Formula sheet

Constants and Factors

Speed of light: c = 299,792,458 m/s exactly (about $3 \times 10^8 \text{ meters/sec}$)

Newton's constant $G = 6.67 \times 10^{-11} \text{ m}^3/\text{s}^2 \text{ kg}$

Earth constants: Acceleration of gravity at surface: $g = 9.8 \text{m/s}^2$, Mass: $M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$,

radius: $r_{\text{Earth}} = 6.37 \times 10^6 \text{m}$

Mass of proton and neutron about 1.67 $\times 10^{-27}~\mathrm{kg}$

Mass of electron: $m_e = 9.11 \times 10^{-31} \text{kg}$

Density of air: $\rho = 1.2 \text{ kg/m}^3$; Density of water: $\rho = 1000 \text{ kg/m}^3$;

 $1 \text{ dyne} = 10^{-5} \text{ Newtons}$

1 mile = 1609 m; 1 foot = 0.3048 m; 1 foot = 12 inches; 1 mile = 5280 ft

1 pound (lb) = 4.448 Newton, corresponding to the weight from mass of 0.454 kg; 1 ton = 2000 lb

Newton = kg m /s²; Joule = Newton m; Watt = Joule/sec

Formulas

Velocity as a derivative of position: $\vec{v} = d\vec{r}/dt$

Acceleration as a derivative of velocity: $\vec{a} = d\vec{v}/dt = d^2\vec{r}/dt^2$

For **constant** acceleration: $\vec{v} = \vec{v}_0 + \vec{a}t$

For **constant** acceleration: $\vec{r} = \vec{r_0} + \vec{v_0}t + \frac{1}{2}\vec{a}t^2$

For **constant** acceleration in a straight line: $v^2 = v_0^2 + 2a(x - x_0)$

Frame of reference: $\vec{v}' = \vec{v} - \vec{V}$; \vec{v} is velocity w.r.t. frame S, \vec{v}' is w.r.t. frame S' which moves at \vec{V}

w.r.t. frame S

Projectile trajectory: (start at x = 0, y = 0, speed v_0 , angle θ_0 : $y = x \tan \theta_0 - gx^2/(2v_0^2 \cos^2 \theta_0)$

Range of projectile above: $x = (v_0^2/g) \sin 2\theta_0$

Circular motion at constant speed: $a = v^2/r$, toward center of circle

Non-uniform circular motion: radial: $a_r = v^2/r$, tangential: $a_t = dv/dt$

Newton's force law: $\vec{F}_{net} = d\vec{p}/dt$, where momentum is $\vec{p} = m\vec{v}$; or if constant mass: $\vec{F}_{net} = m\vec{a}$

Weight: $\vec{W} = m\vec{g}$

Hooke's law for a spring: F = -kx, where k is the spring constant

Friction: Static: $F_s \leq \mu_k N$; Kinetic: $F_k = \mu_k N$; N is the Normal Force

Drag Force: $F_D = \frac{1}{2}C_D\rho Av^2$

Work (constant force in 1-D): $W = F_x \Delta x$; Work (variable force in 3-D): $W = \int_{r_x}^{r_2} \vec{F} \cdot d\vec{r}$

Kinetic Energy: $K = \frac{1}{2}mv^2$, Work-Energy theorem: $W_{\text{net}} = \Delta K$

Power: P = dW/dt; $P = \vec{F} \cdot \vec{v}$

Conservative forces: $\oint \vec{F} \cdot d\vec{r} = 0$; Difference in potential energy is independent of path taken.

Potential Energy: $\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$; in 1-D: $F_x = -dU/dx$

Potential Energy: gravitational near Earth surface: U = mgh; spring elastic: $U = \frac{1}{2}kx^2$; gravita-

tional in general U = -GMm/r

Newton's law of Gravity: $\vec{F} = -\frac{GMm}{r^2}\hat{r}$

Orbital period: $T^2 = 4\pi^2 r^3/(GM)$

Escape velocity: $v_{\rm esc} = \sqrt{2GM/r}$

Rotational equations: $\omega = d\theta/dt$, $\alpha = d\omega/dt$, $v_t = \omega r$, $a_t = \alpha r$

Torque: $\vec{\tau} = \vec{r} \times \vec{F} = d\vec{L}/dt = rF \sin \theta$; direction given by **Right Hand Rule**

Rotational analog of Newton's law: $\tau = I\alpha$, where moment of inertia, $I = \sum m_i r_i^2$ for discrete

masses and $I = \int r^2 dm$ for continuous masses

Rotational kinetic energy: $K_{\rm rot} = \frac{1}{2}I\omega^2$; $W_{\rm rot} = \tau d\theta$

Some moment of inertias: Solid sphere about center: $I = \frac{2}{5}MR^2$;

Hollow sphere about center: $I = \frac{2}{3}MR^2$; Solid cylinder about axis: $I = \frac{1}{2}MR^2$; Hollow cylinder about axis: $I = MR^2$; Thin rod about center: $I = \frac{1}{12}Ml^2$;

Thin rod about end $I = \frac{1}{3}Ml^2$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$; direction given by **Right Hand Rule**

 $\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_z B_x - A_x B_z) + \hat{k}(A_x B_y - A_y B_x) = AB \sin \theta \text{ (direction by RHR)}$

Static Equilibrium; $\sum \vec{F_i} = 0$ and $\sum \vec{\tau_i} = 0$ Center of mass: $\vec{r}_{CM} = (\sum m_i \vec{r}_i)/M_{\rm tot}$