midterm thursday 5/22

Final 8:00–11:00 Thurs June 12th

~~time average Power dissipated / length~~

start ✓

using $\delta << L$

$$\frac{d\langle P\rangle}{dz} = -\frac{1}{2\sigma\delta}\oint_C dl\,\underbrace{\left|\frac{c}{4\pi}H_t\right|^2}_{|K_{eff}|^2}$$

$B_t = H_t$ in vac

$$\frac{d\langle P\rangle}{dz} = -\underbrace{\frac{1}{2\sigma\delta}\frac{c^2}{(4\pi)^2}}_{c\left(\frac{\omega\mu}{\sigma}\right)^{1/2}\frac{1}{(8\pi)^{3/2}}}\oint_C dl\,|H_t|^2$$

for TM mode

at boundary

$$|H_t|^2 = \frac{\omega^2}{c^2\gamma^4}|\nabla_t E_z|^2 = \frac{\omega^2}{c^2\gamma^4}\left|\frac{\partial E_z}{\partial n}\right|^2$$

$$\frac{1}{\langle P\rangle}\frac{d\langle P\rangle}{dz} = \frac{-\frac{c}{(8\pi)^{3/2}}\left(\frac{\omega\mu}{\sigma}\right)^{1/2}\frac{\omega^2}{c^2\gamma^4}\oint dl\left|\frac{\partial E_z}{\partial n}\right|^2}{\frac{\omega k}{\gamma^2}\iint\frac{|E_z|^2}{8\pi}\,dA} \equiv -2\beta$$

$$\therefore \langle P(z)\rangle = \langle P_0\rangle e^{-2\beta z}$$

$$k \to k^{(0)} + \alpha + i\beta$$

read J. sec. 8.6 (Perturbation of b.c.)

Resonant Cavity

Let flat, end conductors be placed at $z = 0, L$ (draw picture)

case 1 TM modes

$$E_z = \psi(x,y)\underbrace{\cos(kz)}_{(e^{ikz}+e^{-ikz})/2} e^{-i\omega t}$$

$$\underset{\sim}{E}_t = -\frac{k \sin kz\, \underset{\sim}{\nabla}_\perp \psi}{\frac{\omega^2}{c^2} - k^2} e^{-i\omega t}$$

$$\underset{\sim}{B}_t = \frac{i\omega}{c}\frac{\cos kz\, \hat{z}\times\underset{\sim}{\nabla}_t \psi}{\frac{\omega^2}{c^2} - k^2} e^{-i\omega t}$$

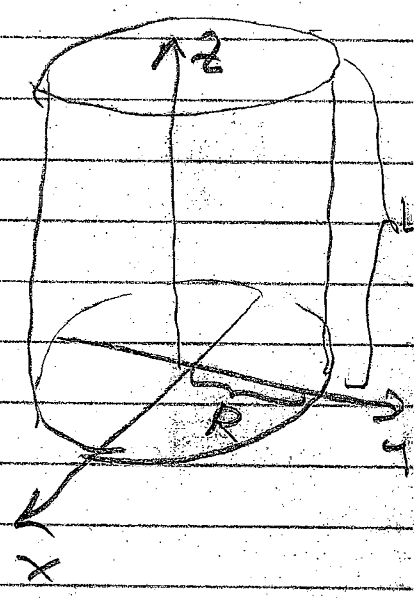
b.c. on end

$0 = \hat{n} \times \underset{\sim}{E} = \hat{n} \times \underset{\sim}{E}_t$ satisfied for $k = \frac{\pi p}{L}$

$0 = \hat{n} \cdot \underset{\sim}{B} = \pm \underbrace{B_z}_{0}$

b.c. on sides require $\psi = 0$

cylindrical cavity



$$\Big[\nabla_t^2 + \underbrace{\left(\frac{\omega^2}{c^2} - k^2\right)}_{\gamma^2}\Big]\psi = 0$$

$$\psi = f_l(r) e^{il\theta}$$

$$\left[\frac{1}{r}\frac{d}{dr} r \frac{d}{dr} - \frac{l^2}{r^2} + \gamma^2\right] f_l(r) = 0$$

$$f_l(r) = J_l(\gamma r)$$

$$0 = f_l(R) \Longrightarrow \gamma R = x_{ln}$$

$$\therefore \quad \frac{\omega^2}{c^2} - \underset{\left(\frac{\pi p}{L}\right)^2}{\overset{\parallel}{k^2}} = \gamma^2 = \frac{x_{ln}^2}{R^2}$$

