Problem

7. Four identical charges q, initially widely separated, are brought to the vertices of a tetrahedron of side a (Fig. 26-26). Find the electrostatic energy of this configuration.

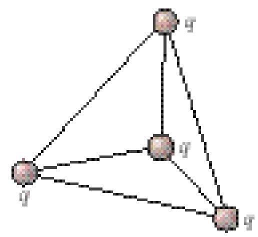

FIGURE 26-26 Problem 7.

Solution

There are six different pairs of equal charges and the separation of any pair is a. Thus, $W = \sum_{\text{pairs}} kq_i q_j/a = 6kq^2/a$. (See Problem 1.)

Problem

8. A charge Q_0 is at the origin. A second charge, $Q_x = 2Q_0$, is brought to the point $x = a$, $y = 0$. Then a third charge Q_y is brought to the point $x = 0$, $y = a$. If it takes twice as much work to bring in Q_y as it did Q_x, what is Q_y in terms of Q_0?

Solution

The work necessary to bring up Q_x is $W_x = kQ_0Q_x/a = 2kQ_0^2/a$, while the work necessary to subsequently bring up Q_y is $W_y = kQ_0Q_y/a + kQ_xQ_y/\sqrt{2}a = kQ_0Q_y(1+\sqrt{2})/a$. If $W_y = 2W_x$, then $Q_y(1+\sqrt{2}) = 4Q_0$, or $Q_y = 4Q_0/(\sqrt{2}+1) = 1.66Q_0$. (Note: $1/(\sqrt{2}+1) = \sqrt{2}-1$.)

Problem

10. Two square conducting plates measure 5.0 cm on a side. The plates are parallel, spaced 1.2 mm apart, and initially uncharged. (a) How much work is required to transfer $7.2\ \mu\text{C}$ from one plate to the other? (b) How much work is required to transfer a second $7.2\ \mu\text{C}$?

Solution

The separation is much smaller than the linear dimensions of the plates, so the discussion in Section 26-2 applies. (a) From Equation 26-2, $W = Q^2d/2\varepsilon_0 A = (7.2\ \mu\text{C})^2(1.2\ \text{mm})/2(8.85\times10^{-12}\ \text{F/m})(5\ \text{cm})^2 = 1.41\ \text{J}$. (b) The additional work required to double the charge on each plate is $\Delta W = (2Q)^2 d/2\varepsilon_0 A - W = 3W = 4.22\ \text{J}$.

Problem

13. A conducting sphere of radius a is surrounded by a concentric spherical shell of radius b. Both are initially uncharged. How much work does it take to transfer charge from one to the other until they carry charges $\pm Q$?

Solution

When a charge q (assumed positive) is on the inner sphere, the potential difference between the spheres is $V = kq(a^{-1} - b^{-1})$. (See the solution to Problem 25-63(a).) To transfer an additional charge dq from the outer sphere requires work $dW = V\,dq$, so the total work required to transfer charge Q (leaving the spheres oppositely charged) is $W = \int_0^Q V\,dq = \int_0^Q kq\,dq\,(a^{-1} - b^{-1}) = \frac{1}{2}kQ^2(a^{-1} - b^{-1})$. (Incidentally, this shows that the capacitance of this spherical capacitor is $1/k(a^{-1} - b^{-1}) = ab/k(b - a)$; see Equation 26-8a.)

Problem

15. Two conducting spheres of radius a are separated by a distance $l \gg a$; since the distance is large, neither sphere affects the other's electric field significantly, and the fields remain spherically symmetric. (a) If the spheres carry equal but opposite charges $\pm q$, show that the potential difference between them is $2kq/a$. (b) Write an expression for the work dW involved in moving an infinitesimal charge dq from the negative to the positive sphere. (c) Integrate your expression to find the work involved in transferring a charge Q from one sphere to the other, assuming both are initially uncharged.

Solution

(a) The potential difference between the two (essentially isolated) spheres is $\Delta V = kq/a - k(-q)/a = 2kq/a$ (see Equation 25-12). (b) ΔV is the work per unit positive charge transferred between the spheres, so $dW = dq\;\Delta V = 2kq\,dq/a$. (c) The integration yields $W = \int dW = \int_0^Q 2kq\,dq/a = kQ^2/a$.

