## Problem

2. A single-turn wire loop is 2.0 cm in diameter and carries a 650-mA current. Find the magnetic field strength (a) at the loop center and (b) on the loop axis, 20 cm from the center.

## Solution

Equation 30-3 gives: (a) at the center,
$x = 0, B = \mu_0 I/2a = (4\pi \times 10^{-7}\ \text{N/A}^2)(650\ \text{mA})/(2 \times 1\ \text{cm}) = 40.8\ \mu\text{T}$; (b) on the axis,
$x = 20\ \text{cm},\ B = \frac{1}{2}\mu_0 I a^2(x^2 + a^2)^{-3/2} = \frac{1}{2}\mu_0 I(1\ \text{cm})^2 \times [(20\ \text{cm})^2 + (1\ \text{cm})^2]^{-3/2} = 5.09\ \text{nT}.$

## Problem

7. A single-turn current loop carrying 25 A produces a magnetic field of 3.5 nT at a point on its axis 50 cm from the loop center. What is the loop area, assuming the loop diameter is much less than 50 cm?

## Solution

If the radius of the loop is assumed to be much smaller than the distance to the field point $(a \ll x = 50\ \text{cm})$, then
Equation 30-4 for the field on the axis of a magnetic dipole can be used to find $\mu = 2\pi x^3 B/\mu_0$. The magnetic moment of a single-turn loop is $\mu = IA$, therefore
$A = \mu/I = 2\pi x^3 B/\mu_0 I = (3.5\ \text{nT})(50\ \text{cm})^3/(2 \times 10^{-7}\ \text{N/A}^2)(25\ \text{A}) = 0.875\ \text{cm}^2.$

## Problem

8. Two identical current loops are 10 cm in diameter and carry 20-A currents. They are placed 1.0 cm apart, as shown in Fig. 30-47. Find the magnetic field strength at the center of either loop when their currents are in (a) the same and
(b) opposite directions.

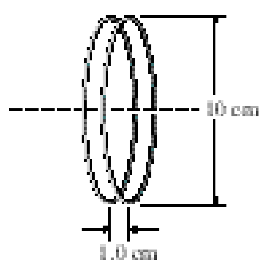



FIGURE 30-47 Problem 8.

## Solution

The magnetic field strength at the center of either loop is the magnitude of the vector sum of the fields due to its own current and the current in the other loop. Equation 30-3 gives $B_{\text{self}} = \mu_0 I/2a$ and $B_{\text{other}} = \mu_0 I a^2/2(a^2 + x^2)^{3/2}$. When the currents are in the same directions, the fields are parallel and $B = B_{\text{self}} + B_{\text{other}}$ while if the currents are in opposite directions, $B = B_{\text{self}} - B_{\text{other}}$. Numerically, $B_{\text{self}} = (4\pi \times 10^{-7})(20)\text{T}/(0.1) = 251\ \mu\text{T}$, and $B_{\text{other}} = B_{\text{self}}(1 + x^2/a^2)^{-3/2} =$ $(251\ \mu\text{T})[1 + (1\ \text{cm}/5\ \text{cm})^2]^{-3/2} = 237\ \mu\text{T}$, so (a) $B_{\text{self}} + B_{\text{other}} = 488\ \mu\text{T}$, and (b) $B_{\text{self}} - B_{\text{other}} = 14.4\ \mu\text{T}$.

## Problem

10. A single piece of wire is bent so that it includes a circular loop of radius $a$, as shown in Fig. 30-48. A current $I$ flows in the direction shown. Find an expression for the magnetic field at the center of the loop.

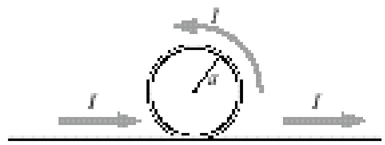


FIGURE 30-48 Problem 10.

## Solution

The field at the center is the superposition of fields due to current in the circular loop and straight sections of wire. The former is $\mu_0 I/2a$ out of the page (Equation 30-3 at $x = 0$ for CCW circulation), and the latter is $\mu_0 I/2\pi a$ out of the page (Equation 30-5 at $y = a$ for the very long, straight sections). Their sum is $B = (1 + \pi)\mu_0 I/2\pi a$ out of the page.

## Problem

12. Four long, parallel wires are located at the corners of a square 15 cm on a side. Each carries a current of 2.5 A, with the top two currents into the page in Fig. 30-49 and the bottom two out of the page. Find the magnetic field at the center of the square.

