PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #6

(1) $\nu = 8$ moles of a diatomic ideal gas are subjected to a cyclic quasistatic process, the thermodynamic path for which is an ellipse in the (V, p) plane. The center of the ellipse lies at $(V_0, p_0) = (0.25\,\text{m}^3, 1.0\,\text{bar})$. The semimajor axes of the ellipse are $\Delta V = 0.10\,\text{m}^3$ and $\Delta p = 0.20\,\text{bar}$.

(a) What is the temperature at $(V, p) = (V_0 + \Delta V, p_0)$?

(b) Compute the net work per cycle done by the gas.

(c) Compute the internal energy difference $E(V_0 - \Delta V, p_0) - E(V_0, p_0 - \Delta p)$.

(d) Compute the heat Q absorbed by the gas along the upper half of the cycle.

(2) Determine which of the following differentials are exact and which are inexact.

(a) $xy\,dx + xy\,dy$

(b) $(x + y^{-1})\,dx - xy^{-2}\,dy$

(c) $xy^3\,dx + 3x^2y^2\,dy$

(d) $(\ln y + \ln z)\,dx + xy^{-1}\,dy + xz^{-1}dz$

(3) Liquid mercury at atmospheric pressure and temperature $T = 0°$ C has a molar volume of $14.72\,\text{cm}^3/\text{mol}$ and a specific heat a constant pressure of $c_p = 28.0\,\text{J/mol·K}$. Its coefficient of expansion is $\alpha = \frac{1}{V}\big(\frac{\partial V}{\partial T}\big)_p = 1.81 \times 10^{-4}/\text{K}$ and its isothermal compressibility is $\kappa_T = -\frac{1}{V}\big(\frac{\partial V}{\partial T}\big)_T = 3.88 \times 10^{-12}\,\text{cm}^2/\text{dyn}$. Find its specific heat at constant volume c_V and the ratio $\gamma = c_p/c_V$. [Reif problem 5.10]

(4) ν moles of an ideal diatomic gas are driven along the cycle depicted in Fig. 1. Section AB is an adiabatic free expansion; section BC is an isotherm at temperature $T_\mathsf{A} = T_\mathsf{B} = T_\mathsf{C}$; CD is an isobar, and DA is an isochore. The volume at B is given by $V_\mathsf{B} = (1 - x)\,V_\mathsf{A} + x\,V_\mathsf{C}$, where $0 \le x \le 1$.

(a) Find an expression for the total work W_cycle in terms of ν, T_A, V_A, V_C, and x.

(b) Suppose $V_\mathsf{A} = 1.0\,\text{L}$, $V_\mathsf{C} = 5.0\,\text{L}$, $T_\mathsf{A} = 500\,\text{K}$, and $\nu = 5$. What is the volume V_B such that $W_\text{cycle} = 0$?

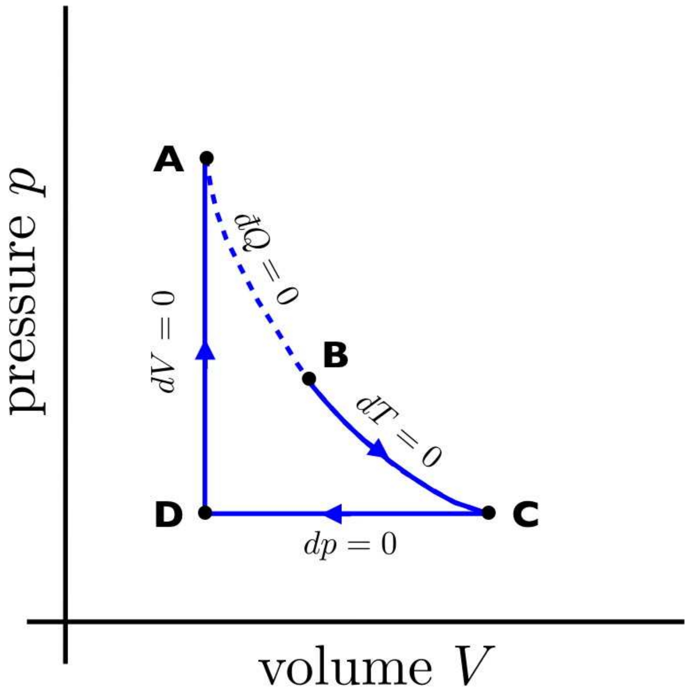

Figure 1: Thermodynamic cycle for problem 4, consisting of adiabatic free expansion (AB), isotherm (BC), isobar (CD), and isochore (DA).

(5) A strange material found stuck to the bottom of a seat in Warren Lecture Hall 2001 obeys the thermodynamic relation $E(S,V,N) = a\,S^6/V^2N^3$, where a is a dimensionful constant.

(a) What are the MKS dimensions of a?

(b) Find the equation of state relating p, V, N, and T.

(c) Find the coefficient of thermal expansion $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$. Express your answer in terms of intensive quantities p, T, and $n = N/V$.

(d) Find the isothermal compressibility $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$. Express your answer in terms of intensive quantities p, T, and $n = N/V$.