PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #7

(1) Using the chain rule from multivariable calculus (see §2.16 of the lecture notes), solve the following:

(a) Find $(\partial N/\partial T)_{S,p}$ in terms of T, N, S, and $C_{p,N}$.

(b) Experimentalists can measure $C_{V,N}$ but for many problems it is theoretically easier to work in the grand canonical ensemble, whose natural variables are (T, V, μ). Show that

$$C_{V,N} = \left(\frac{\partial E}{\partial T}\right)_{V,z} - \left(\frac{\partial E}{\partial z}\right)_{T,V} \left(\frac{\partial N}{\partial T}\right)_{V,z} \Bigg/ \left(\frac{\partial N}{\partial z}\right)_{T,V} ,$$

where $z = \exp(\mu/k_{\rm B}T)$ is the fugacity.

(2) Consider the equation of state,

$$p = \frac{R^2T^2}{a + vRT} ,$$

where $v = N_{\rm A}V/N$ is the molar volume and a is a constant.

(a) Find an expression for the molar energy $\varepsilon(T, v)$. Assume that in the limit $v \to \infty$, where the ideal gas law $pv = RT$ holds, that the gas is ideal with $\varepsilon(v \to \infty, T) = \frac{1}{2}fRT$.

(b) Find the molar specific heat $c_{V,N}$.

(3) A van der Waals gas undergoes an adiabatic free expansion from initial volume $V_{\rm i}$ to final volume $V_{\rm f}$. The equation of state is given in §2.10.3 of the lecture notes. The number of particles N is held constant.

(a) If the initial temperature is $T_{\rm i}$, what is the final temperature $T_{\rm f}$?

(b) Find an expression for the change in entropy ΔS of the gas.