Physics 4C — Quiz 1 Solutions — Fall 2011

Problem 1

(a) Charge of sphere: $q = \frac{4}{3}\pi R^3 \rho$

Field at $r = R$: $E_R = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{4}{3}\frac{\pi R^3 \rho}{4\pi\varepsilon_0 R^2} \Rightarrow \boxed{E_R = \frac{\rho R}{3\varepsilon_0}}$

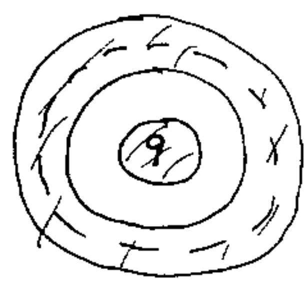
(b) at $r = 2R$:

$$E = \frac{q}{4\pi\varepsilon_0 (2R)^2} = \frac{1}{4}\frac{q}{4\pi\varepsilon_0 R^2} \Rightarrow \boxed{E = \frac{1}{4}E_R}$$

(c) at $r = 3R$, need to use Gauss thm:

$$\oint E \cdot dS = \frac{q_{enc}}{\varepsilon_0} \Rightarrow 4\pi r^2 E = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$$

The enclosed charge is:



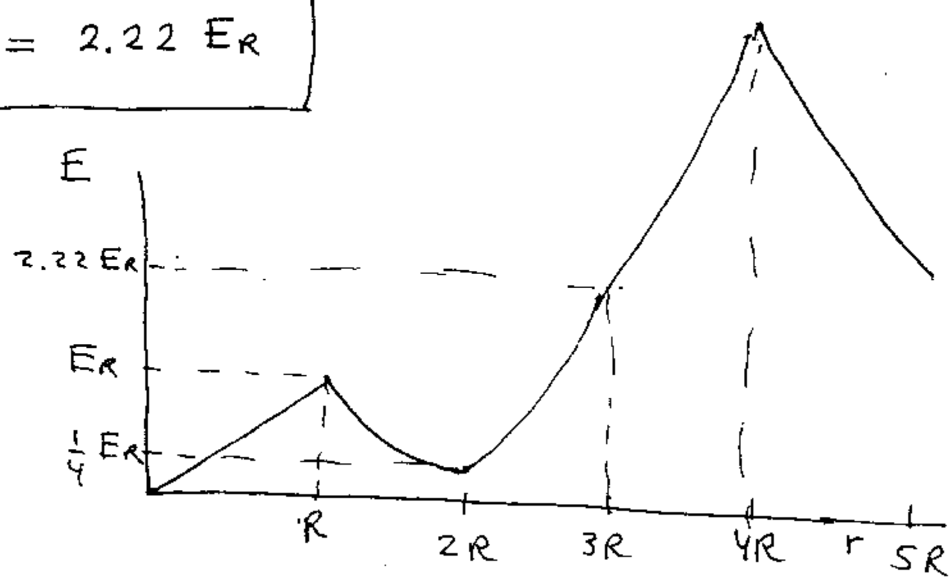
$$q_{enc} = q + \frac{4}{3}\pi\left((3R)^3 - (2R)^3\right)\rho =$$

$$= q + \frac{4}{3}\pi R^3 (27-8)\rho = 20q$$

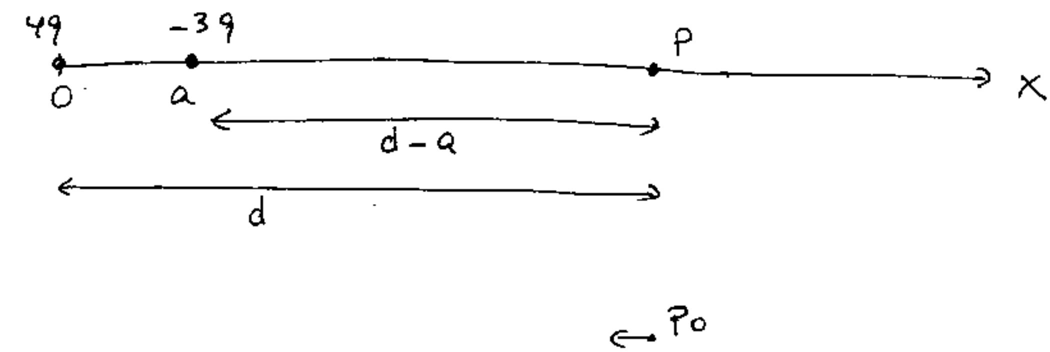
$$\Rightarrow E = \frac{20q}{4\pi\varepsilon_0 (3R)^2} = \frac{20}{9}\frac{q}{4\pi\varepsilon_0 R^2} \Rightarrow$$

$$\boxed{E = \frac{20}{9}E_R = 2.22\,E_R}$$

(d)



