

PHYS 2B Quiz 6 Solutions

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1 Problem 1

A square and circle with the same perimeters have different areas. A side of one square is $p/4$; its area is $A_s = p^2/16$. The radius of the circle is $p/2\pi$; its area is $A_c = \pi(p/(2\pi))^2 = p^2/4\pi$. Since the square and circle are made from the same metal with the same lengths, they have identical resistances R . Call ε_s and ε_c the EMF's induced due to the time-varying flux in the square and circle respectively.

The ratio of the power dissipated is

$$\frac{P_c}{P_s} = \frac{(\varepsilon_c)^2/R}{(\varepsilon_s)^2/R} = \left(\frac{\varepsilon_c}{\varepsilon_s}\right)^2 = \left(\frac{-d\Phi_c/dt}{-d\Phi_s/dt}\right)^2 = \left(\frac{A_c dB/dt}{A_s dB/dt}\right)^2 = \left(\frac{A_c}{A_s}\right)^2 = \left(\frac{p^2/4\pi}{p^2/16}\right)^2 = \left(\frac{4}{\pi}\right)^2 = 1.62 \quad (1)$$

Hence, $P_c = 1.62P_s = 162W$.

2 Problem 2

Suppose a change in magnetic field of $\Delta B = 10$ is brought about in time Δt at a constant rate. The power dissipated is

$$\frac{\varepsilon^2}{R} = \frac{(A dB/dt)^2}{50} = \frac{(2^2 \Delta B / \Delta t)^2}{50} = \frac{(4 * 10)^2}{50(\Delta t)^2} \quad (2)$$

The total energy dissipated is the power multiplied by the time; it has to be less than 0.5J.

$$E = P\Delta t = \frac{160}{50\Delta t} < 0.5 \quad (3)$$

$$\frac{160}{50 * 0.5} < \Delta t \quad (4)$$

$$\Delta t > 6.4 \quad (5)$$

3 Problem 3

The induced currents I_1 and I_2 in the vertical arms of both loops result in the Lorentz forces $F_1 = BI_1a$ and $F_2 = BI_2(2a)$ respectively. These forces must be countered by the external agent pushing the two loops, hence the external agent applies a force equal in magnitude but opposite in direction.

$$\frac{F_2}{F_1} = \frac{BI_2(2a)}{BI_1a} = \frac{2I_2}{I_1} = \frac{2\varepsilon_2/R_2}{\varepsilon_1/R_1} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{d\Phi_2/dt}{d\Phi_1/dt} = \frac{B(2a)v}{Bav} = 2 \quad (6)$$

So $F_2 = 2 * F_1 = 20N$.

4 Problem 4

The current induced in the inner loop is

$$\frac{\varepsilon}{R} = \frac{-d\Phi/dt}{R} = \frac{-d(\int B \cdot da)}{Rdt} \quad (7)$$

In principle, one could calculate the magnetic field due to I_1 at every point inside the inner loop using the Biot-Savart law and do the full area integral to get the flux. However, it is enough to recognize that the magnetic field at all points inside the loop is proportional to the current I_1 . This we recognize from the BS law $dB = I_1 * \text{stuff}$. This means that $B = I_1 * \text{stuff}$ and $\int B \cdot da = I_1 * \text{more stuff}$. Consider what happens when we take the time derivative of the flux. I_1 changes with time, but the rest of the stuff doesn't (it merely depends on the shape and size of the loop). Hence $d\Phi/dt = dI_1/dt * \text{stuff}$. Final result: the current induced in the inner loop is proportional to the rate of change of I_1 . This rate is the same at both $t = 1, 2$, hence the current induced is constant, i.e. $= 1\mu A$.

5 Problem 5

Let's call the direction up (along the wire) \hat{z} . One pictures the magnetic field lines around a current-carrying wire as forming circles in the planes perpendicular to the wire; the magnetic field has no \hat{z} component.

Question: does the circular loop enclose any flux? The standard procedure is: one chooses an arbitrary surface bounded by the loop and does the integral $\Phi = \int B \cdot da$. The simplest surface is a circular drum surface stretched taut, with \hat{z} the normal unit vector of each infinitesimal area element da ; the drum surface lies entirely in the xy plane. B lies entirely in the xy plane. The dot product $B \cdot da$ is zero for every point on that drum surface because the vectors are always perpendicular. So, no, the loop encloses no flux.

No current can be induced in the loop because there is no flux linkage. $\varepsilon = -d\Phi/dt = 0$ because $\Phi = 0$.

6 Problem 6

The flux into the paper is decreasing as a function of time. To oppose this change, an induced current flows clockwise around the solenoid so as to induce a magnetic field into the paper. The current is driven by an induced electric field which points in the same direction as the current so as to accelerate charges in the right direction. At P, clockwise means the electric field points down.

Wait a minute, there is no wire at P! That's right. An induced EMF / electric field exists whether or not there is a wire or charges ready to respond. Lenz's law tells you where it points. The induced field wants to oppose the change in flux. Whether it acts on willing charges or empty space is a moot point.

7 Problem 7

Faraday's law in a situation with cylindrical symmetry says:

$$\varepsilon = \oint E \cdot dl = E(2\pi(2a)) = -\frac{d\Phi}{dt} = -(\pi 0.3^2) \frac{d(B_0 e^{-t/\tau})}{dt} = \frac{0.09\pi B_0 e^{-t/\tau}}{\tau} \quad (8)$$

$$E(t) = \frac{0.09\pi * 10 e^{-t/2}}{1.2\pi * 2} \quad (9)$$

$$E(1) = \frac{0.9 e^{-1/2}}{2.4} = 0.227 \quad (10)$$

8 Problem 8

The rate of change of flux is Blv where l is the effective height of the loop. When the loop is half in, l is half the height of the triangle. When the loop is almost all the way in, l is the full height of the triangle, which is twice as much; hence the EMF induced is also twice as much. The power dissipated is proportional to ε^2 and is 4 times as much. $4 * 100 = 400W$.