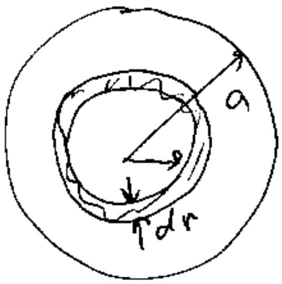
Homework 5

6.1 A $\quad j = j_0 r$



in the small annulus of radius r and thickness dr we have

$$dA = 2\pi r\, dr$$

$$dI = j\, dr = j_0 r \cdot 2\pi r\, dr = 2\pi j_0 r^2 dr$$

Total current:

$$I = \int_0^a 2\pi j_0 r^2 dr = \underline{2\pi j_0 \frac{a^3}{3}}$$

6.1 B $\quad I = 10A$

$$n = 10^{27}/m^3$$

$$A = 1mm^2 = 10^{-6} m^2$$

$$I = nevA$$

$$v = \frac{I}{neA} = \underline{0.0625\ m/s}$$

6.1 C The period is

$$T = \frac{2\pi}{\omega}$$

current $I = \frac{q}{T} = \frac{\omega q}{2\pi}$

6.3 A

$$R = \frac{l}{A\sigma} = \frac{0.2\,m}{2\cdot10^{-4}m^2\cdot 0{,}59\cdot10^{8}(\Omega\cdot m)^{-1}}$$

$$\boxed{R = 1{,}69\cdot10^{-5}\,\Omega}$$

6.3 C $L = 0{,}1\ m$

$A = 2\cdot10^{-4}m^2$

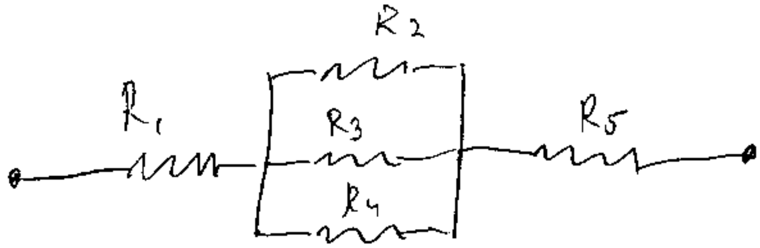
$R = 10^{-4}\,\Omega$

$L \to 0{,}05m = \frac{1}{2}L$

to keep the same volume we need $A \to 2A$

$$R = \frac{\rho L}{A} \to \frac{\rho \frac{L}{2}}{2A} = \frac{R}{4} = 2{,}5\cdot10^{-5}\,\Omega$$

6.4A



$R_1 = 2\,\Omega$

$R_2 = 3\,\Omega$

$R_3 = 4\,\Omega$

$R_4 = 6\,\Omega$

$R_5 = 5\,\Omega$

a. 2, 3, 4 are in parallel;

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right)\Omega^{-1} =$$

$$= \frac{9}{12}\,\Omega^{-1} = \frac{3}{4}\,\Omega^{-1}$$

$$R_{234} = 1.33\,\Omega$$

and is in series with 1 and 5

$$R = R_1 + R_{234} + R_5 = \underline{8.33\,\Omega}$$

8. $V = 10V$

$$I = \frac{V}{R} = 1.2A = I_1 = I_5 = I_{234}$$

$$P_1 = I_1^2 R_1 = \underline{2.88W}$$

$$P_5 = I_5^2 R_5 = \underline{7.2W}$$

Voltage for 2, 3, 4:

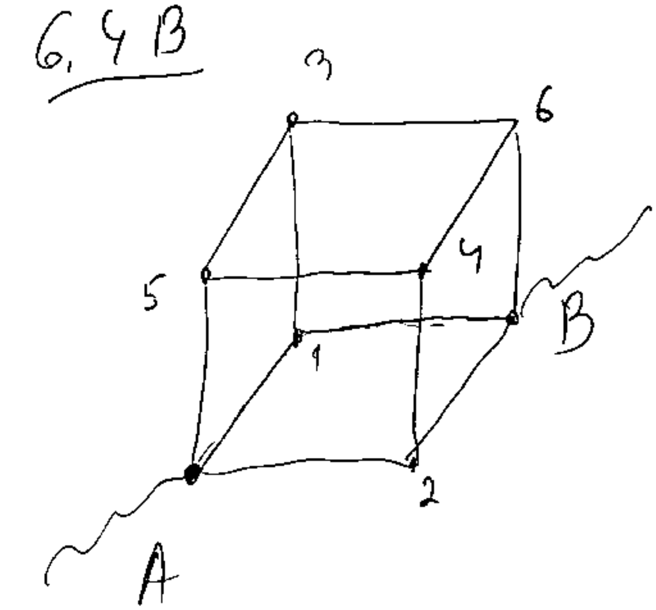
$$V_{234} = R_{234} \cdot I_{234} = 1.6V = V_2 = V_3 = V_4$$

