# Homework 6

**11.4 C** a. $V_R = V_C$

$$V = V_R + V_C = 2V_R \Rightarrow V_R = V_C = \frac{V}{2}$$

$$V_R = V e^{-t/\tau} \Rightarrow e^{-\frac{t}{\tau}} = \frac{1}{2}$$

$$\frac{t}{\tau} = \ln 2$$

$$t = \tau \ln 2 = \underline{RC \ln 2}$$

b. $I_{max} = \frac{V}{R}$ and occurs at time $t = 0$, i.e., when the switch is closed.

c. $Q_{max} = VC$ and occurs after the switch has been closed for a long time; $t \gg RC$.

d. The current drops to half its original value when $V_R$ drops to $\frac{V}{2}$.

The charge drops to half its original value when $V_c$ drops to $\frac{V}{2}$. But these two happen at the same time because at all times

$$V_R + V_c = V$$

This happens at time

$$\underline{t = RC \ln 2}$$

(see part a.)

<u>2)</u> When charging a capacitor, the voltage across the resistor is

$$V_R = V_0 e^{-t/\tau}$$

the power

$$P_R = \frac{V_R^2}{R} = \frac{V_0^2 e^{-2t/\tau}}{R}$$

The total energy dissipated

$$E = \int_0^\infty P_R \, dt = \frac{V_0^2}{R} \int_0^\infty e^{-2t/\tau} dt = \frac{V_0^2}{R} \cdot \frac{\tau}{2}$$

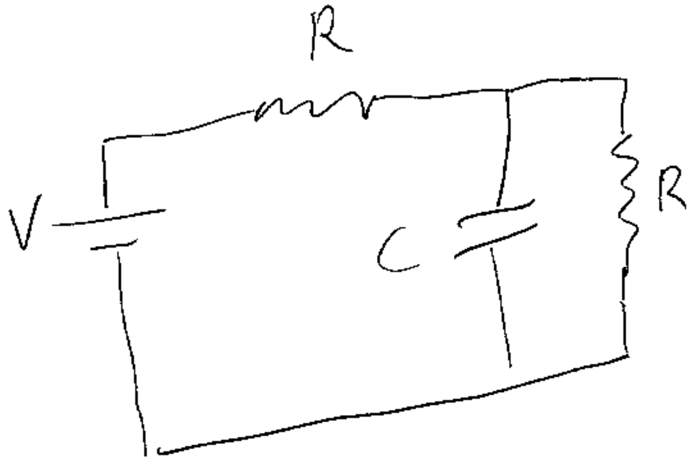
$$E = \frac{V_0^2}{R} \cdot \frac{RC}{2} = \frac{V_0^2 C}{2}$$

which is the energy of the capacitor after it's fully charged.

3) Without the resistor in parallel

$$Q_{wo} = VC$$

with the resistor connected in parallel




after a long time there is no current flowing to the capacitor and we have two resistors in series with the same current.

This means that each resistor gets half of the voltage $\frac{V}{2}$. But the capacitor has the same voltage as the second resistor (they are in parallel):

$$V_c = \frac{1}{2} V$$

$$Q_w = \frac{1}{2} VC = \frac{1}{2} Q_{w0}$$

7.1 A

$$\rho = 1.7 \cdot 10^{-8}\ \Omega \cdot m$$

$$n = 10^{27} / m^3$$

$$\rho = \frac{m}{ne^2 \tau}$$

$$\tau = \frac{m}{ne^2 \rho} = \frac{9.1 \cdot 10^{-31} kg}{10^{27} m^{-3} \cdot (1.6 \cdot 10^{-19} C)^2 \cdot 1.7 \cdot 10^{-8}\ \Omega \cdot m}$$

$$\boxed{\tau = 2.09 \cdot 10^{-12} s}$$

8.1 A For a long straight wire

$$B = \frac{\mu_0 I}{2\pi r}$$

$$F = BIl$$

$$F/l = BI = \frac{\mu_0 I^2}{2\pi r}$$

$$\boxed{F/l = 2 \cdot 10^{-4}\ N/m}$$

8.1 c


If the two loops are very close together they can be approximated around each point as long straight wires, Then

