Physics 2D Quiz 6 Solutions #1 p.1

(a) Normalization condition:

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

Our wavefunction is $\psi(x) = Ae^{-|x|/a}$ so we have

$$\int_{-\infty}^{\infty} (Ae^{-|x|/a})^2 dx = A^2 \int_{-\infty}^{\infty} e^{-2|x|/a} dx = 2A^2 \int_0^{\infty} e^{-2x/a} dx$$

by symmetry about $x = 0$

Change variables, let $u = 2x/a \Rightarrow du = 2dx/a$

$x = 0 \Rightarrow u = 0$ and $x = \infty \Rightarrow u = \infty$, so

$$2A^2 \int_0^{\infty} e^{-u} \left(\frac{a\,du}{2}\right) = \frac{2A^2 a}{2} \int_0^{\infty} e^{-u} du = A^2 a \left[-e^{-u}\right]_0^{\infty} = A^2 a = 1$$

by Normalization

$$\boxed{A = \pm\sqrt{\frac{1}{a}}}$$

You can choose the + sign for convenience.

(b) Probability of being between ① and ② is given by

$$\int_{①}^{②} \psi^* \psi \, dx$$, so we have

$$\frac{1}{a} \int_a^{2a} e^{-2x/a} dx = \frac{1}{2}\left[-e^{-u}\right]_2^4 = \boxed{(-e^{-4} + e^{-2})/2} \simeq 0.117$$

same integral as in part (a)

note: $A^2 a = 1$ and $x = a \Rightarrow u = 2$, $x = 2a \Rightarrow u = 4$

also, the $\frac{1}{2}$ that comes from $dx \to du$ is not cancelled by the "symmetry 2" this time

Physics 2D Quiz 6 Solutions #2 p.1

TISE:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Our wavefunction is $\psi(x) = Ae^{-\alpha x^2}$

Plug in our wavefunction to the TISE:

$$\frac{d\psi}{dx} = -2\alpha x\psi(x)$$

$$\frac{d^2\psi}{dx^2} = -2\alpha\psi(x) + (-2\alpha x)^2\psi(x) = \psi(x)(4\alpha^2x^2 - 2\alpha)$$

$$-\frac{\hbar^2}{2m}\left(\psi(x)(4\alpha^2x^2 - 2\alpha)\right) + U(x)\psi(x) = E\psi(x)$$

$$\psi(x)\left(-\frac{4\hbar^2\alpha^2}{2m}x^2 + \frac{2\hbar^2\alpha}{2m} + U(x)\right) = E\psi(x)$$

E is a constant so the term in parentheses must also be a constant if the equality is to hold for all x.

$$\Rightarrow U(x) = \left(\frac{4\hbar^2\alpha^2}{2m}\right)x^2 \Rightarrow \boxed{C = \left(\frac{2\hbar^2\alpha^2}{m}\right)} \text{ and } \boxed{E = \frac{\hbar^2\alpha}{m}}$$