Section 26-3: Energy and the Electric Field

Problem

16. The energy density in a uniform electric field is $3.0\ \mathrm{J/m^3}$. What is the field strength?

Solution

Equation 26-3 relates the field strength and the electric energy density,

$$E = \sqrt{2u/\varepsilon_0} = \sqrt{\frac{2(3\ \mathrm{J/m^3})}{(8.85\times 10^{-12}\ \mathrm{F/m})}} = 8.23\times 10^5\ \mathrm{V/m}.$$

(Note: the manipulation of units is facilitated by the relations $\mathrm{V} = \mathrm{J/C}$ and $\mathrm{F} = \mathrm{C/V}$. Thus, $(\mathrm{J/m^3})/(\mathrm{F/m}) = (\mathrm{VC/m^3})/(\mathrm{C/V\cdot m}) = (\mathrm{V/m})^2$.)

Problem

22. A sphere of radius R contains charge Q spread uniformly throughout its volume. Find an expression for the electrostatic energy contained within the sphere itself. *Hint:* Consult Example 24-1.

Solution

The radially symmetric field inside the sphere is $E_r = kQr/R^3$, so the energy density is $u(r) = \frac{1}{2}\varepsilon_0 E_r^2 = kQ^2r^2/8\pi R^6$. With thin spherical shells of radius r for volume elements, $dV = 4\pi r^2\,dr$, the integral for the energy is $U = \int_{\text{sphere}} u\,dV = \int_0^R \frac{1}{2}(kQ^2/R^6)r^4\,dr = kQ^2/10R$. (This is just the energy stored inside the sphere. For the energy outside the sphere, and the total energy, see the next two problems.)

Problem

23. A sphere of radius R carries a total charge Q distributed over its surface. Show that the total energy stored in its electric field is $U = kQ^2/2R$.

Solution

The calculation of the electrostatic energy for a sphere with uniform surface charge density is, in fact, given in Example
26-3. We simply set $R_2 = R$, the radius of the sphere, and $R_1 = \infty$ (so the integral covers all the space where the field is non-zero).

Problem

28. A capacitor's plates hold $1.3\ \mu\text{C}$ when charged to 60 V. What is its capacitance?

Solution

From Equation 26-5, $C = Q/V = 1.3\ \mu\text{C}/60\ \text{V} = 0.0217\ \mu\text{F}$.

Problem

33. Find the capacitance of a parallel-plate capacitor consisting of circular plates 20 cm in radius separated by 1.5 mm.

Solution

For a (closely spaced) parallel plate capacitor, with circular plates, Example 26-4 shows that
$C = \varepsilon_0 \pi r^2/d = (8.85\ \text{pF/m})\pi(20\ \text{cm})^2/(1.5\ \text{mm}) = 741\ \text{pF}$.

Problem

34. A parallel-plate capacitor with 1.1-mm plate spacing has $\pm 2.3\ \mu\text{C}$ on its plates when charged to 150 V. What is the plate area?

Solution

From Equation 26-6, $A = Qd/\varepsilon_0 V = (2.3\ \mu\text{C})(1.1\ \text{mm})/(8.85\ \text{pF/m})(150\ \text{V}) = 1.91\ \text{m}^2$.

Problem

35. Find the capacitance of a 1.0-m-long piece of coaxial cable whose inner conductor radius is 0.80 mm and whose outer conductor radius is 2.2 mm, with air in between.

Solution

The capacitance of air-filled $(\kappa = 1)$ cylindrical capacitor was found in Example 26-5:
$C = 2\pi\varepsilon_0 \ell/\ln(b/a) = 2\pi(8.85\ \text{pF/m})\ (1\ \text{m})/\ln(2.2/0.8) = 55.0\ \text{pF}$.

Problem

40. A certain capacitor stores 40 mJ of energy when charged to 100 V. (a) How much would it store when charged to 25 V? (b) What is its capacitance?

