## Problem

4. A circular wire loop 40 cm in diameter has 100-Ω resistance and lies in a horizontal plane. A uniform magnetic field points vertically downward, and in 25 ms it increases linearly from 5.0 mT to 55 mT Find the magnetic flux through the loop at (a) the beginning and (b) the end of the 25 ms period. (c) What is the loop current during this time? (d) Which way does this current flow?

## Solution

(a) As in the previous solution,

$\phi_B = \mathbf{B}\cdot\mathbf{A} = \frac{1}{4}\pi d^2 B = \frac{1}{4}\pi(40\text{ cm})^2(5\text{ mT}) = 6.28\times 10^{-4}$ Wb at $t_1 = 0$, and

(b) $6.91\times 10^{-3}$ Wb at $t_2 = 25$ ms. (c) Since the field increases linearly,

$d\phi_B/dt = \Delta\phi_B/\Delta t = (6.91 - 0.628)\times\ 10^{-3}\text{ Wb}/25\text{ ms} = 0.251\text{ V}$. From Faraday's law, this is equal to the magnitude of the induced emf, which causes a current $I = \mathcal{E}/R = 0.251\text{ V}/100\ \Omega = 2.51\text{ mA}$ in the loop. (d) The direction must oppose the increase of the external field downward, hence the induced field is upward and $I$ is CCW when viewed from above the loop.

## Problem

6. A conducting loop of area $A$ and resistance $R$ lies at right angles to a spatially uniform magnetic field. At time $t = 0$ the magnetic field and loop current are both zero. Subsequently, the current increases according to $I = bt^2$, where $b$ is a constant with the units A/s$^2$. Find an expression for the magnetic field strength as a function of time.

## Solution

The induced current and the derivative of the magnetic field strength are related as in the previous problem, $|dB/dt| = IR/A = (bR/A)t^2$. Integration yields $B(t) = (bR/A)t^3/3$, where $B(0) = 0$ was specified.

## Problem

9. A square wire loop of side $\ell$ and resistance $R$ is pulled with constant speed v from a region of no magnetic field until it is fully inside a region of constant, uniform magnetic field **B** perpendicular to the loop plane. The boundary of the field region is parallel to one side of the loop. Find an expression for the total work done by the agent pulling the loop.

## Solution

The loop can be treated analogously to the situation analyzed in Section 31-3, under the heading "Motional EMF and Lenz's Law"; instead of exiting, the loop is entering the field region at constant velocity. All quantities have the same magnitudes, except the current in the loop is CCW instead of CW, as in Fig. 31-13. Since the applied force acts over a displacement equal to the side-length of the loop, the work done can be calculated directly: $W_{\text{app}} = \mathbf{F}_{\text{app}}\cdot\ell = (I\ell B)\ell = I\ell^2 B$. But,

$I = \mathcal{E}/R = |d\phi_B/dt|/R = d/dt(B\ell x)/R = B\ell \text{v}/R$, as before, so $W_{\text{app}} = B^2\ell^3\text{v}/R$. [Alternatively, the work can be calculated from the conservation of energy:

$I = B\ell\text{v}/R$, $\mathcal{P}_{\text{diss}} = I^2R = (B\ell\text{v})^2/R$, and $W_{\text{app}} = \mathcal{P}_{\text{diss}}t = [(B\ell\text{v})^2/R](\ell/\text{v})$.]

## Problem

14. A square wire loop 3.0 m on a side is perpendicular to a uniform magnetic field of 2.0 T. A 6-V light bulb is in series with the loop, as shown in Fig. 31-45. The magnetic field is reduced steadily to zero over a time $\Delta t$. (a) Find $\Delta t$ such that the light will shine at full brightness during this time. (b) Which way will the loop current flow?

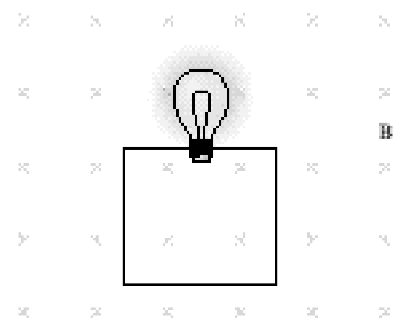



FIGURE 31-45 Problem 14.