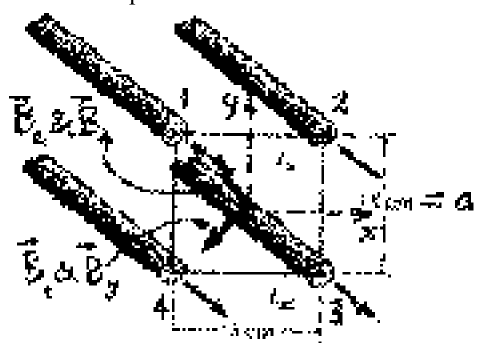


FIGURE 30-49 Problem 12 Solution.

## Solution

The magnetic field from each wire has magnitude $\mu_0 I/2\pi(a/\sqrt{2})$ (from Equation 30-5, with $y = \sqrt{2}a/2$, the distance
from a corner of a square of side $a$ to the center). The right-hand rule gives the field direction along one of the diagonals, as shown superposed on Fig. 30-49, such that the fields from currents at opposite corners are parallel. The net field is

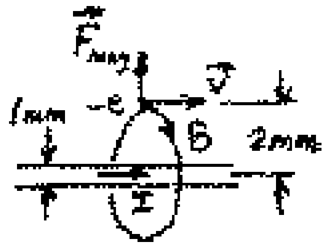
$\mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3 + \mathbf{B}_4 = 2(\mathbf{B}_1 + \mathbf{B}_2) = 2[\mu_0 I/2\pi(a/\sqrt{2})](-2\cos 45°\hat{\mathbf{i}}) = -2\mu_0 I\hat{\mathbf{i}}/\pi a = -(8 \times 10^{-7}\ \text{T})(2.5)\hat{\mathbf{i}}/(0.15) = -13.3\ \mu\text{T}\,\hat{\mathbf{i}}$.

## Problem

14. An electron is moving at $3.1 \times 10^6$ m/s parallel to a 1.0-mm-diameter wire carrying 20 A. If the electron is 2.0 mm from the center of the wire, with its velocity in the same direction as the current, what are the magnitude and direction of the force it experiences?

## Solution

The magnetic field from a long, straight, current-carrying wire (or very close to the given wire) is $\mu_0 I/2\pi r$ (Equation 30-8) and encircles the wire (as in Fig. 30-10b). An electron with velocity **v**, parallel to ***I*** and perpendicular to **B**, experiences a force $\mathbf{F}_{\text{mag}} = -e\mathbf{v} \times \mathbf{B}$, with magnitude $ev\mu_0 I/2\pi r = (1.6 \times 10^{-19}\ \text{C})(3.1 \times 10^6\ \text{m/s})(2 \times 10^{-7}\ \text{N/A}^2)(20\ \text{A})/(2\ \text{mm}) = 9.92 \times 10^{-16}\ \text{N}$, and direction away from the wire. (The electron represents a current element $I'd\boldsymbol{\ell}' = (dq/dt)d\boldsymbol{\ell}' = dq\,(d\boldsymbol{\ell}'/dt) = -e\mathbf{v}$, antiparallel to $I$, so Equation 30-6 could have been used.)




Problem 14 Solution.

## Problem

15. Part of a long wire is bent into a semicircle of radius $a$, as shown in Fig. 30-50. A current $I$ flows in the direction shown. Use the Biot-Savart law to find the magnetic field at the center of the semicircle (point $P$).

## Solution

The Biot-Savart law (Equation 30-2) written in a coordinate system with origin at $P$, gives $\mathbf{B}(P) = (\mu_0 I/4\pi)\int_{\text{wire}} d\mathbf{l} \times \hat{\mathbf{r}}/r^2$, where $\hat{\mathbf{r}}$ is a unit vector from an element $d\boldsymbol{\ell}$ on the wire to the field point $P$. On the straight segments to the left and right of the semicircle, $d\boldsymbol{\ell}$ is parallel to $\hat{\mathbf{r}}$ or $-\hat{\mathbf{r}}$, respectively, so $d\mathbf{l} \times \hat{\mathbf{r}} = 0$. On the semicircle, $d\boldsymbol{\ell}$ is perpendicular to $\hat{\mathbf{r}}$ and the radius is constant, $r = a$. Thus,

$$B(P) = \frac{\mu_0 I}{4\pi}\int_{\text{semicircle}} \frac{d\ell}{a^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{\pi a}{a^2} = \frac{\mu_0 I}{4a}.$$

The direction of $\mathbf{B}(P)$, from the cross product, is into the page.

FIGURE 30-50 Problem 15 Solution.

## Problem

17. Figure 30-51 shows a conducting loop formed from concentric semicircles of radii $a$ and $b$. If the loop carries a current $I$ as shown, find the magnetic field at point $P$, the common center.

