Problem

2. An electron moving at right angles to a 0.10-T magnetic field experiences an acceleration of 6.0×10^{15} m/s^2. (a) What is the electron's speed? (b) By how much does its *speed* change in 1 ns ($= 10^{-9}$ s)?

Solution

(a) If the magnetic force is the only one of significance acting in this problem, then $F = ma = evB\sin\theta$. Thus,
$v = ma/eB\sin\theta = (9.11 \times 10^{-31}\ \text{kg})(6 \times 10^{15}\ \text{m/s}^2)/(1.6 \times 10^{-19}\ \text{C})(0.1\ \text{T})\sin 90° = 3.42 \times 10^5\ \text{m/s}.$ (b) Since $\mathbf{F} \propto \mathbf{v} \times \mathbf{B}$ is perpendicular to **v**, the magnetic force on a charged particle changes its direction, but not its speed.

Problem

5. A particle carrying a $50\text{-}\mu\text{C}$ charge moves with velocity $\mathbf{v} = 5.0\hat{\mathbf{i}} + 3.2\hat{\mathbf{k}}$ m/s through a uniform magnetic field $\mathbf{B} = 9.4\hat{\mathbf{i}} + 6.7\hat{\mathbf{j}}$ T. (a) What is the force on the particle? (b) Form the dot products $\mathbf{F} \cdot \mathbf{v}$ and $\mathbf{F} \cdot \mathbf{B}$ to show explicitly that the force is perpendicular to both **v** and **B**.

Solution

(a) From Equation 27-2,
$\mathbf{F} = q\mathbf{v} \times \mathbf{B} = (50\ \mu\text{C})(5\hat{\mathbf{i}} + 3.2\hat{\mathbf{k}}\ \text{m/s}) \times (9.4\hat{\mathbf{i}} + 6.7\hat{\mathbf{j}}\ \text{T}) = (50 \times 10^{-6}\ \text{N})(5 \times 6.7\hat{\mathbf{k}} + 3.2 \times 9.4\hat{\mathbf{j}} - 3.2 \times 6.7\hat{\mathbf{i}}) = (-1.072\hat{\mathbf{i}} + 1.504\hat{\mathbf{j}} + 1.675\hat{\mathbf{k}}) \times 10^{-3}\ \text{N}.$ (The magnitude and direction can be found from the components, if desired.) (b) The dot products $\mathbf{F} \cdot \mathbf{v}$ and $\mathbf{F} \cdot \mathbf{B}$ are, respectively, proportional to $(-1.072)(5) + (1.675)(3.2) = 0$, and $(-1.072)(9.4) + (1.504)(6.7) = 0$, since the cross product of two vectors is perpendicular to each factor. (We did not round off the components of **F**, so that the vanishing of the dot products could be exactly confirmed.)

Problem

12. A velocity selector uses a 60-mT magnetic field and a 24 kN/C electric field. At what speed will charged particles pass through the selector undeflected?

Solution

The condition for zero deflection is $v = E/B = (24\ \text{kN/C})/(0.06\ \text{T}) = 400\ \text{km/s}.$

Problem

13. A region contains an electric field $\mathbf{E} = 7.4\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}}$ kN/C and a magnetic field $\mathbf{B} = 15\hat{\mathbf{j}} + 36\hat{\mathbf{k}}$ mT. Find the electromagnetic force on (a) a stationary proton, (b) an electron moving with velocity $\mathbf{v} = 6.1\hat{\mathbf{i}}$ Mm/s.

Solution

The force on a moving charge is given by Equation 29-2 (called the Lorentz force) $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. (a) For a stationary proton,
$q = e$ and $\mathbf{v} = 0$, so $\mathbf{F} = e\mathbf{E} = (1.6 \times 10^{-19}\ \text{C})(7.4\hat{\mathbf{i}} + 2.8\hat{\mathbf{j}})\ \text{kN/C} = (1.18\hat{\mathbf{i}} + 0.448\hat{\mathbf{j}})\ \text{fN}$. (b) For the electron, $q = -e$ and $\mathbf{v} = 6.1\hat{\mathbf{i}}$ Mm/s, so the electric force is the negative of the force in part (a) and the magnetic force is $-e\mathbf{v} \times \mathbf{B} = (-1.6 \times 10^{-19}\ \text{C})(6.1\hat{\mathbf{i}}\ \text{Mm/s})(15\hat{\mathbf{j}} + 36\hat{\mathbf{k}})\ \text{mT} = (-14.6\hat{\mathbf{k}} + 35.1\hat{\mathbf{j}})\ \text{fN}.$ The total Lorentz force is the sum of these, or $(-1.18\hat{\mathbf{i}} + 34.7\hat{\mathbf{j}} - 14.6\hat{\mathbf{k}})$ fN.