## Solution

(a) To shine at full brightness, the potential drop across the bulb must be 6 V. This is equal to the induced emf, if we neglect the resistance of the rest of the loop circuit. From Faraday's law, $|\mathcal{E}| = |-d\phi_B/dt| = |-d(BA)/dt| = A|\Delta B|/\Delta t.$ Thus, $\Delta t = A|\Delta B|/|\mathcal{E}| = (3\text{ m})^2(2\text{ T})/6\text{ V} = 3\text{ s}.$ (b) The direction of current opposes the decrease of **B** into the page, and thus must act to increase **B** into the page. From the right-hand rule, this corresponds to a clockwise current in Fig. 31-45.

## Problem

17. A square conducting loop of side $s = 0.50$ m and resistance $R = 5.0\ \Omega$ moves to the right with speed $v = 0.25$ m/s. At time $t = 0$ its rightmost edge enters a uniform magnetic field $B = 1.0$ T pointing into the page, as shown in Fig. 31-46. The magnetic field covers a region of width $w = 0.75$ m. Plot (a) the current and (b) the power dissipation in the loop as functions of time, taking a clockwise current as positive and covering the time until the entire loop has exited the field region.

## Solution

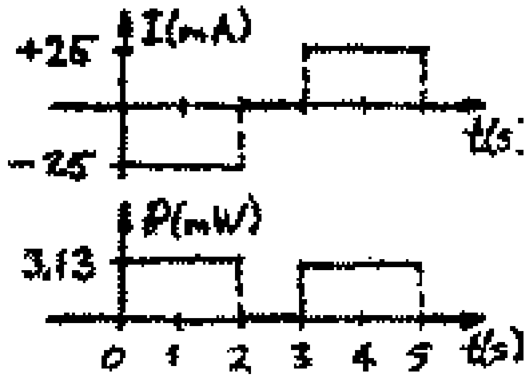
Let $x$ be the distance between the right side of the loop and the left edge of the field region. Take $t = 0$ when $x = 0$, so that $x = vt$. The loop enters the field region at $t = 0$, is completely within the region for $t$ between $\ell/v = 2$ s and $w/v = 3$ s, and is out of the region for $t \geq (w + \ell)/v = 5$ s.


FIGURE 31-46 Problem 17 Solution.

The area of loop overlapping the field region increases linearly from 0 to $\ell^2$, stays constant at $\ell^2$, then decreases to 0 between these times. (We use $\ell$ for side length to avoid confusion with time units.). Thus,

$$\phi_B = BA = B\ell^2 \begin{cases} 0 \\ vt/\ell \\ 1 \\ (w + \ell - vt)/\ell \\ 0 \end{cases} = 0.25\ \text{Wb} \begin{cases} 0, & t \le 0 \\ 0.5t, & 0 \le t \le 2 \\ 1, & 2 \le t \le 3 \\ 0.5(5 - t), & 3 \le t \le 5 \\ 0, & 5 \le t \end{cases}$$

(We substituted the given numerical values and used SI units for flux, with time $t$ in seconds, see solution to Problem 3.)




Problem 17 Solution.

(a) The induced current (positive clockwise) is given by Faraday's and Ohm's laws:

$$I = -\frac{1}{R}\frac{d\phi_B}{dt} = 25\ \text{mA} \begin{cases} 0, & t \le 0 \\ -1, & 0 \le t \le 2 \\ 0, & 2 \le t \le 3 \\ +1, & 3 \le t \le 5 \\ 0, & 5 \le t \end{cases}$$

(b) The power dissipated, $I^2R$, is $(\pm 25\ \text{mA})^2(5\ \Omega) = 3.13\ \text{mW}$ when the current is not zero.

## Problem

22. A magnetic field is described by $\mathbf{B} = B_0 \sin \omega t\, \hat{\mathbf{k}}$, where $B_0 = 2.0$ T and $\omega = 10\ \text{s}^{-1}$. A conducting loop with area $150\ \text{cm}^2$ and resistance $5.0\ \Omega$ lies in the $x$-$y$ plane. Find the induced current in the loop (a) at $t = 0$ and (b) at $t = 0.10$ s.

## Solution

Using the current given in the next solution, we find (a)

$I(0) = -\omega B_0 A/R = -(10\ \text{s}^{-1})(2\ \text{T})(150\ \text{cm}^2)/(5\ \Omega) = -60\ \text{mA}$, and (b)

$I(0.1\ \text{s}) = -(60\ \text{mA})\cos[(10\ \text{s}^{-1})(0.1\ \text{s})] = -60\ \text{mA} \cos(1\ \text{radian}) = -32.4\ \text{mA}$.

## Problem

24. A car alternator consists of a 250-turn coil 10 cm in diameter in a magnetic field of 0.10 T. If the alternator is turning at 1000 revolutions per minute, what is its peak output voltage?

