chapter 33

Problem

32. An LC circuit includes a $20\text{-}\mu\text{F}$ capacitor and has a period of 5.0 ms. The peak current is 25 mA. Find (a) the inductance and (b) the peak voltage.

Solution

(a) The inductance can be calculated from Equation 33-11:

$L = 1/\omega^2 C = (T/2\pi)^2/C = (5\text{ ms}/2\pi)^2/20\ \mu\text{F} = 31.7\ \text{mH}.$

(b) Fig. 33-12 and the expressions for the electric and magnetic energies for the LC circuit in the text imply that $\frac{1}{2}CV_p^2 = \frac{1}{2}LI_p^2$, so $V_p = I_p\sqrt{L/C} = (25\text{ mA})\sqrt{31.7\text{ mH}/20\ \mu\text{F}} = 995\text{ mV}.$

Problem

37. One-eighth of a cycle after the capacitor in an LC circuit is fully charged, what are each of the following as fractions of their peak values: (a) capacitor charge, (b) energy in the capacitor, (c) inductor current, (d) energy in the inductor?

Solution

The equations in Section 33-3 give the desired quantities, which we evaluate when

$\omega t = \omega(T/8) = 2\pi/8 = \frac{1}{4}\pi = 45°$

(i.e., $\frac{1}{8}$ cycle). (Note that phase constant zero corresponds to a fully charged capacitor at $t = 0$.) (a) From Equation 33-10, $q/q_p = \cos 45° = 1/\sqrt{2}$. (b) From the equation for electric energy, $U_E/U_{E,p} = \cos^2 45° = 1/2$. (c) From Equation 33-12, $I/I_p = -\sin 45° = -1/\sqrt{2}$. (The direction of the current is away from the positive capacitor plate at $t = 0$.) (d) From the equation for magnetic energy, $U_B/U_{B,p} = \sin^2 45° = 1/2$.

Problem

38. Show from conservation of energy that the peak voltage and current in an LC circuit are related by $I_p = V_p\sqrt{C/L}$.

Solution

When all the energy is stored in the capacitor, $U_{\text{tot}} = \frac{1}{2}CV_p^2$, and when all is stored in the inductor, $U_{\text{tot}} = \frac{1}{2}LI_p^2$. Therefore, $I_p = V_p\sqrt{C/L}$. (See Figure 33-10, parts (a), (c), (e), and (g).)

Problem

39. The $2000\text{-}\mu\text{F}$ capacitor in Fig. 33-30 is initially charged to 200 V. (a) Describe how you would manipulate switches A and B to transfer all the energy from the $2000\text{-}\mu\text{F}$ capacitor to the $500\text{-}\mu\text{F}$ capacitor. Include the times you would throw the switches. (b) What will be the voltage across the $500\text{-}\mu\text{F}$ capacitor once you've finished?

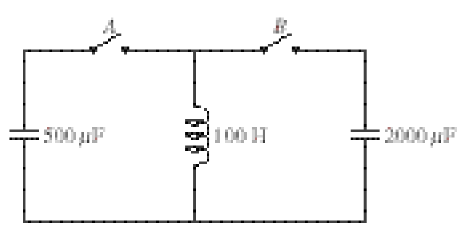

FIGURE 33-30 Problem 39.

Solution

(a) The energy initially stored in the first capacitor is $\frac{1}{2}(2\ \text{mF})(200\ \text{V})^2 = 40\ \text{J}$. First close switch B for one
quarter of a period of the LC circuit containing the $2000\ \mu\text{F}$ capacitor, or
$t_B = \frac{1}{4}T_B = \frac{1}{4}(2\pi/\omega_B) = \frac{1}{2}\pi\sqrt{LC_B} = \frac{1}{2}\pi\sqrt{(100\ \text{H})(2\ \text{mF})} = 702\ \text{ms}$. This transfers 40 J to the inductor.
Then open switch B and close switch A for one
quarter of a period of the LC circuit containing the $500\ \mu\text{F}$ capacitor, or
$t_A = \frac{1}{2}\pi\sqrt{(100\ \text{H})(0.5\ \text{mF})} = \frac{1}{2}t_B = 351\ \text{ms}$.
This transfers 40 J to the second capacitor from the inductor. Finally, open switch A. (b) When the second capacitor
has 40 J of stored energy, its voltage is $\sqrt{2(40\ \text{J})/(0.5\ \text{mF})} = 400\ \text{V}$.