Problem

15. What is the radius of the circular path described by a proton moving at 15 km/s in a plane perpendicular to a 400-G magnetic field?

Solution

From Equation 29-3, the radius of the orbit is
$r = m\text{v}/eB = (1.67 \times 10^{-27} \text{ kg})(15 \text{ km/s})/(1.6 \times 10^{-19} \text{ C})(4 \times 10^{-2} \text{ T}) = 3.91 \text{ mm}$. (SI units and data for the proton are summarized in the appendices and inside front cover.)

Problem

19. Electrons and protons with the same kinetic energy are moving at right angles to a uniform magnetic field. How do their orbital radii compare?

Solution

It is convenient to anticipate the result of Problem 22 for the orbital radius of a non-relativistic charged particle in a plane perpendicular to a uniform magnetic field. From Equation 29-3, $r = m\text{v}/qB$. For a non-relativistic particle, $K = \frac{1}{2}m\text{v}^2$, or $\text{v} = \sqrt{2K/m}$, therefore $r = \sqrt{2Km}/qB$. (Note: All quantities can, of course, be expressed in standard SI units, but in many applications, atomic units are more convenient. The conversion factor for electron volts to joules is just the numerical magnitude of the electronic charge, so if K is expressed in MeV, m in MeV/c^2, q in multiples of e, and B in teslas, we obtain

$$r = \frac{\sqrt{2K(e \times 10^6)m(e \times 10^6/c^2)}}{(qe)B} = \frac{10^6\sqrt{2Km}}{(3 \times 10^8)qB} = \frac{\sqrt{2Km}}{300qB}.\Bigg)$$

From this expression, it follows that protons and electrons with the same kinetic energy have radii in the ratio $r_p/r_e = \sqrt{m_p/m_e} = \sqrt{1836} \approx 43$, in the same magnetic field. Heavier particles are more difficult to bend.

Problem

22. Show that the orbital radius of a charged particle moving at right angles to a magnetic field B can be written

$$r = \frac{\sqrt{2Km}}{qB},$$

where K is the kinetic energy in joules, m the particle mass, and q its charge.

Solution

See solution to Problem 19.

Problem

30. An electron is moving in a uniform magnetic field of 0.25 T; its velocity components parallel and perpendicular to the field are both equal to $3.1 \times 10^6 \text{ m/s}$. (a) What is the radius of the electron's spiral path? (b) How far does it move along the field direction in the time it takes to complete a full orbit about the field direction?

Solution

(a) The radius depends only on the perpendicular velocity component,
$r = m\text{v}_\perp/eB = (9.11 \times 10^{-31} \text{ kg})(3.1 \times 10^6 \text{ m/s}) \div (1.6 \times 10^{-19} \text{ C})(0.25 \text{ T}) = 70.6 \ \mu\text{m}$. (b) The distance

moved parallel to the field is $d = v_{\parallel}T$, where T is the cyclotron period (Equation 29-4). Since $v_{\parallel} = v_{\perp}$ in this case, $d = v_{\parallel}(2\pi m/eB) = 2\pi(70.6\ \mu\text{m}) = 444\ \mu\text{m}$.

Problem

36. A wire of negligible resistance is bent into a rectangle as shown in Fig. 29-40, and a battery and resistor are connected as shown. The right-hand side of the circuit extends into a region containing a uniform magnetic field of $38\ \text{mT}$ pointing into the page. Find the magnitude and direction of the net force on the circuit.

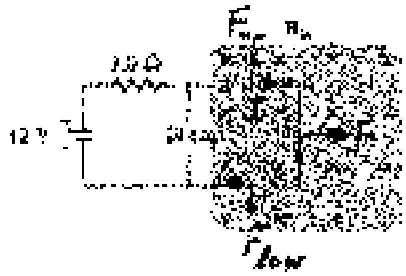

FIGURE 29-40 Problem 36 Solution.

Solution

The forces on the upper and lower horizontal parts of the circuit are equal in magnitude, but opposite in direction and thus cancel (see Fig. 29-40), leaving the force on the righthand wire, $I\ell B = (\mathcal{E}/R)\ell B = (12\ \text{V}/3\ \Omega)(0.1\ \text{m})(38\ \text{mT}) = 15.2\ \text{mN}$ toward the right, as the net force on the circuit.