## Solution

The peak output voltage of an electric generator, like the one depicted in Fig. 31-15, was found in Example 31-6 to be $\mathcal{E}_{peak} = 2\pi fNBA$, where $A = \frac{1}{4}\pi d^2$ is the loop area in this case. Numerically, $\mathcal{E}_{peak} = 2\pi(1000/60\text{ s})(250) \times (0.1\text{ T})\frac{1}{4}\pi(0.1\text{ m})^2 = 20.6\text{ V}$.

## Problem

27. Figure 31-49 shows a pair of parallel conducting rails a distance $\ell$ apart in a uniform magnetic field **B**. A resistance $R$ is connected across the rails, and a conducting bar of negligible resistance is being pulled along the rails with velocity **v** to the right. (a) What is the direction of the current in the resistor? (b) At what rate must work be done by the agent pulling the bar?

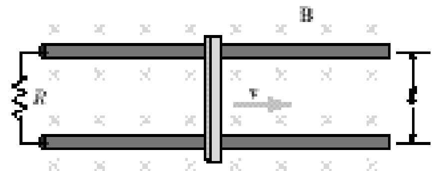



FIGURE 31-49 Problem 27.

## Solution

(a) The force on a (hypothetical) positive charge carrier in the bar, $q\mathbf{v} \times \mathbf{B}$, is upward in Fig. 31-49, so current will circulate CCW around the loop containing the bar, the resistor, and the rails (i.e., downward in the resistor). (The force per unit positive charge is the motional emf in the bar.) Alternatively, since the area enclosed by the circuit, and the magnetic flux through it, are increasing, Lenz's law requires that the induced current oppose this with an upward induced magnetic field. Thus, from the right-hand rule, the induced current must circulate CCW. (Take the positive sense of circulation around the circuit CW, so that the normal to the area is in the direction of **B**, into the page.) (b) In Example 31-4, which analyzed the same situation, the current in the bar was found to be $I = |\mathcal{E}|/R = B\ell v/R$. Since this is perpendicular to the magnetic field, the magnetic force on the bar is $F_{mag} = I\ell B$ (to the left in Fig. 31-49). The agent pulling the bar at constant velocity must exert an equal force in the direction of **v**, and therefore does work at the rate $\mathbf{F} \cdot \mathbf{v} = I\ell Bv = (B\ell v)^2/R$. (Note: The conservation of energy requires that this equal the rate energy is dissipated in the resistor (we neglected the resistance of the bar and the rails), $I^2R = (B\ell v/R)^2 R$.)

## Problem

28. The resistor in the preceding problem is replaced by an ideal voltmeter. (a) To which rail should the positive meter terminal be connected to if the meter is to indicate a positive voltage? (b) At what rate must work be done by the agent pulling the bar?

## Solution

(a) The motional emf (mentioned in part (a) of the previous solution) is upward in the moving bar, and so acts like the positive terminal of a battery. Thus, the positive terminal of the voltmeter should be connected to the top rail in Fig. 31-49. (b) When an ideal voltmeter replaces the resistor, no current flows (since its resistance is infinite), and no work must be done moving the bar. (However, during a brief instant when charge is separating in the bar, work is done.)

## Problem

30. A toroidal coil of square cross section has inner radius $a$ and outer radius $b$. It consists of $N$ turns of wire and carries a time-varying current $I = I_0 \sin\omega t$. A single-turn wire loop encircles the toroid, passing through its center hole as shown in Fig. 31-50. Find an expression for the peak emf induced in the loop.

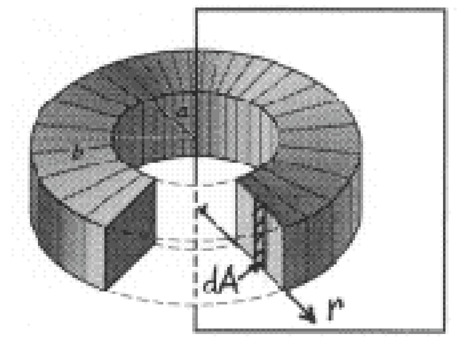



FIGURE 31-50 Problem 30.

## Solution

The magnetic flux through the loop is the same as the flux in one turn of the toroid. From Equation 30-12, $B = \mu_0 NI/2\pi r$, and we can take strips of area, $dA = (b - a)\,dr$, over the square cross-section of the toroid, so

$$\phi_B = \frac{\mu_0 NI(b-a)}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 NI}{2\pi}(b-a)\ln\left(\frac{b}{a}\right).$$

From Faraday's law and the given sinusoidal current, the induced emf is proportional to $dI/dt = \omega I_0 \cos\omega t$, so $(\mathcal{E}_i)_{\text{peak}} = [\mu_0 N\omega I_0(b-a)/2\pi]\ln(b/a)$.

