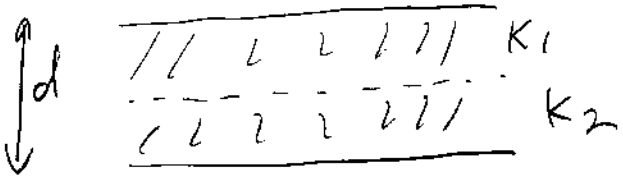
Homework 4, more problems

5.3B



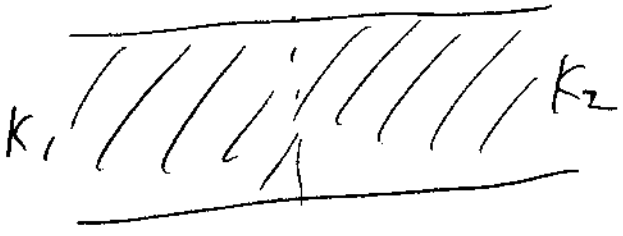
This can be considered as two capacitors in series:

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} \qquad C_2 = \frac{K_2 \varepsilon_0 A}{d/2}$$

$$\frac{1}{C_B} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\varepsilon_0 A} + \frac{d}{2K_2\varepsilon_0 A} =$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{d(K_1 + K_2)}{2\varepsilon_0 A\, K_1 K_2}$$

$$C_B = \frac{2\varepsilon_0 A K_1 K_2}{d(K_1 + K_2)}$$



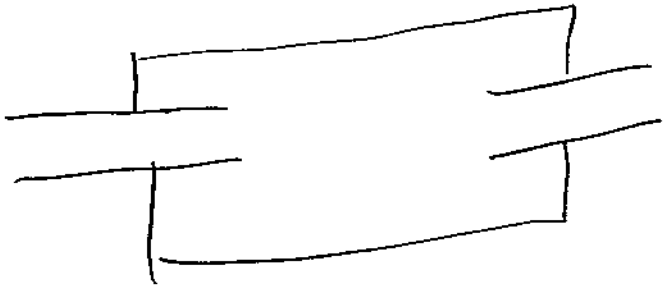
This can be considered as two capacitors in parallel

$$C_1 = \frac{K_1 \varepsilon_0 \frac{A}{2}}{d} \qquad C_2 = \frac{K_2 \varepsilon_0 \frac{A}{2}}{d}$$

$$C_a = C_1 + C_2 = \frac{\varepsilon_0 A (K_1 + K_2)}{2d}$$

$$\frac{C_b}{C_a} = \frac{2K_1K_2}{K_1+K_2} \cdot \frac{2}{K_1+K_2} = \frac{4K_1K_2}{(K_1+K_2)^2}$$

5.3 C



Initially each has charge

$Q_0 = CV_1$

After inserting the dielectric, the capacitance of one of them becomes

$C' = KC$

total capacitance will be

$C_{tot} = KC + C = C(K+1)$

The total charge $2Q_0$ will not change, so

$$V_2 = \frac{2Q_0}{C_{tot}} = \frac{2CV_1}{C(K+1)} = \frac{2V_1}{K+1}$$

new charges will be

$$Q_1 = KC \cdot V_2 = \frac{2KCV_1}{K+1}$$

$$Q_2 = C \cdot V_2 = \frac{2CV_1}{K+1}$$

The charge transferred is

$$\Delta Q = Q_1 - Q_0 = CV_1\left(\frac{2K}{K+1} - 1\right) = CV_1\frac{K-1}{K+1}$$

5.4A The total charge

$$Q = CV_1$$

stays the same, capacitance becomes

$$C' = KC \Rightarrow V_2 = \frac{Q}{C'} = \frac{CV_1}{KC} = \frac{V_1}{K}$$

The energy before:

$$U = \frac{Q^2}{2C}$$

and after

$$U' = \frac{Q^2}{2C'} = \frac{Q^2}{2kC} = \frac{U}{k} < U$$

so the electrostatic forces do work when the slab is inserted, i.e, they pull the slab in.

5.4B Now the voltage stays the same. The energy

$$U = \frac{V^2 C}{2}$$

$$U' = \frac{V^2 C'}{2} = \frac{V^2 kC}{2} = kU > U$$

The argument of the previous problem cannot be used since now the capacitor is connected to a battery which is a source of energy. In

fact, the direction of the force is the same, because when the slab is not moving it does not matter if the capacitor is connected to a battery or not, the force is the same,