Problem

38. A 20-cm-long conducting rod with mass $18\ \text{g}$ is suspended by wires of negligible mass, as shown in Fig. 29-41. The rod is in a region containing a uniform magnetic field of $0.15\ \text{T}$ pointing horizontally into the page, as shown. An external circuit supplies current between the support points A and B. (a) What is the minimum current necessary to move the bar to the upper position shown? (b) Which direction should the current flow?

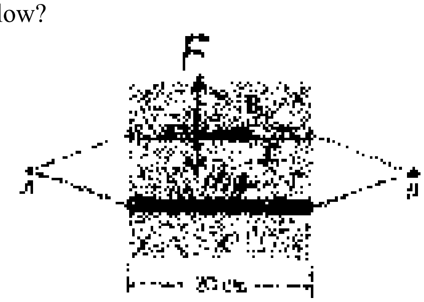

FIGURE 29-41 Problem 38 Solution.

Solution

An upward magnetic force on the rod equal (in magnitude) to its weight is the minimum force necessary. (a) Since the rod is perpendicular to B, $I\ell B = mg$ implies $I = mg/\ell B = (0.018 \times 9.8\ \text{N})/(0.2 \times 0.15\ \text{T}\cdot\text{m}) = 5.88\ \text{A}$. (b) The force is upward for current flowing from A to B, consistent with the right hand rule for the cross product.

Problem

47. A single-turn square wire loop 5.0 cm on a side carries a 450-mA current. (a) What is the magnetic moment of the loop? (b) If the loop is in a uniform 1.4-T magnetic field with its dipole moment vector at 40° to the field direction, what is the magnitude of the torque it experiences?

Solution

(a) Equation 29-10 for the magnetic moment of a loop gives $\mu = \text{NIA} = (1)(450\ \text{mA})(5\ \text{cm})^2 = 1.13 \times 10^{-3}\text{A} \cdot \text{m}^2$. (b) Equation 29-11 gives the torque on a magnetic dipole moment in a uniform magnetic field, $\tau = |\mu \times \mathbf{B}| = \mu B \sin\theta =$ $(1.13 \times 10^{-3}\ \text{A} \cdot \text{m}^2)(1.4\ \text{T}) \sin 40° = 1.01 \times 10^{-3}\ \text{N} \cdot \text{m}$.

Problem

49. A bar magnet experiences a 12-mN · m torque when it is oriented at 55° to a 100-mT magnetic field. What is the magnitude of its magnetic dipole moment?

Solution

Equation 29-11, solved for the magnitude of the dipole moment, gives $\mu = \tau/B \sin\theta = (12 \times 10^{-3}\ \text{N} \cdot \text{m})/(0.1\ \text{T}) \sin 55° = 0.146\ \text{A} \cdot \text{m}^2$.

Problem

53. Nuclear magnetic resonance (NMR) is a technique for analyzing chemical structures and is also the basis of magnetic resonance imaging used for medical diagnosis. The NMR technique relies on sensitive measurements of the energy needed to flip atomic nuclei upside-down in a given magnetic field. In an NMR apparatus with a 7.0-T magnetic field, how much energy is needed to flip a proton $(\mu = 1.41 \times 10^{-26}\text{A} \cdot \text{m}^2)$ from parallel to antiparallel to the field?

Solution

From Equation 29-12, the energy required to reverse the orientation of a proton's magnetic moment from parallel to antiparallel to the applied magnetic field is $\Delta U = 2\mu B = 2(1.41 \times 10^{-26}\ \text{A} \cdot \text{m}^2)(7.0\ \text{T}) = 1.97 \times 10^{-25}\ \text{J} = 1.23 \times 10^{-6}\ \text{eV}$. (This amount of energy is characteristic of radio waves of frequency 298 MHz, see Chapter 39.)

Problem

54. A wire of length ℓ carries a current *I*. (a) Find an expression for the magnetic dipole moment that results when the wire is wound into an *N*-turn circular coil. (b) For what integer value of *N* is this moment a maximum?

Solution

(a) The number of turns of radius *r* that can be formed from a wire of length ℓ is $N = \ell/2\pi r$, so $r = \ell/2\pi N$. The magnitude of the magnetic dipole moment of such a coil is $\mu = NI\pi(\ell/2\pi N)^2 = I\ell^2/4\pi N$. (b) This is clearly a maximum when *N* is a minimum, and the smallest value of *N* is, of course, one.