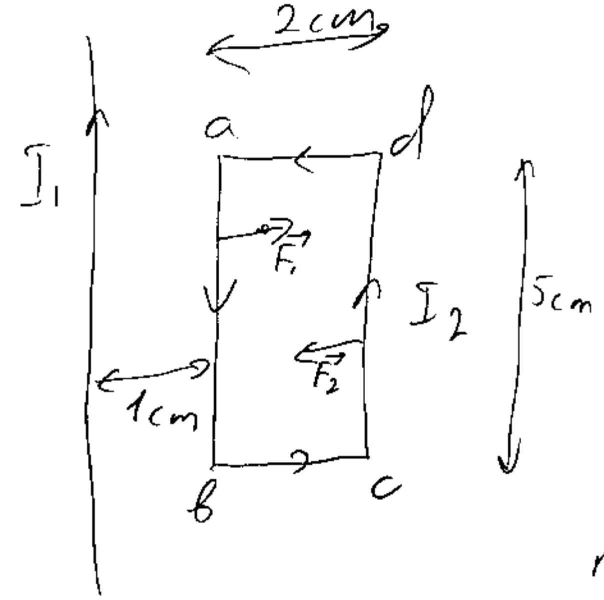
$$F/\ell = \frac{\mu_0 I_c^2}{2\pi \cdot b}$$

where $I_c$ is the total current in each coil; $I_c = N I$

$$\ell = 2\pi a$$

$$F = \frac{\mu_0 I_c^2 \cdot \cancel{2\pi} a}{\cancel{2\pi}\, b} = \frac{\mu_0 N^2 I^2 \cdot a}{b}$$

8.1 D




The forces on bc and ad exactly cancel each other. The magnetic field near ab is

$$B_1 = \frac{\mu_0 I_1}{2\pi (0{,}01 m)} = 2 \cdot 10^{-4} T \quad \text{into the page.}$$

$$F_1 = B_1 I_2 \cdot (0{,}05 m) = 5 \cdot 10^{-5} N \quad \text{to the right}$$

Near dc

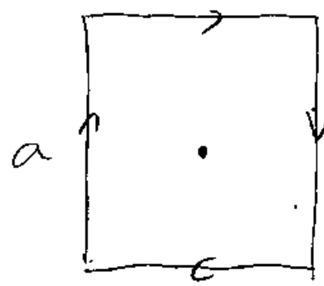
$$B_2 = \frac{\mu_0 I_1}{2\pi (0{,}03 m)} = 6{,}67 \cdot 10^{-5} T \quad \text{into the page}$$

$$F_2 = B_2 I_2 (0{,}05 m) = 1{,}67 \cdot 10^{-5} N \quad \text{to the left}$$

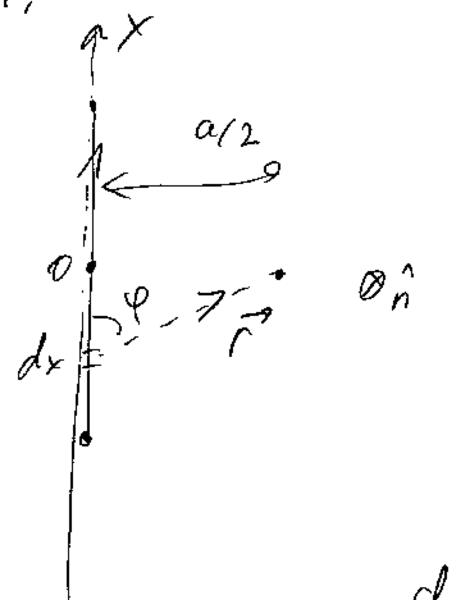
Total force:

$$F = F_1 - F_2 = \underline{3{,}33 \cdot 10^{-5} N \quad \text{to the right}}$$

8.2 A




Each side creates the same magnetic field (magnitude <u>and</u> direction) at the center, so we can calculate the magnetic field from one side and multiply by 4. Consider an element $dx$




$$d\vec{B} = \frac{\mu_0}{4\pi} \, \frac{I \, d\vec{\ell} \times \hat{r}}{r^2}$$

$$r^2 = x^2 + \left(\frac{a}{2}\right)^2$$

$$d\vec{\ell} \times \hat{r} = dx \cdot \sin\varphi \, \hat{n} = dx \, \frac{a/2}{r} \, \hat{n}$$

where $\hat{n}$ is a unit vector into the page.

