Homework 9

11.1 A

$V = V_0 \sin \omega t$

a. $Z = \sqrt{R^2 + \omega^2 L^2}$

$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$I(t) = I_0 \cos(\omega t - \theta)$

$$\boxed{I(t) = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \theta)}$$

where $\tan \theta = \frac{\omega L}{R}$

b. The inductance and the resistance are in series, so the current is the same, the phase angle is 0.

c. V_R is $\frac{\pi}{2}$ behind V_L.

d. $V_R = R I_0 = \frac{R V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$$V_L = X_L I_0 = \frac{\omega L V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

e. $\vec{V}_R$ and $\vec{V}_L$ are perpendicular,

that is why $|V_R| + |V_L| \neq V_0$

but

$$V_0^2 = V_R^2 + V_L^2$$

$$V_0^2 = \frac{R^2 V_0^2}{R^2 + \omega^2 L^2} + \frac{\omega^2 L^2 V_0^2}{R^2 + \omega^2 L^2}$$

11.1 B a. $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$I(t) = I_0 \cos(\omega t - \theta)$$

$$\boxed{I(t) = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos(\omega t - \theta)}$$

where $\tan\theta = \frac{-\frac{1}{\omega C}}{R} = -\frac{1}{\omega C R}$

b. The currents are the same, phase angle is 0.

c. V_R is $\frac{\pi}{2}$ ahead of V_C.

d. $V_R = R I_0 = \frac{R V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

$V_C = X_C I_0 = \frac{V_0}{\omega C \sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

e. $\vec{V_R}$ and $\vec{V_C}$ are perpendicular

so $|V_R| + |V_C| \neq V_0$ but

$V_R^2 + V_C^2 = V_0^2$

$V_0^2 = \frac{R^2 V_0^2}{R^2 + \frac{1}{\omega^2 C^2}} + \frac{V_0^2}{\omega^2 C^2 \left(R^2 + \frac{1}{\omega^2 C^2}\right)}$ ✓

11.1 C a. $L = 2H$

$$f = 60\,Hz$$

$$X_L = \omega L = 2\pi f L = \underline{754\,\Omega}$$

b. $C = 50 \cdot 10^{-6} F$

$$f = 60\,Hz$$

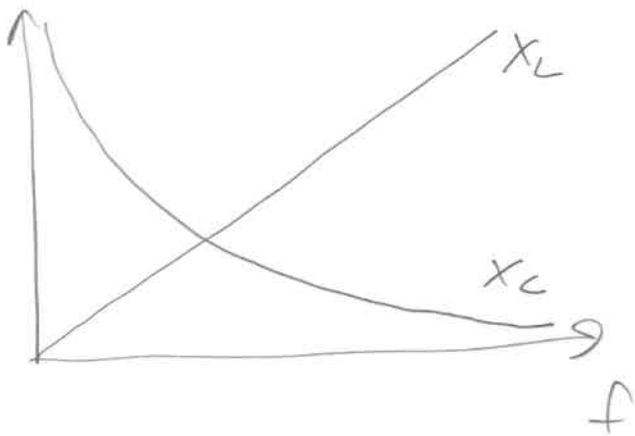
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \underline{53\,\Omega}$$

c. $2\pi f L = \dfrac{1}{2\pi f C}$

$$(2\pi f)^2 = \frac{1}{LC}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \underline{15,9\,Hz}$$

d.



11.1 E $R = 400\,\Omega$

$f = 60\,Hz$

$V_{eff} = 80\,V$

$I_{eff} = 0{,}1\,A$

a. $Z = \frac{V_{eff}}{I_{eff}} = \underline{800\,\Omega}$

b. $P = V_{eff}\, I_{eff} \cos\theta$

$\cos\theta = \frac{R}{Z} = \frac{1}{2}$

$P = \underline{4\,W}$

c. $Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

$\frac{1}{\omega^2 C^2} = Z^2 - R^2$

$C = \frac{1}{\omega\sqrt{Z^2 - R^2}} = \frac{1}{2\pi f \sqrt{Z^2 - R^2}}$

$\underline{C = 3{,}83 \cdot 10^{-6}\,F}$

11.3 A a. $\omega_0 = \sqrt{\frac{1}{LC}}$

b. $Z_0 = \sqrt{R^2 + (\omega_0 L - \frac{1}{\omega_0 C})^2} = R \quad \underline{(R = R_1 + R_2)}$

