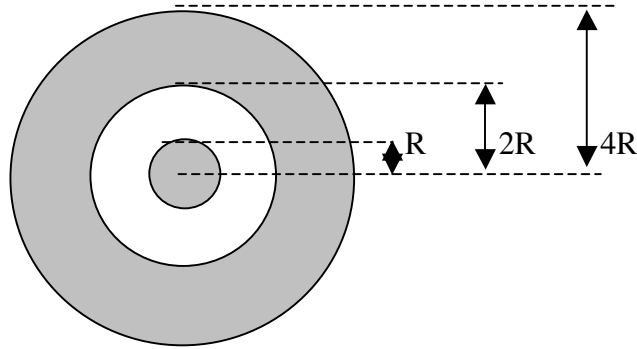


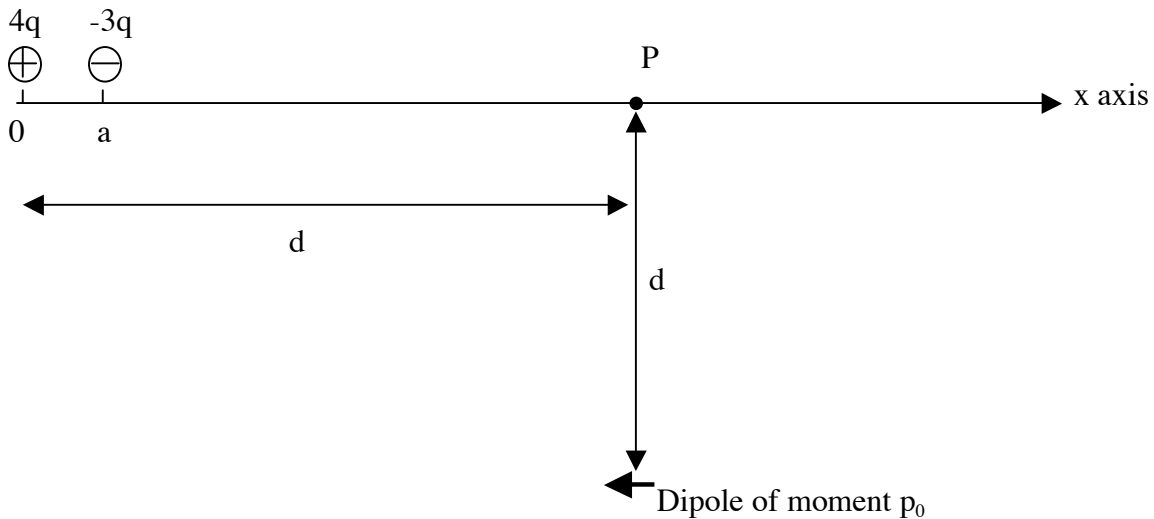
Problem 1 (10 pts)



The figure shows a non-conducting sphere of radius R and uniform charge density ρ , surrounded by a concentric non-conducting spherical shell of inner radius $2R$, outer radius $4R$, and the same uniform charge density ρ . In-between there is vacuum.

- Give an expression for the electric field at radius R , E_R , in terms of ρ , R , and ϵ_0 .
- Give the value of the electric field at radius $2R$, $E(2R)$, in terms of E_R only.
- Give the value of the electric field at radius $3R$, $E(3R)$, in terms of E_R . (Note that $3R$ is inside the outer shell halfway between its surfaces).
- Make a plot of $E(r)$ versus r for r ranging from 0 to $5R$ indicating on the r axis the points where there are changes in the behavior (slope) of $E(r)$.

Problem 2 (10 pts)



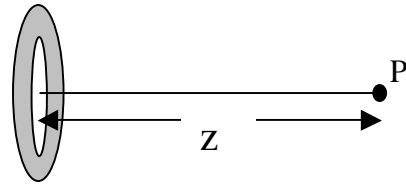
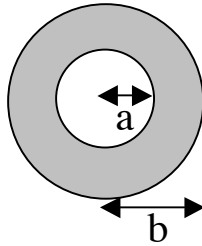
There is a charge $4q$ at the origin and a charge $-3q$ at position a along the x axis. The point P is on the x axis at distance d from the origin. At a distance d from P in direction perpendicular to the x axis there is a dipole of moment p_0 pointing in the $-x$ direction, as shown in the figure. The electric field at point P is found to be $E_P = \frac{q}{4\pi\epsilon_0} \frac{1}{d^2} + O(\frac{1}{d^4})$

pointing in the positive x direction. Assume $d \gg a$.

Find the value of the dipole p_0 in terms of q and a .

Hint: don't guess. Use the binomial expansion $(1+\alpha)^m \sim 1+m\alpha$. Show your work.

Problem 3 (10 pts)



The annulus in the figure has inner radius a and outer radius b and charge per unit area σ . We would like to calculate the electric field at a point P at distance z from the center along the axis perpendicular to the annulus (see figure on the right).

We derived in class that for a ring of radius r and total charge q the electric field at distance z along the perpendicular axis is

$$E = \frac{q}{4\pi\epsilon_0} \frac{z}{(z^2 + r^2)^{3/2}}$$

Use that result to do this problem. First, write an expression for the contribution to the electric field at point P from the part of the annulus that has radius between r and $r+dr$.

(a) Then, do an integral and find a formula for the electric field at point P , $E(z)$, in terms of σ , z , a , b , and ϵ_0 . Your answer should not contain the letters q nor r .

(b) Find from your formula the value $E(z=0)$. Does it make sense? Explain.

(c) Take in your formula the limit $a \rightarrow 0$ and then the limit $z \rightarrow 0$ and give the value of E in that case. Does it make sense? Explain why.

(d) Find from your formula the limit $E(z)$ for z very large ($z \gg a$, $z \gg b$). (not $E=0!$). (Use that $(1+x)^m \sim 1+mx$ for small x). Explain how you can tell that the answer you get is correct.

Justify all your answers to all problems