## Problem

34. A rectangular conducting loop of resistance $R$, mass $m$, and width $w$ falls into a uniform magnetic field **B**, as shown in Fig. 31-52. If the loop is long enough and the field region has a great enough vertical extent, the loop will reach a terminal speed. (a) Why? (b) Find an expression for the terminal speed. (c) What will be the direction of the loop current as the loop enters the field?

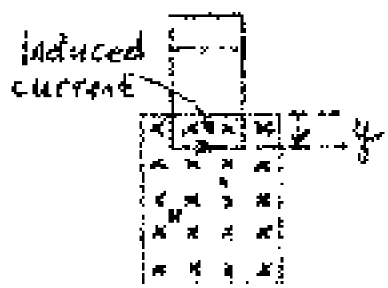



FIGURE 31-52 Problem 34.

## Solution

(a) As long as the flux through the loop is changing, there is an upward magnetic force on the induced current in the bottom wire, which may cancel the downward force of gravity on the loop. (b) The flux through the loop is proportional to the vertical distance it falls into the field region (as shown added to Fig. 31-52), $\phi_B = BA = Bwy$, so Faraday's and Ohm's laws give a magnetic force proportional to the vertical speed, $F = IwB = (|\mathcal{E}_i|/R)wB = |-d\phi_B/dt|wB/R = vw^2B^2/R$. When this equals $mg$, the terminal speed is $v_t = mgR/w^2B^2$. (c) The flux of $B$ is positive (into the plane of Fig. 31-52) for clockwise circulation around the loop, so the induced current must be negative, or counterclockwise. (The motional emf, $dq\mathbf{v}\times\mathbf{B}$, on positive charge carriers in the bottom wire, is to the right, and the magnetic force on them, $Id\boldsymbol{\ell}\times\mathbf{B}$, is upward.)

## Problem

36. The induced electric field 12 cm from the axis of a solenoid with 10 cm radius is 45 V/m. Find the rate of change of the solenoid's magnetic field.

## Solution

The geometry of the induced electric field from the solenoid is described in Example 31-9, where $2\pi r|E| = |-d(\pi R^2 B)/dt| = \pi R^2|dB/dt|$. Thus, $|dB/dt| = 2r|E|/R^2 = 2(12\text{ cm})(45\text{ V/m})/(10\text{ cm})^2 = 1.08\times 10^3\text{ T/s} = 1.08\text{ T/ms}$. (The sign of $dB/dt$ and the direction of the induced electric field are related by Lenz's law.)

## Problem

38. Figure 31-53 shows a top view of a tokamak. The magnetic field in the center is confined to a circular area of radius 50 cm, and during a pulse it increases at the rate of 5.1 T/ms. (a) What is the magnitude of the induced electric field in the tokamak, 1.2 m from the center of the field region in Fig. 31-53? (b) What is the field direction? (c) If a proton circles the tokamak once at this radius, going with the electric field, how much energy does it gain?

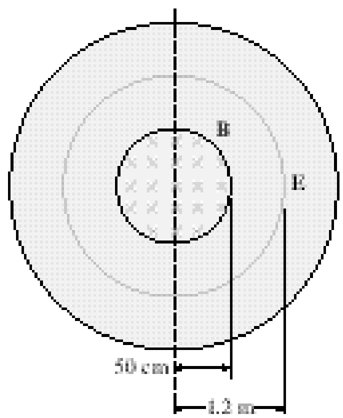



FIGURE 31-53 Problem 38.

## Solution

(a) Assume that the induced electric field is axially symmetric, so that the lines of $E$ encircle the magnetic field region. Then Faraday's law (as in Example 31-9) gives $\oint \mathbf{E}\cdot d\boldsymbol{\ell} = 2\pi rE = -d\phi_B/dt = -d(\pi R^2 B)/dt = -\pi R^2 dB/dt$, so $|E| = (R^2/2r)|dB/dt| = (50\text{ cm})^2(5.1\text{ T/ms})/(2\times 1.2\text{ m}) = 531\text{ V/m}$. (b) **E** must be CCW in Fig. 31-53 in order to oppose the increase of magnetic field into the page. (c) The induced emf is $|\mathcal{E}| = 2\pi r|E| = 2\pi(1.2\text{ m})(531\text{ V/m}) = 4.01\text{ kV}$ so a proton (of charge $e$) would gain energy $e|\mathcal{E}| = 4.01\text{ keV} = 6.41\times 10^{-16}\text{ J}$ in one complete circuit.