$I_0 = \frac{V_0}{Z_0} = \underline{\frac{V_0}{R} = \frac{V_0}{R_1 + R_2}}$

c. $I = \frac{I_0}{2} \rightarrow Z = 2 Z_0 = 2R$

$$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = 2R$$

$$\omega L - \frac{1}{\omega C} = \pm\sqrt{3} R$$

$$\omega^2 LC \mp \sqrt{3} R \omega C - 1 = 0$$

$$\omega = \frac{\pm\sqrt{3} RC \pm \sqrt{3R^2C^2 + 4LC}}{2LC}$$

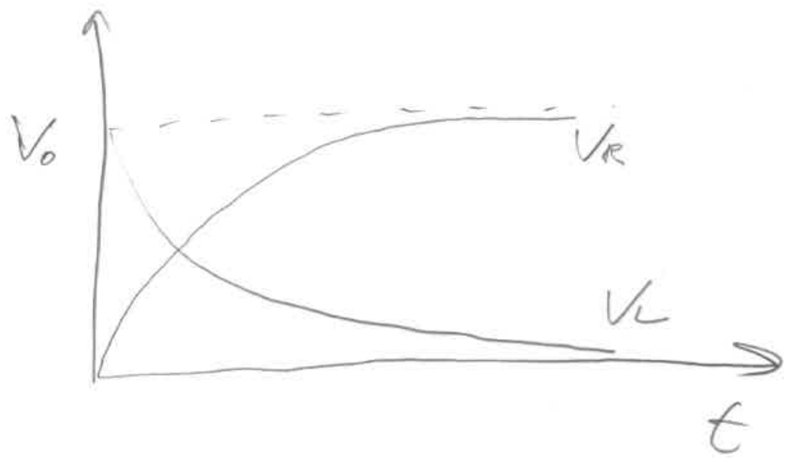
Need to choose only positive solutions:

$$\boxed{\omega = \frac{\pm\sqrt{3} RC + \sqrt{3R^2C^2 + 4LC}}{2LC}}$$

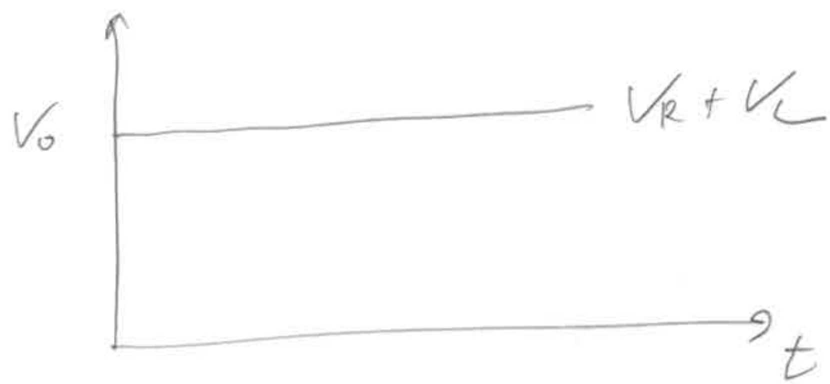
d. $Z = \infty$ for $\underline{\omega = 0 \quad \text{or} \quad \omega = \infty}$

for which $I = 0$.

11.4 A



$V_R + V_L$ is always V_0:



b. $V_L(t) = V_0 e^{-t/\tau}$

$V_R(t) = V_0(1 - e^{-t/\tau})$

where $\tau = \dfrac{L}{R}$

$V_L = V_R \Rightarrow e^{-t/\tau} = 1 - e^{-t/\tau}$

$$e^{-t/\tau} = \frac{1}{2}$$

$$t/\tau = \ln 2$$

$$t = \tau \ln 2 = \underline{\frac{L}{R} \ln 2}$$

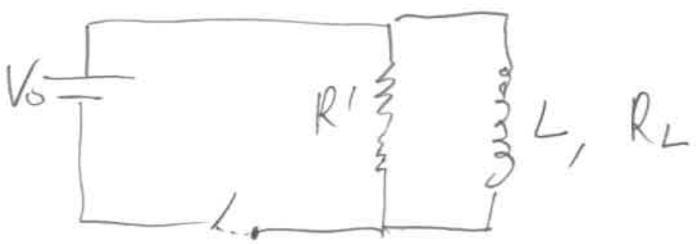
C. $\underline{\tau = \frac{L}{R}}$

<u>11.4 C</u> See week 6!

<u>11.4 D</u> $L = 10H$

$R_L = 100\,\Omega$

$V_0 = 20V$, $V_{max} = 100V$



After opening the switch we have a circuit:

with the same initial current as before opening the switch:

$$I_0 = \frac{V_0}{R_L} = 0.2\ A$$

The voltage across the coil is the same as the voltage across R', which initially is

$$V = I_0 R' \leq V_{max}$$

$$R' \leq \frac{V_{max}}{I_0} = \underline{500\ \Omega}$$

$$I(t) = I_0 e^{-t/\tau}$$

with $\tau = \frac{L}{R_{total}} = \frac{L}{R_L + R'}$

$$\frac{dI}{dt} = -\frac{I_0}{\tau} e^{-t/\tau}$$

Initially:

$$\left|\frac{dI}{dt}\right|_{t=0} = \frac{I_0}{\tau} = \frac{I_0 R_{total}}{L} = \underline{12\ \frac{A}{s}}$$