$$P_2 = \frac{V_2^2}{R_2} = \underline{0.85W}$$

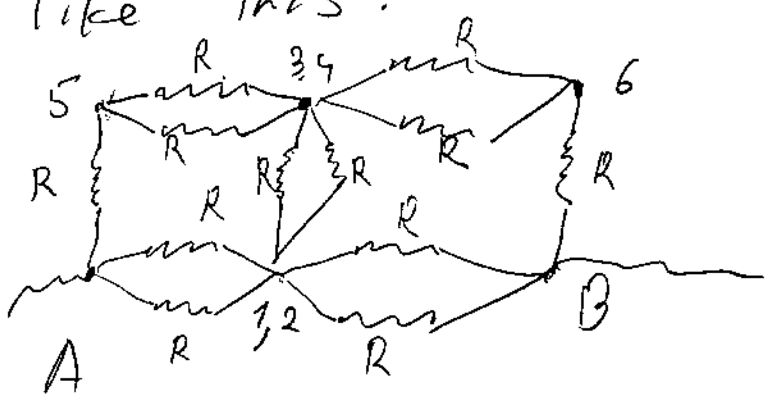
$$P_3 = \frac{V_3^2}{R_3} = \underline{0.64W}$$

$$P_4 = \frac{V_4^2}{R_4} = \underline{0.43W}$$

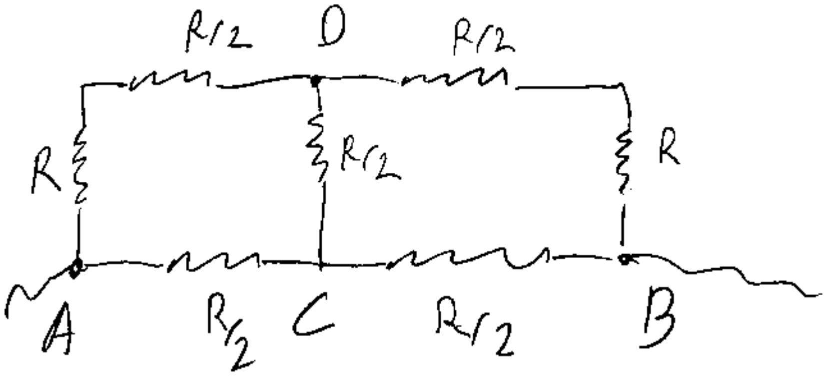
6.4 B



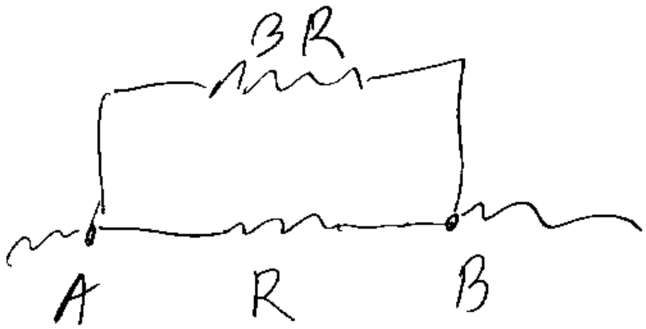
By symmetry, points 1 and 2 have the same potential, so they can be connected without changing the resistance. The same applies to points 3 and 4. The circuit can then be redrawn like this:



Then pairs of parallel R-s can be replaced by R/2:



By symmetry, the current from A to C must be the same as the current from C to B, which means that there is on current from D to C, i.e. the resistance DC can be removed without changing anything, Then we have