## Problem

41. Figure 31-56 shows a magnetic field pointing into the page; the field is confined to a layer of thickness $h$ in the vertical direction but extends infinitely to the left and right. The field strength is increasing with time: $B = bt$, where $b$ is a constant. Find an expression for the electric field at all points outside the field region. *Hint:* Consult Example 30-5.

FIGURE 31-56 Problem 41.

## Solution

A changing magnetic field acts as a source for an induced electric field, just like a current density is a source for a magnetic field. In fact, Faraday's law (for a loop fixed in space), $\oint_{\text{loop}} \mathbf{E}\cdot d\boldsymbol{\ell} = -d\phi_B/dt = \int_{\text{surface}}(\partial\mathbf{B}/\partial t)\cdot d\mathbf{A}$, is analogous to Ampère's law, $\oint_{\text{loop}} \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I_{\text{encircled}} = \int_{\text{surface}} \mu_0\mathbf{J}\cdot d\mathbf{A}$ (compare **E** and $\partial\mathbf{B}/\partial t$ with **B** and $\mu_0\mathbf{J}$). The geometry of the source and symmetry of the field in Fig. 31-56 is similar to that in Fig. 30-25 for an infinite current sheet. The induced electric field, **E**, should have the same magnitude above and below the source region for $\partial\mathbf{B}/\partial t$, and should circulate in a CCW sense so as to oppose the increase of flux into the page (i.e., CCW circulation is out of the page, opposite to the normal to an area into the page). For the rectangular loop shown added to Fig. 31-56, $\oint_{\text{loop}} \mathbf{E}\cdot d\boldsymbol{\ell} = 2E\ell = \int_{\text{area}}(\partial B/\partial t)\,dA = (\partial B/\partial t)\ell h$, or $E = \frac{1}{2}|\partial B/\partial t|h = \frac{1}{2}bh$.

## Problem

50. A copper disk 90 cm in diameter is spinning at 3600 rpm about a conducting axle through its center, as shown in Fig. 31-59. A uniform 1.5-T magnetic field is perpendicular to the disk, as shown. A stationary conducting brush maintains contact with the disk's rim, and a voltmeter is connected between the brush and the axle. (a) What does the voltmeter read? (b) Which voltmeter lead is positive?

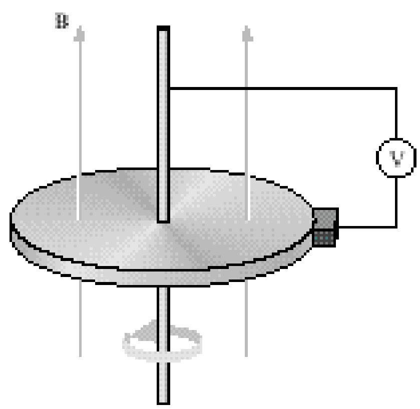



FIGURE 31-59 Problem 50.

## Solution

The result of the previous problem can be used, although the motional emf argument makes more sense in the case of a disk. (b) Since $\omega$ is parallel to **B** (rather than anti-parallel as in the previous problem) the rim is positive relative to the axle. ($\mathbf{v}\times\mathbf{B} = +vB\hat{\mathbf{r}}$ in Fig. 31-59.) (a) The voltmeter reading equals the magnitude of the induced emf (since no current flows if the voltmeter's resistance is very large) $\mathcal{E} = \frac{1}{2}\omega R^2 B = \frac{1}{2}(3600 \times 2\pi/60\text{ s})(0.45\text{ m})^2(1.5\text{ T}) = 57.3\text{ V}$.

## Problem

58. A circular wire loop of resistance $R$ and radius $a$ lies with its plane perpendicular to a uniform magnetic field. The field strength changes from an initial value $B_1$ to a final value $B_2$. Show, by integrating the loop current over time, that the total charge that moves around the ring is

$$q = \frac{\pi a^2}{R}(B_2 - B_1).$$

Note that this result is independent of how the field changes with time.

## Solution

The magnitude of the induced current is $I = |\mathcal{E}/R| = |-d\phi_B/dt|/R = (\pi a^2/R)|dB/dt|$, since for a fixed circular loop perpendicular to **B**, $\phi_B = BA$. In terms of the charge moving through the circuit during the interval the magnetic field is changing, $I = dq/dt$, therefore $q = \int_1^2 dq = (\pi a^2/R) \times \int_1^2 dB = (\pi a^2/R)(B_2 - B_1)$.