Physics 4C **Quiz 4 Solutions** Fall 2011

Problem 1

For a wire carrying current i, $B = \frac{\mu_0 i}{2\pi r}$ at distance r.

For a solid cylinder, use Ampere's law. If total current is i,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_{enclosed} \Rightarrow B \cdot 2\pi r = \mu_0 i_{enclosed}$$

If the current density is j, $i_{enclosed} = j \cdot \pi r^2$. For total current i,

$j = i/\pi b^2$, with b = radius. So $i_{enclosed} = i r^2/b^2$. So,

$$B \cdot 2\pi \cancel{r} = \mu_0 i r^{\cancel{2}}/b^2 \Rightarrow \quad B = \frac{\mu_0 i}{2\pi}\frac{r}{b^2}$$

Here, the current of the cylinder is $\frac{5}{3} i$, so $B_{cylinder} = \frac{5}{3}\frac{\mu_0 i}{2\pi}\frac{r}{b^2}$

The magnetic fields of the wire and the cylinder point in opposite directions.

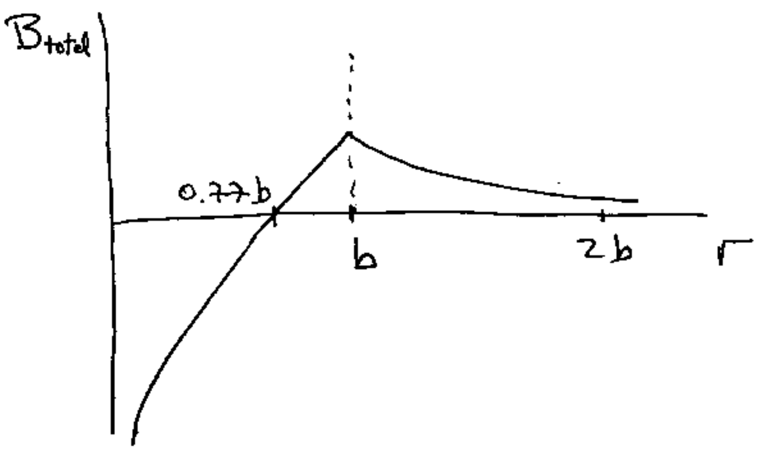
They cancel if $\frac{\cancel{\mu_0 i}}{\cancel{2\pi} r} = \frac{5}{3}\frac{\cancel{\mu_0 i}}{\cancel{2\pi}}\frac{r}{b^2} \Rightarrow r^2 = \frac{3}{5}b^2 \Rightarrow \boxed{r = \sqrt{\frac{3}{5}}\,b = 0.77b}$

(a)

(b) For $r > b$, $\boxed{B_{total} = \left(\frac{5}{3}i - i\right)\frac{\mu_0}{2\pi r} = \frac{2}{3}\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{3\pi r}}$

(c) For $r < b$, $\boxed{B_{total} = \frac{\mu_0 i}{2\pi}\left(\frac{5}{3}\frac{r}{b^2} - \frac{1}{r}\right)}$

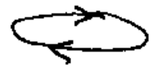
So plot is



Problem 2

The B-field generated by $i(t)$ of the large loop at its center is

$$B(t) = \frac{\mu_0 \, i(t)}{2b}$$ pointing ↑. It creates a flux through the small loop, by Lenz's law the small loop generates a current that creates a magnetic field pointing ↓, i.e.


current induced in small loop (a)

(b) The emf induced is

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$ The flux through the small loop is $\Phi(t) = \pi a^2 B(t)$, so

$$\Phi(t) = \frac{\pi a^2}{2b}\mu_0 i(t) = \frac{\pi a^2}{2b}\mu_0 i_0 \frac{t^3}{\tau^3} = Ct^3$$ with C a constant.

So $\mathcal{E} = -3Ct^2$; if R is the resistance of the small loop,

$$i_{\text{induced}} = \frac{\mathcal{E}}{R} = -\frac{3Ct^2}{R}.$$ So if at $t = 1\,s$ the induced current is $2\,A$, at $t = 3\,s$ it is $2\,A \cdot \left(\frac{3\,s}{1\,s}\right)^2 = \boxed{18\,A}$ (b)

(c) If $i(t)$ flows in the small loop, the emf induced in the large loop is the same as in (b). This is because

$$\mathcal{E}_2 = -M_{21}\frac{di_1}{dt}, \quad \mathcal{E}_1 = -M_{12}\frac{di_2}{dt},$$ and $M_{12} = M_{21}$ always.

In this case, the current in the loop of radius b is $i_b = \frac{\mathcal{E}}{R_b}$.

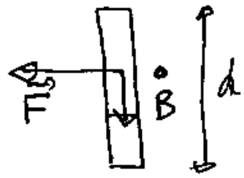
The resistance of the large loop $R_b = \frac{b}{a}R$, with R the resistance of the small loop, because the ratio of their lengths is b/a.

Hence, $$\boxed{i_b = \frac{a}{b}\cdot(2\,A) = \frac{1}{100}\cdot 2A = 0.02\,A}$$

Problem 3

Current along the rod flows down, B is out of the paper, force is

$\vec{F} = i\,\vec{d}\times\vec{B}$



rod starts moving to the left due to force on wire carrying current in a B field.

(b) Immediately after closing the switch, current is

$$\boxed{i_0 = i(t=0^+) = \frac{\mathcal{E}_0}{R}}$$

(c) As the rod starts moving, a counter emf $\mathcal{E}$ is induced.

Current is $\quad i = \dfrac{\mathcal{E}_0 - \mathcal{E}}{R} = \dfrac{5}{7} i_0 = \dfrac{5}{7}\dfrac{\mathcal{E}_0}{R} \Rightarrow \mathcal{E} = \mathcal{E}_0 - \dfrac{5}{7}\mathcal{E}_0$

$$\Rightarrow \boxed{\mathcal{E} = \frac{2}{7}\mathcal{E}_0}$$

(d) The counter-emf $\mathcal{E}$ results from Faraday law

$$\mathcal{E} = -\frac{d\phi}{dt}\ ; \quad \phi = B\cdot d\cdot x, \quad \frac{d\phi}{\partial t} = B\cdot d\cdot v \Rightarrow$$

$$\mathcal{E} = B\cdot d\cdot v = \frac{2}{7}\mathcal{E}_0 \Rightarrow \boxed{v = \frac{2\mathcal{E}_0}{7Bd}}$$

(e) Power supplied by battery is $P_b = \mathcal{E}_0 i = \dfrac{5}{7}\mathcal{E}_0 i_0 = \dfrac{35}{49}\mathcal{E}_0 i_0$

Power dissipated in resistor is $P_R = i^2 R = \dfrac{25}{49} i_0^2 R = \dfrac{25}{49}\mathcal{E}_0 i_0$

So $P_b > P_R$, $\boxed{P_b - P_R = \dfrac{10}{49}\mathcal{E}_0 i_0 = \dfrac{10}{49}\dfrac{\mathcal{E}_0^2}{R}}$

The difference is the power being used to accelerate the rod.