Physics 4C Quiz 5 Solutions Fall 2011

Problem 1

(a) $\boxed{i = \dfrac{\mathcal{E}}{R} = \dfrac{100\ V}{5\,\Omega} = 20A}$

(b) $\boxed{Q = 0}$

(c) $U = \frac{1}{2} L i^2 = \frac{1}{2} \cdot 10 \cdot 10^{-3} \cdot 20^2\ J = \frac{1}{2} \cdot \frac{400}{100}\ J$

$\Rightarrow \boxed{U = 2\,J}$

(d) When all the energy is in the capacitor

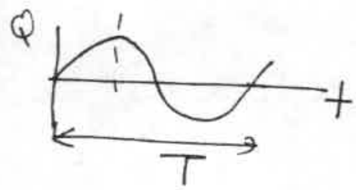
$$U = \frac{Q^2}{2C} \Rightarrow Q^2 = 2CU = 2 \cdot 10 \cdot 10^{-3} \cdot 2\,C^2 = \frac{4}{100}\,C$$

$\Rightarrow \boxed{Q = 0.2\,C}$

(e) Frequency of oscillation is $\omega = \dfrac{1}{\sqrt{LC}}$

$\omega = 2\pi f = \dfrac{2\pi}{T}$. Charge does

$T = \dfrac{2\pi}{\omega}$



So maximum charge is at $T/4 \equiv t$

$$t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{2\pi}{4}\sqrt{LC} = \frac{\pi}{2}\sqrt{10 \cdot 10^{-3} \cdot 10 \cdot 10^{-3}}\ s =$$

$$= \frac{\pi}{2} \cdot 10^{-2}\ s = \frac{\pi}{200}\ s \Rightarrow \boxed{t = 0.0157\,s}$$

Problem 2

$\mathcal{E} = V_0 \cos\omega t$

$i = I_0 \cos(\omega t - \theta)$

$\tan\theta = \dfrac{\omega L - 1/\omega C}{R}$

$I_0 = \dfrac{V_0}{Z}$, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

(a) $\theta = 0 \Rightarrow$

$\omega L = \dfrac{1}{\omega C} \Rightarrow C = \dfrac{1}{\omega^2 L} = \dfrac{1}{100^2 . 50 . 10^{-3}} F =$

$= \dfrac{1}{500} F \Rightarrow \boxed{C = 2 mF}$

(b) $\boxed{Z = R = 2\Omega}$ $\boxed{I_0 = \dfrac{V_0}{R} = 50 A}$

(c) $V_L = X_L I_0 = \omega L I_0 =$

$= 100 . 50 . 10^{-3} . 50 V \Rightarrow \boxed{V_L = 250 V}$

Problem 3

The Ampere-Maxwell law is

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{j}\cdot d\vec{S} + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{S}$$

For point P_1: $B_1 \cdot 2\pi r = \mu_0 i \Rightarrow \boxed{B_1 = \frac{\mu_0 i}{2\pi r}}$

For point P_2: $B_2 \cdot 2\pi r = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\cdot \pi r^2 \Rightarrow$

$$B_2 = \frac{\mu_0 \varepsilon_0}{2}\frac{\partial E}{\partial t}\cdot r \quad ; \quad E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \Rightarrow \frac{\partial E}{\partial t} = \frac{i}{A\varepsilon_0} = \frac{i}{\pi R^2 \varepsilon_0}$$

where $A = \pi R^2$ is the area of the capacitor plate. So,

$$B_2 = \frac{\mu_0 \varepsilon_0}{2}\frac{i}{\pi R^2 \varepsilon_0} r \Rightarrow \boxed{B_2 = \frac{\mu_0 i}{2\pi}\frac{r}{R^2}}$$

So for $r = \frac{R}{2}$, $B_1 = \frac{\mu_0 i \cdot 2}{2\pi R} = \frac{\mu_0 i}{\pi R}$, $B_2 = \frac{\mu_0 i}{2\pi}\frac{1}{2R} = \frac{\mu_0 i}{4\pi R} \Rightarrow$

$\Rightarrow B_2 = \frac{1}{4} B_1$, since $B_1 = 0.4\,mT \Rightarrow \boxed{B_2 = 0.1\,mT}$

(b) At point P_3: $r = R \Rightarrow \boxed{B_3 = \frac{\mu_0 i}{2\pi R}} \Rightarrow B_3 = \frac{B_1}{2} \Rightarrow$

$$\boxed{B_3 = 0.2\,mT}$$

(c) $B_2 = \frac{\mu_0 \varepsilon_0}{2}\frac{\partial E}{\partial t}\cdot r \Rightarrow \frac{\partial E}{\partial t} = \frac{2B_2}{\mu_0 \varepsilon_0 r}$; for we get r from

$$B_1 = \frac{\mu_0 i}{2\pi r} \Rightarrow r = \frac{\mu_0 i}{2\pi B_1} = \frac{\overset{2}{\cancel{4\pi}}\cdot 10^{-7}\cdot 200}{\cancel{2\pi}\cdot 0.4\times 10^{-3}} = \frac{10^{-5}}{0.1\cdot 10^{-3}}\,m = 0.1\,m = 10\,cm$$

$$\frac{\partial E}{\partial t} = \frac{2\cdot 0.1\times 10^{-3}}{4\pi\cdot 10^{-7}\cdot 8.854\cdot 10^{-12}\cdot 0.1}\frac{V}{m\cdot s} \Rightarrow \boxed{\frac{\partial E}{\partial t} = 1.8\times 10^{14}\frac{V}{m\cdot s}}$$