Physics 4C — Quiz 2 Solutions — Fall 2011

Problem 1

Electric potential at P_1 and at P_2:

$$V_1 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{10b} - \frac{1}{11b}\right) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{110b}$$

$$V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{20b} - \frac{1}{21b}\right) = \frac{q}{4\pi\varepsilon_0}\,\frac{1}{420b}$$

hence $V_2 = \frac{110}{420} V_1$. $V_1 = 5\,V \Rightarrow \boxed{V_2 = 1.31\,V}$ (a)

(b) The potential of a dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}\cos\theta$$

Here we have approximately a dipole potential since $10b \gg b$. The dipole moment is $p = qb$, and $\theta = 0$.

Potential decreases as $1/r^2$, so when distance is doubled (from $10b$ to $20b$) the potential should decrease by a factor of $2^2 = 4$. So if $V_1 = 5\,V$, $V_2 \approx \frac{5}{4}\,V$

$\Rightarrow \boxed{V_2 \approx 1.25\,V}$, similar to answer in (a).

(c) The electric potential at point P_3 is zero, since

$$V_3 = \frac{q}{4\pi\varepsilon_0 a/2} - \frac{q}{4\pi\varepsilon_0 a/2} = 0.$$

The work you do in bringing a charge Q from P_1 to P_3 along any path is

$$W = Q\cdot V_3 - Q\cdot V_1 = -QV_1 = -(-3C)\cdot 5V = \boxed{+15\,J}$$

Work is positive, the negative ($-3C$) charge prefers to be at P_1 rather than at P_3 because it is closer to $+q$ than to $-q$ there.

Problem 2

$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$; energy $\boxed{U = \frac{q^2}{2C} = \frac{q^2}{8\varepsilon_0} \frac{b-a}{ab}}$ (a)

(b) For a single sphere of radius R, capacitance is $C = 4\pi\varepsilon_0 R$,

so energy is $U = \frac{q^2}{8\pi\varepsilon_0 R}$. So the energy is the sum of the

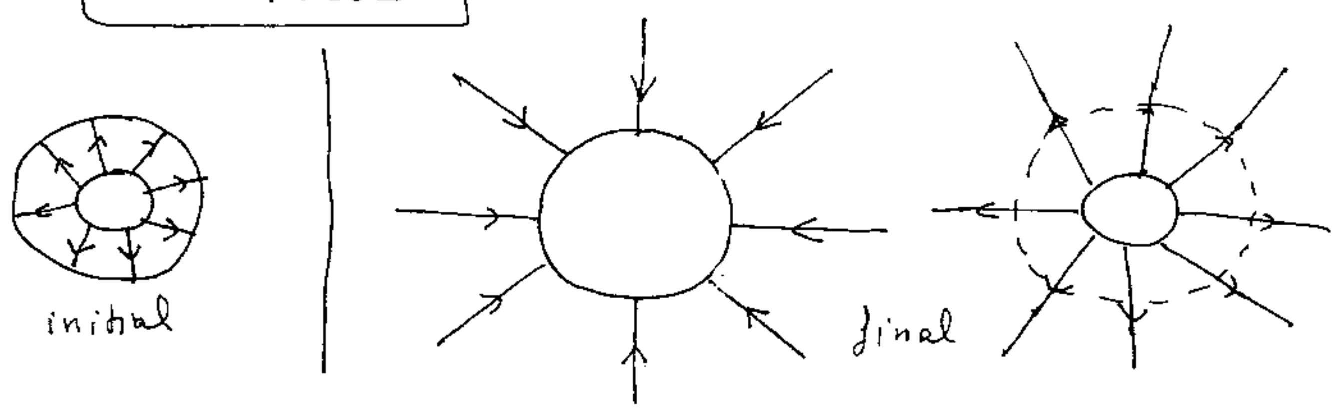
energies of both spheres: $U' = \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{q^2}{8\pi\varepsilon_0} \frac{a+b}{ab}$

So the work done in pulling shells away is difference

$$W = U' - U = \frac{q^2}{8\varepsilon_0\pi}\left(\frac{b+a}{ab} - \frac{b-a}{ab}\right) = \frac{q^2}{8\pi\varepsilon_0 ab} \cdot 2a = \frac{q^2}{4\pi\varepsilon_0 b}$$

So: $\boxed{W = \frac{q^2}{4\pi\varepsilon_0 b}}$

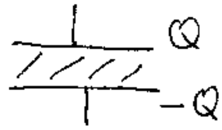
(c)



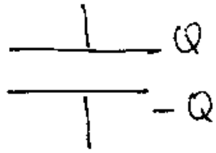
The energy in the initial state comes from the E-field lines between the shells only. So in the final state we substract that part, which is the part in the dashed circle in the figure to the right. What remains then is 2 spheres of radius b contributing the same E^2 outside b (the sign of E doesn't matter)

hence $\boxed{\Delta U = 2 \cdot \frac{q^2}{8\pi\varepsilon_0 b} = \frac{q^2}{4\pi\varepsilon_0 b}}$ agrees with (b), as it should

Problem 3

(a) When the slab is fully inserted 

, $Q = KCV$;

without the slab, 

$Q = CV_0 \Rightarrow V = \frac{Q}{KC} = \frac{V_0}{K}$

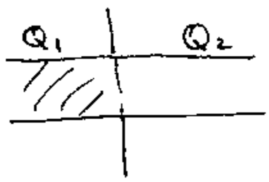
since $V_0 = 30V$, $K = 4 \Rightarrow \boxed{V = 7.5\,V}$

(b) Initial energy: $U_0 = \frac{1}{2} Q V_0$, final energy $U = \frac{1}{2} Q V = \frac{U_0}{K}$

Work done = final energy − initial energy $= \frac{U_0}{K} - U_0 = U_0\left(1 - \frac{1}{K}\right)$

For $U_0 = 10\,J$, $K = 4$, $\boxed{\text{Work} = -\frac{3}{4} U_0 = -7.5\,J}$

The work done by the person inserting the slab is <u>negative</u> because the slab gets polarized and is attracted into the plates.

(c) 

dielectric is half-way between the plates. Each half has capacitance $\frac{C}{2}$ without dielectric

$Q_1 = K\frac{C}{2}V$
$Q_2 = \frac{C}{2}V$
$\Rightarrow \frac{Q_1}{K} = Q_2 \Rightarrow Q_1 = KQ_2$

and $Q_1 + Q_2 = Q \Rightarrow (1+K)Q_2 = Q \Rightarrow$

$\Rightarrow Q_2 = \frac{1}{1+K}Q$, $Q_1 = \frac{K}{1+K}Q$

$$V = 2\frac{Q_2}{C} = \frac{2}{1+K}\frac{Q}{C} = \frac{2}{1+K}V_0 = \frac{2}{5}V_0 = 12\,V$$

$\boxed{V = 12\,V}$ if initial voltage was 30V.

Note: the answer should be in-between 30V and 7.5V, but it is not the average.