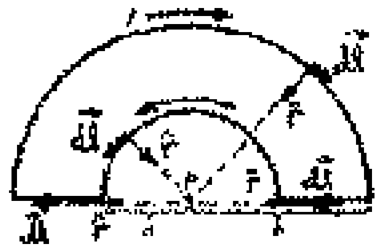


FIGURE 30-51 Problem 17 Solution.

## Solution

The Biot-Savart law gives $\mathbf{B}(P) = (\mu_0/4\pi)\int_{\text{loop}} I d\mathbf{l} \times \hat{\mathbf{r}}/r^2$. $I d\mathbf{l} \times \hat{\mathbf{r}}/r^2$ on the inner semicircle has magnitude $I d\ell/a^2$ and direction out of the page, while on the outer semicircle, the magnitude is $I d\ell/b^2$ and the direction is into the page. On the straight segments, $d\mathbf{l} \times \hat{\mathbf{r}} = 0$, so the total field at $P$ is $(\mu_0/4\pi)[(I\pi a/a^2) - (I\pi b/b^2)] = \mu_0 I(b - a)/4ab$ out of the page. (Note: the length of each semicircle is $\int d\ell = \pi r$.)

## Problem

21. The structure shown in Fig. 30-52 is made from conducting rods. The upper horizontal rod is free to slide vertically on the uprights, while maintaining electrical contact with them. The upper rod has mass 22 g and length 95 cm. A battery connected across the insulating gap at the bottom of the left-hand upright drives a 66-A current through the structure. At what height $h$ will the upper wire be in equilibrium?

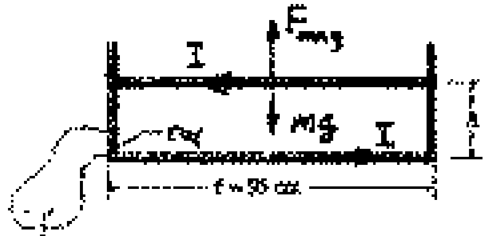


FIGURE 30-52 Problem 21 Solution.

## Solution

If $h$ is small compared to the length of the rods, we can use Equation 30-6 for the repulsive magnetic force between the horizontal rods (upward on the top rod) $F = \mu_0 I^2 \ell/2\pi h$. The rod is in equilibrium when this equals its weight, $F = mg$, hence

$h = \mu_0 I^2 \ell/2\pi mg = (2 \times 10^{-7}\ \text{N/A}^2)(66\ \text{A})^2(0.95\ \text{m})/(0.022 \times 9.8\ \text{N}) = 3.84\ \text{mm}$ (This is indeed small compared to 95 cm, as assumed.)

## Problem

24. A long, straight wire carries 20 A. A 5.0-cm by 10-cm rectangular wire loop carrying 500 mA is located 2.0 cm from the wire, as shown in Fig. 30-53. Find the net magnetic force on the loop.

## Solution

At any given distance from the long, straight wire, the force on a current element in the top segment cancels that on a corresponding element in the bottom. The force on the near side (parallel currents) is attractive, and that on the far side (antiparallel currents) is repulsive. The net force is the sum, which can be found from Equation 30-6 (+ is attractive):

$$F = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{2\text{ cm}} - \frac{1}{7\text{ cm}}\right) = \left(2\times10^{-7}\ \frac{\text{N}}{\text{A}^2}\right)(20\text{ A})\left(\frac{1}{2}\text{ A}\right)(10\text{ cm})\left(\frac{5}{14\text{ cm}}\right) = 7.14\times10^{-6}\text{ N.}$$

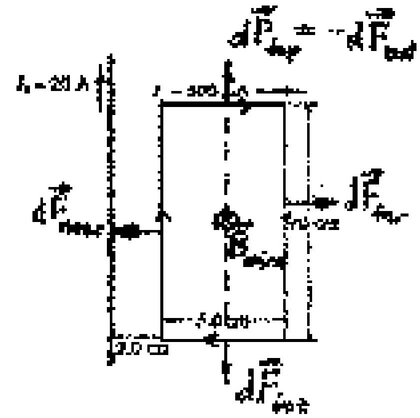


FIGURE 30-53 Problem 24 Solution.

## Problem

28. In Fig. 30-55, $I_1 = 2$ A flowing out of the page; $I_2 = 1$ A, also out of the page, and $I_3 = 2$ A, into the page. What is the line integral of the magnetic field taken counterclockwise around each loop shown?