Problem 2



Ignoring po, the electric field at point P is along the x axis and is

$$E_P = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q}{d^2} - \frac{3q}{(d-a)^2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{d^2} - \frac{3}{d^2\left(1-\frac{a}{d}\right)^2}\right) =$$

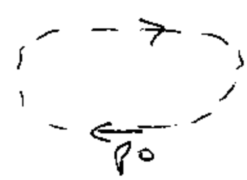
$$\cong \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{d^2} - \frac{3}{d^2}\left(1 + \frac{2a}{d}\right)\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{4}{d^2} - \frac{3}{d^2} - \frac{6a}{d^3}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0 d^2} - \frac{6qa}{4\pi\varepsilon_0 d^3} \Rightarrow \boxed{E_P = \frac{q}{4\pi\varepsilon_0 d^2} - \frac{6qa}{4\pi\varepsilon_0 d^3}}$$

without the field of $\vec{p}_0$

Now we know that the field of $\vec{p}_0$ at point P is (book, Eq. 2.6)

$$E_{p_0} = \frac{1}{4\pi\varepsilon_0}\frac{p_0}{d^3}$$

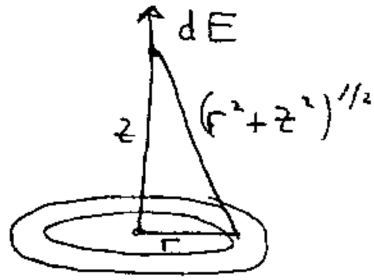


pointing in the +x direction.

It has to cancel the $\frac{1}{d^3}$ term in E_P, therefore

$$\boxed{p_0 = 6qa}$$

Problem 3



For a ring of charge dq,

$$dE = \frac{dq}{4\pi\varepsilon_0}\frac{z}{(z^2+r^2)^{3/2}}$$

We have

$$dq = \sigma \cdot 2\pi r\,dr$$

$$\Rightarrow \quad dE = \frac{\cancel{2\pi}\, r\,dr\,\sigma\, z}{\cancel{4\pi}\,\varepsilon_0\,(z^2+r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\frac{r\,dr}{(z^2+r^2)^{3/2}} \Rightarrow$$

$$E = \int_a^b \frac{\sigma z}{2\varepsilon_0}\frac{r\,dr}{(z^2+r^2)^{3/2}} = -\frac{\sigma z}{2\varepsilon_0}\frac{1}{(z^2+r^2)^{1/2}}\Bigg]_a^b = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{(z^2+a^2)^{1/2}} - \frac{1}{(z^2+b^2)^{1/2}}\right]$$

$$\boxed{E(z) = \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{(z^2+a^2)^{1/2}} - \frac{1}{(z^2+b^2)^{1/2}}\right]}$$

(b) For $z=0$, $E(z)=0$, makes sense by symmetry.

(c) For $a \to 0$, $E(z) = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+b^2)^{1/2}}\right]$; then for

$z \to 0$, $E(z) = \frac{\sigma}{2\varepsilon_0}$, field of a sheet, makes sense in

the limit $z \ll b$ and $a = 0$.

(d) For large z, $\frac{1}{(z^2+a^2)^{1/2}} = \frac{1}{z\left(1+\frac{a^2}{z^2}\right)^{1/2}} = \frac{1}{z}\left(1 - \frac{a^2}{2z^2}\right) \Rightarrow$

$$\Rightarrow E(z) = \frac{\sigma \cancel{z}}{2\varepsilon_0}\frac{1}{\cancel{z}}\left(1 - \frac{a^2}{2z^2} - 1 + \frac{b^2}{2z^2}\right) = \frac{\sigma}{2\varepsilon_0}\frac{(b^2-a^2)}{2z^2} =$$

$$= \frac{\sigma\pi(b^2-a^2)}{4\varepsilon_0 z^2} = \boxed{\frac{q}{4\pi\varepsilon_0 z^2}}$$

$q = \pi(b^2-a^2)\sigma$ = total charge

makes sense (Coulomb's law)