Solution

(a) Equation 26-8b, expressed as a ratio for the same capacitor charged to two different voltages, gives $U_2/U_1 = (V_2/V_1)^2$. Therefore, $U_2 = (25/100)^2(0.04\ \text{J}) = 2.5$ mJ. (b) From the same Equation 26-8b, $C = 2U_1/V_1^2 = 2(0.04\ \text{J})/(100\ \text{V})^2 = 8\ \mu\text{F}$. ($C = 2U_2/V_2^2$, of course.)

Problem

47. A solid conducting slab is inserted between the plates of a charged capacitor, as shown in Fig. 26-29. The slab thickness is 60% of the plate spacing, and its area is the same as the plates. (a) What happens to the capacitance? (b) What happens to the stored energy, assuming the capacitor is not connected to anything?

Solution

(a) The charge on the plates remains the same, and so does the electric field $(E = \sigma/\varepsilon_0)$ in the gaps between either plate and the slab. However, the separation (i.e., the thickness of the field region) between the plates is reduced to 40% of its original value $d' = d_1 + d_2 = 0.4d$, therefore the capacitance is increased, $C' = \varepsilon_0 A/d' = \varepsilon_0 A/0.4d = 2.5$ C. (The equations $V = E\ell$ and $C = Q/V$ lead to the same result.) In fact, the configuration behaves like a series combination of two parallel plate capacitors, $1/C' = C_1^{-1} + C_2^{-1} = (d_1/\varepsilon_0 A) + (d_2/\varepsilon_0 A) = (d_1 + d_2)/\varepsilon_0 A = 0.4d/\varepsilon_0 A = 1/2.5$ C. (b) When the charge is constant (no connections to anything isolates the system), the energy stored is inversely proportional to the capacitance, $U = Q^2/2C$. Thus $U' = Q^2/2C' = Q^2/2(2.5C) = 0.4U$, or the energy decreases to 40% of its original value. (With the slab inserted, there is less field region and less energy stored. While the slab is being inserted, work is done by electrical forces to conserve energy.)

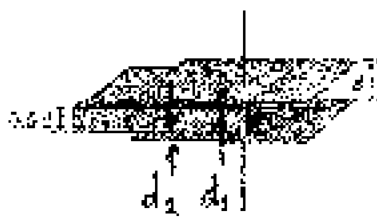

FIGURE 26-29 Problem 47 Solution.

Problem

52. (a) What is the equivalent capacitance of the combination shown in Fig. 26-30? (b) If a 100-V battery is connected across the combination, what is the charge on each capacitor? (c) What is the voltage across each?

Solution

(a) C_1 is in series with the parallel combination of C_2 and C_3. Thus, $C = C_1(C_2 + C_3)/(C_1 + C_2 + C_3) = (0.02\ \mu\text{F}) \times (1 + 2)/(2 + 1 + 2) = 0.012\ \mu\text{F}$. (b) The net charge on the entire combination is $Q = CV = (0.012\ \mu\text{F})(100\ \text{V}) = 1.2\ \mu\text{C}$. Since C_1 is in series with the capacitors in parallel, $Q = 1.2\ \mu\text{C} = Q_1 = Q_2 + Q_3$. Moreover, for the parallel capacitors, $V_2 = Q_2/C_2 = V_3 = Q_3/C_3$, so $Q_3/Q_2 = C_3/C_2 = 2$. Thus, $Q_2 = (1/3)Q = 0.4\ \mu\text{C}$ and $Q_3 = (2/3)Q = 0.8\ \mu\text{C}$. (In general, for two capacitors in parallel, $Q_2 = C_2Q/(C_2 + C_3)$ etc.) (c) Equation 26-5, applied to each capacitor, gives $V_1 = Q_1/C_1 = 1.2\ \mu\text{C}/0.02\ \mu\text{F} = 60$ V, and $V_2 = V_3 = 40$ V. (Alternatively, one can first use the general result in the solution to
Problem 51 (with C_2 replaced by $C_2 + C_3$) to obtain the voltages,

$V_1 = (C_2 + C_3)V/(C_1 + C_2 + C_3) = (3/5)(100\text{ V})$, $V_2 = V_3 = C_1V/(C_1 + C_2 + C_3) = (2/5)(100\text{ V})$, and then use Equation 26-5 to find the charges.)