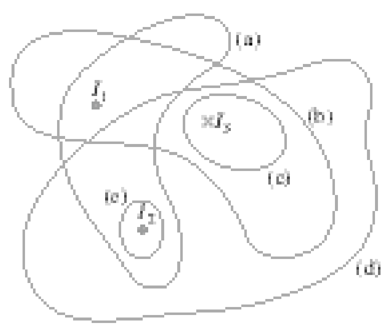



FIGURE 30-55 Problem 28.

## Solution

Ampère's law says that the line integral of **B** is equal to $\mu_0 I_{\text{encircled}}$, where, for CCW circulation, currents are positive

out of the page. (a) $\oint_a \mathbf{B}\cdot d\ell = \mu_0(I_1 + I_2) = (4\pi\times10^{-7}\ \text{N/A}^2)(3\text{ A}) = 12\pi\times10^{-7}\ \text{T}\cdot\text{m}$. (b)

$\oint_b \mathbf{B}\cdot d\ell\square\mu_0(I_1 - I_3) = 0$. (c) $\oint_c \mathbf{B}\cdot d\mathsf{l} - \mu_0 I_3 = -8\pi\times10^{-7}\ \text{T}\cdot\text{m}$. (d)

$\oint_d \mathbf{B}\cdot d\mathsf{l}\ \mu_0(I_2 - I_3) = -4\pi\times10^{-7}\ \text{T}\cdot\text{m}$. (e) $\oint_e \mathbf{B}\cdot d\mathsf{l} = \mu_0 I_2 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m}$.

## Problem

31. Figure 30-58 shows a magnetic field pointing in the $x$ direction. Its strength, however, varies with position in the $y$ direction. At the top and bottom of the rectangular loop shown the field strengths are $3.4\ \mu\text{T}$ and $1.2\ \mu\text{T}$, respectively. How much current flows through the area encircled by the loop?

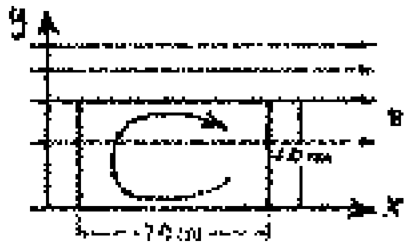


FIGURE 30-58 Problem 31 Solution.

## Solution

Equation 30-7 applied to the loop shown (going around in the direction of **B** at the top, i.e., clockwise) gives $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = B_{top}\ell - B_{bot}\ell = (3.4 - 1.2)\,\mu\text{T}(7\text{ cm}) = \mu_0 I_{encircled}$, so

$I_{encircled} = (2.2\ \mu\text{T})(0.07\text{ m}) \div (0.4\pi\,\mu\text{T}\cdot\text{m/A}) = 123\text{ mA}$ (positive current into the page in Fig. 30-58). (Note: $\mathbf{B}\cdot d\boldsymbol{\ell} = 0$ on the 4 cm sides of the amperian loop and a right-hand screw turned clockwise advances into the page.)

## Problem

33. A solid wire 2.1 mm in diameter carries a 10-A current with uniform current density. What is the magnetic field strength (a) at the axis of the wire, (b) 0.20 mm from the axis, (c) at the surface of the wire, and (d) 4.0 mm from the wire axis?

## Solution

The magnetic field strength is given by Equation 30-9 inside the wire $(r \le R)$ and Equation 30-8 outside $(r \ge R)$ as shown in Fig. 30-24. (a) For $r = 0$, $B = 0$. (b) For $r = 0.2\text{ mm} < R = \frac{1}{2}\times 2.1\text{ mm} = 1.05\text{ mm}$, $B = \mu_0 Ir/2\pi R^2 =$ $(2\times 10^{-7}\text{ N/A}^2)(10\text{ A})(0.2\text{ mm}) \div (1.05\text{ mm})^2 = 3.63\text{ G}$. (c) For $r = R$, $B = \mu_0 I/2\pi R = (2\times 10^{-7}\text{ N/A}^2)(10\text{ A}) \div (1.05\text{ mm}) = 19.0\text{ G}$. (d) For $r = 4\text{ mm} > R$, $B = \mu_0 I/2\pi r = (2\times 10^{-7}\text{ N/A}^2)(10\text{ A}) \div (4\text{ mm}) = 5\text{ G}$.

## Problem

36. A long, hollow conducting pipe of radius $R$ carries a uniform current $I$ along the pipe, as shown in Fig. 30-59.
    Use Ampère's law to find the magnetic field strength (a) inside and (b) outside the pipe.

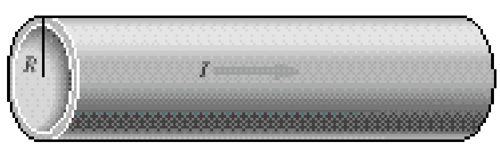



FIGURE 30-59 Problem 36.