$$d\vec{B} = \frac{\mu_0}{4\pi} \, \frac{I \, dx \, a/2}{\left(x^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}} \, \hat{n}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{a}{2} \hat{n} \int_{-\frac{a}{2}}^{a/2} \left(x^2 + \left(\frac{a}{2}\right)^2\right)^{-3/2} dx =$$

$$= \frac{\mu_0}{4\pi} I \frac{a}{2} \hat{n} \left(\frac{a}{2}\right)^{-3} \cdot \frac{a}{2} \int_{-1}^{1} (y^2+1)^{-3/2} dy$$

let $y = \tan\theta$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{a/2} \hat{n} \int_{-\frac{\pi}{4}}^{\pi/4} \cos^3\theta \cdot \frac{1}{\cos^2\theta} d\theta =$$

$$= \frac{\mu_0}{4\pi} \frac{2I}{a} \hat{n} \sin\theta \Big|_{\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\mu_0}{2\pi} \frac{I}{a} \cdot \sqrt{2} \hat{n}$$

multiplying by 4

$B = \frac{2\mu_0}{\pi} \frac{I}{a} \sqrt{2}$ into the page

$\underline{B = 1{,}13 \cdot 10^{-4}\, T}$

8.2 C


The magnetic field from the straight parts of the wire is 0 since

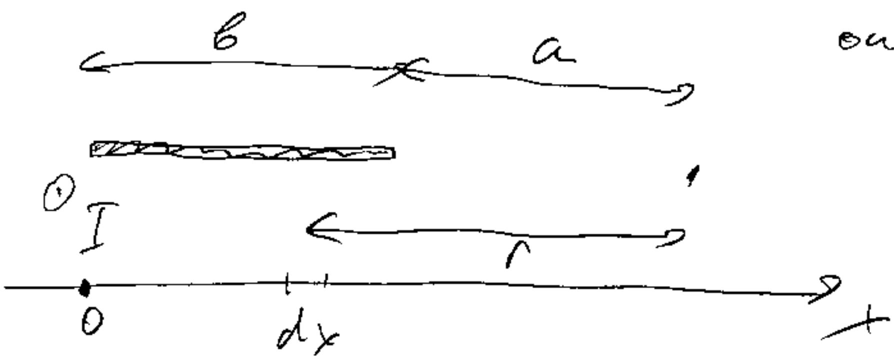
$d\vec{l} \times \hat{r} = 0$

From the semicircular loop, all the points are at the same distance, so

$$B = \frac{\mu_0}{4\pi} \frac{I \cdot \pi r}{r^2} = \frac{\mu_0 I}{4r}$$

out of the page.

8.2 D




The current is flowing out of the page,

From an element $dx$ we get

$$dB = \frac{\mu_0 \, dI}{2\pi r}$$

as for a long straight wire but with current

$$dI = \frac{I\,dx}{b}$$

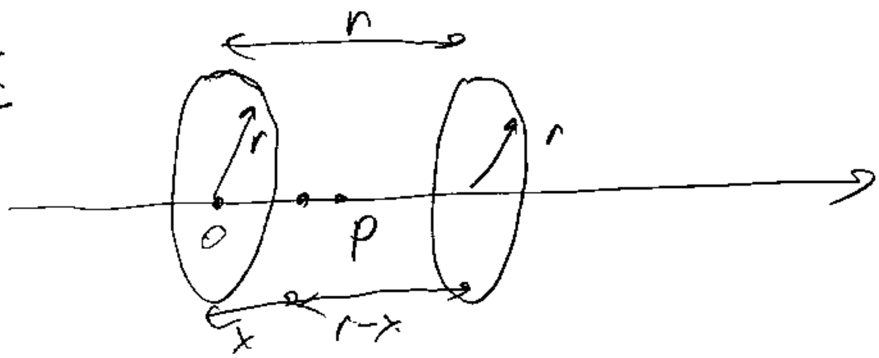
also $r = b + a - x$

$$dB = \frac{\mu_0 I\,dx}{2\pi b(b+a-x)}$$

$$B = \int_0^b \frac{\mu_0 I\,dx}{2\pi b(b+a-x)} = \frac{\mu_0 I}{2\pi b}\left(-\ln(b+a-x)\right)\Big|_0^b =$$