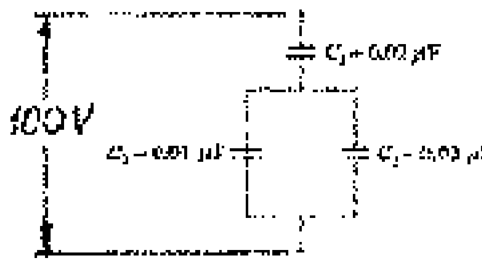

FIGURE 26-30 Problem 52 Solution.

Problem

54. What is the equivalent capacitance of the four identical capacitors in Fig. 26-31, measured between *A* and *B*?

Solution

Relative to points *A* and *B*, the combination of capacitors 2, 3, and 4 is in parallel with 1 (see numbering added to
Fig. 26-28), so $C_{tot} = C_1 + C_{234}$. However, C_{234} consists of 2 in series with the parallel combination of 3 and 4, so $C_{234} = C_2C_{34}/(C_2 + C_{34}) = C_2(C_3 + C_4)/(C_2 + C_3 + C_4)$. Since each individual capacitance is equal to C,
$C_{234} = \frac{2}{3}C$ and $C_{tot} = \frac{5}{3}C$.

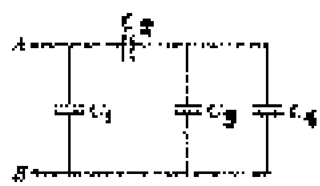

FIGURE 26-31 Problem 54 Solution.

Problem

63. A $5.0\text{-}\mu\text{F}$ capacitor is charged to 50 V, and a $2.0\text{-}\mu\text{F}$ capacitor is charged to 100 V. The two are disconnected from their charging batteries and connected in parallel, positive to positive. (a) What is the common voltage across each after they are connected? *Hint:* Charge is conserved. (b) Compare the total electrostatic energy before and after the capacitors are connected. Speculate on the discrepancy.

Solution

(a) The charge on the parallel combination is the sum of the original charges,
$Q_\parallel = Q_1 + Q_2 = C_1V_1 + C_2V_2 = (5\ \mu\text{F})(50\text{ V}) + (2\ \mu\text{F})(100\text{ V}) = 450\ \mu\text{C}$, while the capacitance is $C_\parallel = C_1 + C_2 = 7\ \mu\text{F}$. Thus, the voltage is $V_\parallel = Q_\parallel/C_\parallel = 450\ \mu\text{C}/7\ \mu F = 64.3\text{ V}$. (b) The total energy stored in both capacitors before they are connected is
$\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{5}(5\ \mu\text{F})(50\text{ V})^2 + \frac{1}{2}(2\ \mu\text{F})(100\text{ V})^2 = 16.3\text{ mJ}$. After the connection, $U_\parallel = \frac{1}{2}C_\parallel V_\parallel^2 =$
$\frac{1}{2}(7\ u\text{F})(64.3\text{ V})^2 = 14.5\text{ mJ}$, a difference of 1.79 mJ. It takes work to redistribute the original charges when the capacitors are connected. (The new charges are
$Q_1' = (5\ \mu\text{F})(64.3\text{ V}) = 321\ \mu\text{C}$, and $Q_2' = 129\ \mu\text{C}$, respectively.)

Problem

65. A 470-pF capacitor consists of two circular plates 15 cm in radius, separated by a sheet of polystyrene. (a) What is the thickness of the sheet? (b) What is the working voltage?

Solution

(a) With reference to Equations 26-6, 26-11, and Table 26-1, one finds that $C = \kappa C_0 = \kappa \varepsilon_0 A/d$, or $d = \kappa\varepsilon_0 A/C = (2.6)(8.85\text{ pF/m})\pi(0.15\text{ m})^2 \div 470\text{ pF} = 3.46\text{ mm}$. (Since this is much less than the radius of the plates, the parallel plate approximation (plane symmetry) is a good one.) (b) The dielectric breakdown field for polystyrene is $E_{\max} = 25\text{ kV/mm}$, so the maximum voltage for this capacitor is $V_{\max} = E_{\max} d = (25\text{ kV/mm})(3.46\text{ mm}) = 86.5\text{ kV}$. (Note: in practice, the working voltage would be less than this by a comfortable safety margin.)