## Solution

(b) Since the pipe is assumed to have cylindrical symmetry, the field outside is given by Equation 30-8. (a) For an
amperian loop with $r < R$, $I_{encircled} = 0$, hence $B = 0$ inside. (The thickness of the pipe is considered negligible.)

## Problem

41. A hollow conducting pipe of inner radius $a$ and outer radius $b$ carries a current $I$ parallel to its axis and distributed uniformly through the pipe material (Fig. 30-61). Find expressions for the magnetic field for (a) $r < a$, (b) $a < r < b$, and (c) $r > b$, where $r$ is the radial distance from the pipe axis.

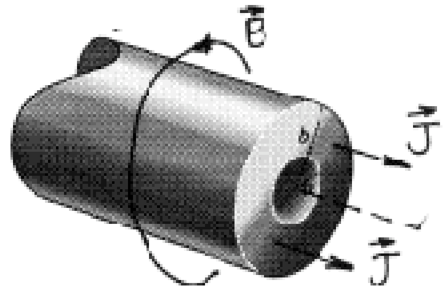



FIGURE 30-61 Problem 41.

## Solution

The symmetry argument in the text, for the field of a straight wire, shows that the magnetic field lines (from a very long pipe) are concentric circles, counterclockwise for current out of the page. Ampere's law, for loops along the field lines, gives $2\pi rB = \mu_0 I_{\text{encircled}}$. For uniform current density, $I_{\text{encircled}}$ is proportional to the cross-sectional area of conducting material. Therefore,

$$B(r) = \frac{\mu_0}{2\pi r}\begin{cases} 0, & r < a \\ I\dfrac{(r^2 - a^2)}{(b^2 - a^2)}, & a \le r \le b \\ I, & r > b \end{cases}$$

Problem 41 Solution.

## Problem

44. A solenoid used in a plasma physics experiment is 10 cm in diameter, 1.0 m long, and carries a 35-A current to produce a 100-mT magnetic field. (a) How many turns are in the solenoid? (b) If the solenoid resistance is $2.7\ \Omega$, how much power does it dissipate?

## Solution

A length:diameter ratio of 10:1 is large enough for Equation 30-11 to be a good approximation to the field near the solenoid's center. (a) $n = B/\mu_0 I = (10^{-1}\ \text{T})/(4\pi \times 10^{-7}\ \text{N/A}^2)(35\ \text{A}) = 2.27 \times 10^3\ \text{m}^{-1}$. Since $n$ is the number per unit length, $N = n\ell = 2.27 \times 10^3$ turns. (b) A direct current is used in the solenoid, so the power dissipated (Joule heat) is $P = I^2R = (35\ \text{A})^2(2.7\ \Omega) = 3.31\ \text{kW}$ (see Equation 27-9a).

## Problem

46. A toroidal coil of inner radius 15 cm and outer radius 17 cm is wound from 1200 turns of wire. What are (a) the minimum and (b) the maximum magnetic field strengths within the toroid when it carries a 10-A current?

## Solution

The magnetic field strength inside a toroid is given by Equation 30-12, and shows a $1/r$ variation. For this particular toroid, therefore (a) the minimum $B$ is
$\mu_0 NI/2\pi r_{max} = (2 \times 10^{-7}\ \text{N/A}^2)(1200)(10\ \text{A})/(0.17\ \text{m}) = 141\ \text{G}$, and (b) the maximum field strength is $B_{max} = (17/15)B_{min} = 160\ \text{G}$.

## Problem

56. A thin conducting washer of inner radius $a$ and outer radius $2a$ carries a current $I$ distributed uniformly with radial position, as suggested in Fig. 30-63. Find an expression for the magnetic field strength at its center.

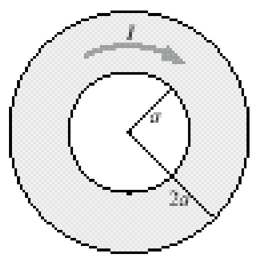



FIGURE 30-63 Problem 56.

## Solution

We suppose that uniform radial current means that the current per unit radial width, $I/a$, is a constant. We can regard the washer as being composed of circular loops of radius $r$, width $dr$, carrying current $dI = (I/a)\,dr$, and producing an axial magnetic field strength $dB = \mu_0 dI/2r$ at the center. The total field is therefore

$$B = \int_a^{2a} dB = \frac{\mu_0 I}{2a}\int_a^{2a}\frac{dr}{r} = \frac{\mu_0 I}{2a}\ln 2.$$