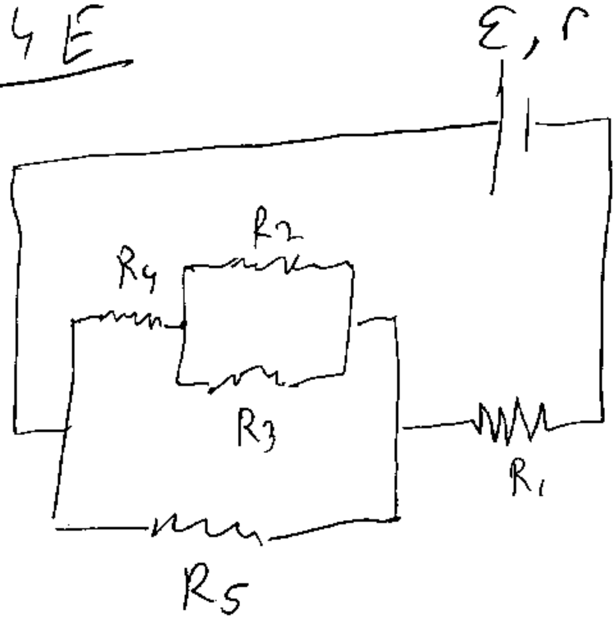


which gives

$$\frac{1}{R_{total}} = \frac{1}{R} + \frac{1}{3R} = \frac{4}{3R}$$

$$\boxed{R_{total} = \frac{3R}{4}}$$

6.4 E



$\mathcal{E} = 12\,V$

$r = 1\,\Omega$

$R_1 = 1\,\Omega$

$R_2 = 6\,\Omega$

$R_3 = 12\,\Omega$

$R_4 = 4\,\Omega$

$R_5 = 8\,\Omega$

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \left(\frac{1}{6} + \frac{1}{12}\right)\Omega^{-1} = \frac{1}{4\,\Omega}$$

$$R_{23} = 4\,\Omega$$

$$R_{234} = R_{23} + R_4 = 8\,\Omega$$

$$\frac{1}{R_{2345}} = \frac{1}{R_{234}} + \frac{1}{R_5} = \left(\frac{1}{8} + \frac{1}{8}\right)\Omega^{-1} = \frac{1}{4\,\Omega}$$

$$R_{2345} = 4\,\Omega$$

$$R_{12345} = R_1 + R_{2345} = 5\,\Omega$$

$$R_{total} = R_{12345} + r = 6\,\Omega$$

a. $I = \dfrac{\mathcal{E}}{R_{total}} = \underline{2\,A}$

b, c. $I = I_1 = I_{2345} = \underline{2A}$

$P_1 = I_1^2 \cdot R_1 = \underline{4W}$

$V_{2345} = I_{2345} \cdot R_{2345} = 8V = V_{234} = V_5$

$I_5 = \frac{V_5}{R_5} = \underline{1A}$

$P_5 = I_5^2 \cdot R_5 = \underline{8W}$

$I_{234} = \frac{V_{234}}{R_{234}} = \underline{1A = I_4} = I_{23}$

$P_4 = I_4^2 \cdot R_4 = \underline{4W}$

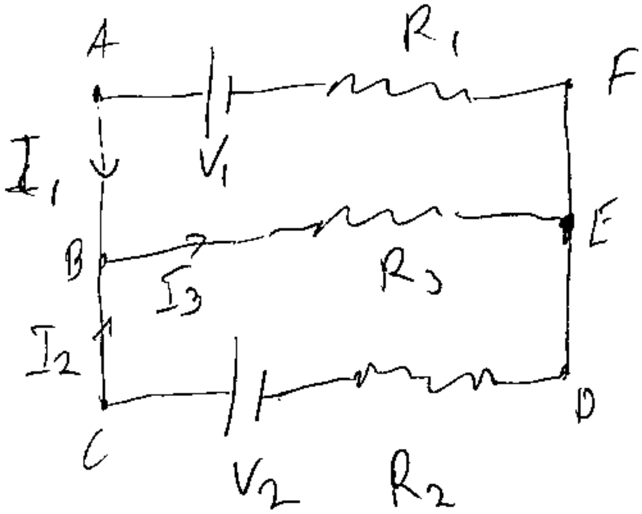
$V_{23} = I_{23} \cdot R_{23} = 4V = V_2 = V_3$

$I_2 = \frac{V_2}{R_2} = \underline{0{,}67\ A}$ $\quad P_2 = I_2^2 \cdot R_2 = \underline{2{,}67\ W}$

$I_3 = \frac{V_3}{R_3} = \underline{0{,}33\ A}$ $\quad P_3 = \underline{1{,}33\ W}$

$P_{Battery} = I^2 \cdot r = \underline{4W}$

6. 5 A



$V_1 = 5V$

$V_2 = 2V$

$R_1 = 3\,\Omega$

$R_2 = 2\,\Omega$

$R_3 = 4\,\Omega$

$I_1 + I_2 = I_3$

from loop ABEF:

$-I_3 R_3 - I_1 R_1 + V_1 = 0$

from loop CBED:

$-I_3 R_3 - I_2 R_2 + V_2 = 0$

putting together:

$$\begin{cases} I_1 + I_2 = I_3 \\ 3I_1 + 4I_3 = 5 \\ 2I_2 + 4I_3 = 2 \end{cases} \Rightarrow \begin{cases} I_1 + I_2 = I_3 \\ I_1 = \dfrac{5-4I_3}{3} \\ I_2 = \dfrac{2-4I_3}{2} \end{cases}$$

$$\frac{5-4I_3}{3} + \frac{2-4I_3}{2} = I_3$$

$$10 - 8I_3 + 6 - 12I_3 = 6I_3$$

$$16 = 26I_3$$

$$\boxed{I_3 = \frac{8}{13} A}$$

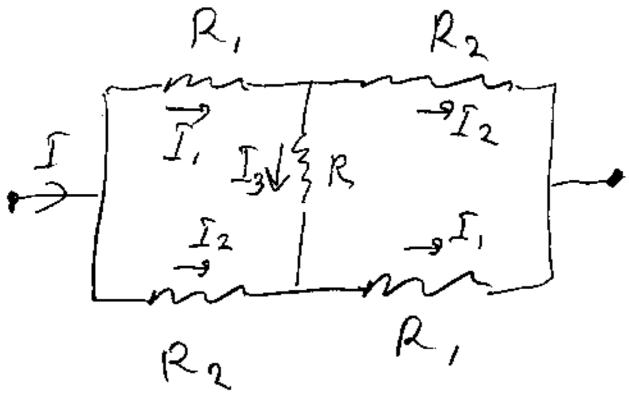
$$I_1 = \frac{5 - 4I_3}{3} = \underline{\frac{11}{13} A}$$

$$I_2 = \frac{2 - 4I_3}{2} = \underline{-\frac{3}{13} A}$$

this means that the direction of I_2 is <u>opposite</u> to what we initially guessed.

<u>6.5 B</u>

a.



By symmetry, the current is the same through both R_1-s, also through both R_2-s. We have then

$$\begin{cases} I_3 = I_1 - I_2 \\ V = I_1 R_1 + I_2 R_2 \\ -I_1 R_1 - I_3 \cdot R + I_2 R_2 = 0 \end{cases}$$

plugging the first equation into the last one:

$$-I_1 R_1 - (I_1 - I_2) R + I_2 R_2 = 0$$

$$-I_1 (R_1 + R) + I_2 (R_2 + R) = 0$$

$$I_2 = \frac{I_1 (R_1 + R)}{R_2 + R}$$

from the second equation

$$V = I_1 \left(R_1 + R_2 \frac{R_1 + R}{R_2 + R} \right) = I_1 \frac{2 R_1 R_2 + R_1 R + R_2 R}{R_2 + R}$$

$$I_1 = V \frac{R_2 + R}{2 R_1 R_2 + R_1 R + R_2 R}$$

$$I_2 = V \frac{R_1 + R}{2 R_1 R_2 + R_1 R + R_2 R}$$

$$I = I_1 + I_2 = V \frac{R_1 + R_2 + 2R}{2 R_1 R_2 + R_1 R + R_2 R}$$

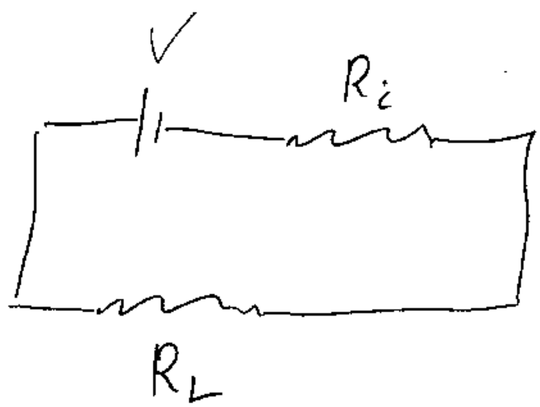
$$R_{total} = \frac{V}{I} = \frac{2R_1 R_2 + R_1 R + R_2 R}{R_1 + R_2 + 2R}$$

b. $$R_{total} = \frac{2 \cdot 4 \cdot 2 + 4 + 2}{4 + 2 + 2}\,\Omega = \underline{2.75\,\Omega}$$

c. If R is removed, we have two $(R_1 + R_2)$'s in parallel, i.e.

$$R_{total} = \frac{R_1 + R_2}{2} = \underline{3\,\Omega}$$

6.6 B



$$I = \frac{V}{R_i + R_L}$$

$$P_L = I^2 R_L = \frac{V^2}{(R_i + R_L)^2} \cdot R_L = \frac{V^2}{R_L^2 (1+x)^2} \cdot R_L =$$

$$= \frac{V^2}{(1+x)^2} \cdot \frac{R_i}{R_L} \cdot \frac{1}{R_i} = \frac{V^2}{R_i} \frac{x}{(1+x)^2}$$

where $x = \frac{R_i}{R_L}$

since V and R_i are fixed, we just need to maximize

$$f(x) = \frac{x}{(1+x)^2} = x(1+x)^{-2}$$

$$f'(x) = (1+x)^{-2} - 2x(1+x)^{-3} = 0$$

$$\frac{1}{(1+x)^2} = \frac{2x}{(1+x)^3}$$

$$1+x = 2x$$

$$\boxed{x = 1}$$