Problem

68. An air-insulated parallel-plate capacitor has plate area 76 cm^2 and spacing 1.2 mm. It is charged to 900 V and then disconnected from the charging battery. A plexiglass sheet is then inserted to fill the space between the plates. What are (a) the capacitance, (b) the potential difference between the plates, and (c) the stored energy both before and after the plexiglass is inserted?

Solution

Before the plexiglass is inserted, (a) the capacitance is
$C_0 = \varepsilon_0 A/d = (8.85\text{ pF/m})(76\text{ cm}^2) \div (1.2\text{ mm}) = 56.1\text{ pF}$,
(b) the voltage is $V_0 = 900\text{ V}$, and (c) the stored energy is $U_0 = \frac{1}{2}C_0V_0^2 = 22.7\ \mu\text{J}$. With the plexiglass insulation inserted, (a) the capacitance is $C = \kappa C_0 = (3.4)(56.1\text{ pF}) = 191\text{ pF}$. Since the capacitor was disconnected before the process of insertion, i.e., the plates are isolated and their charge Q is constant, (b) the voltage is reduced by a factor of $1/\kappa$, $V = V_0/\kappa = 900\text{ V}/3.4 = 265\text{ V}$ (see the discussion in the text preceding Equation 26-11), and (c) so is the stored energy, $U = U_0/\kappa = 22.7\ \mu\text{J}/3.4 = 6.68\ \mu\text{J}$ (see Equation 26-12).

Problem

69. The capacitor of the preceding problem is connected to its 900-V charging battery and left connected as the plexiglass sheet is inserted, so the potential difference remains at 900 V. What are (a) the charge on the plates and (b) the stored energy both before and after the plexiglass is inserted?

Solution

(a) The capacitances before and after the insertion of the plexiglass insulation are
$C_0 = \varepsilon_0 A/d = (8.85\text{ pF/m})(76\text{ cm}^2) \div (1.2\text{ mm}) = 56.1\text{ pF}$, and $C = \kappa C_0 = (3.4)(56.1\text{ pF}) = 191\text{ pF}$, as found previously. Therefore, since the voltage stays at
900 V in this case (due to the battery), $Q_0 = C_0(900\text{ V}) = 50.4\text{ nC}$, and $Q = C(900\text{ V}) = \kappa Q_0 = 172\text{ nC}$, before and after insertion, respectively. (b) The stored energy is $U_0 = \frac{1}{2}C_0(900\text{ V})^2 = 22.7\ \mu\text{J}$ before, and $U = \frac{1}{2}C(900\text{ V})^2 = \kappa U_0 = 77.2\ \mu\text{J}$ after. (The difference between this situation and the one in the previous problem is that the battery does additional work moving more charge to the capacitor plates, while maintaining the constant voltage. Equation 26-12 applies to an isolated capacitor only.)

Problem

73. A 20-μF air-insulated parallel-plate capacitor is charged to 300 V. The capacitor is then disconnected from the charging battery, and its plate separation is doubled. Find the stored energy (a) before and (b) after the plate separation increases. Where does the extra energy come from?

Solution

(a) Initially, the stored energy is $U_0 = \frac{1}{2}C_0V_0^2 = \frac{1}{2}(20\ \mu\text{F})(300\ \text{V})^2 = 0.9\ \text{J}$. (b) Disconnected from the battery, the charge stays constant, but the capacitance is halved when the separation is doubled $(C = \varepsilon_0 A/2d = C_0/2)$. Therefore, the stored energy is doubled, since $U = Q^2/2C = Q^2/2(C_0/2) = 2U_0 = 1.8\ \text{J}$. Work must be done, against the attractive force between the oppositely charged plates, to increase their separation.