt{0.01/1.20} = 9.13 \times 10^{-2}\, a$. For $a = 20$ cm, $x < 1.83$ cm. (b) For large x, the point charge field, $E_{pt} = kq/x^2 = k\pi\sigma a^2/x^2$, is good to 10% for $|E_{pt} - E|/E < 0.1$. The solution of this inequality is simplified by defining an angle ϕ, such that $\cos\phi = x/\sqrt{x^2 + a^2}$ and $\tan\phi = a/x$. In terms of ϕ, one finds $E = 2\pi k\sigma(1 - \cos\phi)$, $E_{pt} = k\pi\sigma\tan^2\phi$, and $E_{pt}/E = \tan^2\phi/2(1 - \cos\phi)$. Furthermore, $\tan^2\phi = \sin^2\phi/\cos^2\phi = (1 - \cos\phi)(1 + \cos\phi)/\cos^2\phi$, so $E_{pt}/E = (1 + \cos\phi)/2\cos^2\phi$. The range $0 \le x < \infty$ corresponds to $0 < \phi \le \pi/2$, so $E_{pt}/E > 1$ and the inequality becomes $E_{pt}/E = (1 + \cos\phi) \div 2\cos^2\phi < 1.1$, or $2.2\cos^2\phi - \cos\phi - 1 > 0$. The quadratic formula for the positive root gives $\cos\phi > (1 + \sqrt{1 + 8.8})/4.4 = 0.939$, or $\phi < 20.2°$. This implies $x = a/\tan\phi > a/\tan 20.2° = 2.72\, a$. For $a = 20$ cm, $x > 54.5$ cm.

Problem

46. A point charge $+q$ lies at the center of a spherical conducting shell carrying a net charge $\frac{3}{2}q$. Sketch the field lines both inside and outside the shell, using 8 field lines to represent a charge of magnitude q.

Solution

The field inside the shell is just due to the point charge (8 lines of force radiating outward). The field outside is like that of
a point charge $q + \frac{3}{2}q = \frac{5}{2}q$ (20 lines of force radiating outward). (Note: there is a charge $-q$, spread uniformly over the inner surface of the shell, and the field inside the conducting material is zero.)



Problem 46 Solution.

Problem

47. A 250-nC point charge is placed at the center of an uncharged spherical conducting shell 20 cm in radius. (a) What is the surface charge density on the outer surface of the shell? (b) What is the electric field strength at the shell's outer surface?

Solution

(a) There is a non-zero field outside the shell, because the net charge within is not zero. Therefore, there is a surface charge density $\sigma = \varepsilon_0 E$ on the outer surface of the shell, which is uniform, if we ignore the possible presence of other charges and conducting surfaces outside the shell. Gauss's law (with reasoning similar to Example 24-7) requires that the charge on the shell's outer surface is equal to the point charge within, so $\sigma = q/4\pi R^2 = 250\ \text{nC}/4\pi(0.20\ \text{m})^2 = 497\ \text{nC/m}^2$. (b) Then the field strength at the outer surface is $E = \sigma/\varepsilon_0 = 56.2\ \text{kN/C}$.

Problem

51. A total charge of $18\ \mu\text{C}$ is applied to a thin, square metal plate 75 cm on a side. Find the electric field strength near the plate's surface.

Solution

The net charge of $18\ \mu\text{C}$ must distribute itself over the outer surface of the plate, in accordance with Gauss's law for conductors. The outer surface consists of two plane square surfaces on each face, plus the edges and corners. Symmetry arguments imply that for an isolated plate, the charge density on the faces is the same, but not necessarily uniform because the edges and corners also have charge. If the plate is thin, we could assume that the edges and corners have negligible charge and that the density on the faces is approximately uniform. Then the surface charge density is the total charge divided by the area of both faces, $\sigma = 18\ \mu\text{C}/2(75\ \text{cm})^2 = 16.0\ \mu\text{C}$, and the field strength near the plate (but not near an edge) is $E = \sigma/\varepsilon_0 = 1.81\ \text{MN/C}$.

Problem

52. Two closely spaced parallel metal plates carry surface charge densities $\pm 95\ \text{nC/m}^2$ on their facing surfaces, with no charge on their outer surfaces. Find the electric field strength (a) between the plates and (b) outside the plates. Treat the plates as infinite in extent.

Solution

The last paragraph of Section 24-6 explains why (a) between the plates, $E = \sigma/\varepsilon_0 = (95\ \text{nC/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 10.7\ \text{kN/C}$, and (b) outside, but not near an edge, $E = 0$.

Problem

54. A coaxial cable consists of an inner wire and a concentric cylindrical outer conductor (Fig. 24-49). If the conductors carry equal but opposite charges, show that there is no surface charge density on the *outside* of the outer conductor.

Solution

Assume line symmetry, and apply Gauss's law, as in Equation 24-8, to the outer cylindrical conducting surface, $2\pi r\ell E_{\text{surf}} = q_{\text{enclosed}}/\varepsilon_0$. Since the conductors in the cable carry opposite charges of equal magnitude, there is zero charge enclosed, so the field and the charge density there $(\sigma = \varepsilon_0 E_{\text{surf}})$ vanish.