$$= \frac{\mu_0 I}{2\pi b}\ln\frac{b+a}{a}$$

8.2K




At any point $x$ the magnetic field is (see eq. 8.8)

$$B = \frac{\mu_0 I}{2}\frac{r^2}{(r^2+x^2)^{3/2}} + \frac{\mu_0 I}{2}\frac{r^2}{(r^2+(r-x)^2)^{3/2}}$$

which gives

$$\frac{dB}{dx} = \frac{\mu_0 I r^2}{2}\left(-\frac{3}{\cancel{2}}(r^2+x^2)^{-5/2}\cdot \cancel{2}x -\right.$$

$$\left. -\frac{3}{\cancel{2}}\left(r^2+(r-x)^2\right)^{-5/2}\cdot \cancel{2}(r-x)\cdot(-1)\right) =$$

$$= \frac{\mu_0 I r^2}{2}\cdot 3\cdot\left(\left(r^2+(r-x)^2\right)^{-5/2}(r-x) - (r^2+x^2)^{-5/2}\cdot x\right)$$

For point $P$, $x = \frac{r}{2}$ and

$$\left.\frac{dB}{dx}\right|_{r/2} = 0 \quad \checkmark$$

$$\frac{d^2B}{dx^2} = \frac{3\mu_0 I r^2}{2}\left(-\frac{5}{2}\left(r^2+(r-x)^2\right)^{-7/2}\cdot 2(r-x)\cdot(-1)\cdot(r-x) -\right.$$

$$-\left(r^2+(r-x)^2\right)^{-5/2} + \frac{5}{\cancel{2}}(r^2+x^2)^{-7/2}\cancel{2}x\cdot x -$$

$$\left. -(r^2+x^2)^{-5/2}\right) =$$

$$= \frac{3\mu_0 I r^2}{2}\left(5(r^2+x^2)^{-7/2}\cdot x^2 + 5\left(r^2+(r-x)^2\right)^{-7/2}(r-x)^2 -\right.$$

$$\left. -(r^2+x^2)^{-5/2} - \left(r^2+(r-x)^2\right)^{-5/2}\right)$$

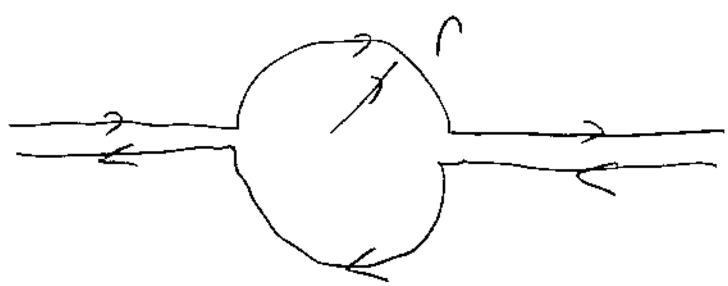
At point P:

$$\left.\frac{d^2B}{dx^2}\right|_{r/2} = \frac{3\mu_0 I r^2}{2}\left(5\left(r^2+\left(\frac{r}{2}\right)^2\right)^{-7/2}\cdot\frac{r^2}{4} + 5\left(r^2+\left(\frac{r}{2}\right)^2\right)^{-7/2}\frac{r^2}{4} - \left(r^2+\left(\frac{r}{2}\right)^2\right)^{-5/2} - \left(r^2+\left(\frac{r}{2}\right)^2\right)^{-5/2}\right) =$$

$$= \frac{3\mu_0 I r^3}{2}\left(10\frac{r^2}{4}\cdot\frac{\left(r^2+\left(\frac{r}{2}\right)^2\right)^{-5/2}}{r^2+\frac{r^2}{4}} - 2\left(r^2+\left(\frac{r}{2}\right)^2\right)^{-5/2}\right) =$$

$$= 0$$

8.2 M




The straight portions in opposite directions exactly cancel each other and we have a circular loop with

$$\boxed{B = \frac{\mu_0 I}{2r} \text{ into the page}}$$