ω's are specified by cavity geometry

$$\omega_{l,n,p}^2 = c^2\left[\frac{x_{ln}^2}{R^2} + \left(\frac{\pi p}{L}\right)^2\right] = c^2 k_{eff}^2$$

$$E_z = J_l\left[\frac{x_{ln}}{R} r\right] e^{il\theta - i\omega t} \cos\left[\frac{\pi}{L} p z\right]$$

$$\underset{\sim}{E_t} = -\frac{\frac{\pi p}{L}\sin\left(\frac{\pi p z}{L}\right) e^{il\theta - i\omega t}}{\left(x_{lm}/R\right)^2}\left[\frac{il}{r} J_l\left(\frac{x_{ln} r}{R}\right)\hat{\theta} + \frac{x_{ln}}{R} J_l'\left(\frac{x_{ln} r}{R}\right)\hat{r}\right]$$

$$\underset{\sim}{B_t} = \frac{i\omega}{c}\frac{\cos\left(\frac{\pi p z}{L}\right) e^{il\theta - i\omega t}}{\left(x_{lm}/R\right)^2}\left[-\frac{il}{r} J_l\left(\frac{x_{ln}}{R} r\right)\hat{r} + \frac{x_{ln}}{R} J_l'\left(\frac{x_{ln} r}{R}\right)\hat{\theta}\right]$$

$\tau_c/\hbar$ about damping rate of cavity mode

Θ of the cavity, eg walls, radiation

Homework

(37) Repeat for T.E fields

Canonical coordinates for the radiation field in vacuum

$$\underset{\sim}{B} = \underset{\sim}{\nabla}\times\underset{\sim}{A}\ ,\quad \underset{\sim}{E} = -\underset{\sim}{\nabla}\phi - \frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t}$$

use Coulomb gauge: $\underset{\sim}{\nabla}\cdot\underset{\sim}{A} = 0$

use periodic b.c. in cubic region of volume L^3

$$\underset{\sim}{A}(\underset{\sim}{r},t) = \sum_{\underset{\sim}{k}} \underset{\sim}{A}_{\underset{\sim}{k}}(t)\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}\ ;\quad \underset{\sim}{k} = \frac{2\pi}{L}\underset{\sim}{n}$$

$$0 = \underset{\sim}{\nabla}\cdot\underset{\sim}{A} = \sum_{\underset{\sim}{k}} i\underset{\sim}{k}\cdot\underset{\sim}{A}_{\underset{\sim}{k}}\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}$$

$\therefore\ \underset{\sim}{k}\cdot\underset{\sim}{A}_{\underset{\sim}{k}} = 0$ for all $\underset{\sim}{k}$

$$\phi(\underset{\sim}{r},t) = \sum_{\underset{\sim}{k}} \phi_{\underset{\sim}{k}}(t)\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}$$

$$\nabla \cdot \mathbf{A} = 0$$

$$0 = 4\pi\rho = \nabla \cdot \mathbf{E} = -\nabla^2\phi = -\sum_{k} k^2 \phi_k e^{i\mathbf{k}\cdot\mathbf{r}}$$

$\therefore \phi_k = 0$ for $\mathbf{k} \neq 0$

$$\phi = \phi_{\mathbf{k}=0} = \text{const.} = 0$$

$$\underbrace{\nabla\times\nabla\times\mathbf{A}}_{\nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}} = \nabla\times\mathbf{B} = +\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} = -\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}$$

$$\nabla\cdot\mathbf{A} = 0$$

$$0 = \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} = -\frac{1}{c^2}\sum_{\mathbf{k}} \underbrace{\left[\ddot{\mathbf{A}}_{\mathbf{k}} + k^2c^2\mathbf{A}_{\mathbf{k}}\right]}_{=0} e^{i\mathbf{k}\cdot\mathbf{r}}$$

let $\mathbf{A}_{\mathbf{k}}(t) \propto e^{\pm i\omega_k t}$ $\qquad \omega_k = |\mathbf{k}|c$

reality condition

$$\mathbf{A}_{\mathbf{k}}^* = \mathbf{A}_{-\mathbf{k}}$$

Reality condition is explicitly satisfied for

$$\underset{\sim}{A}_{\underset{\sim}{k}} = \underset{\sim}{a}_{\underset{\sim}{k}} + \underset{\sim}{a}^*_{-k}$$

where

$$\underset{\sim}{a}_{\underset{\sim}{k}} \propto e^{-i\omega_k t}$$

$$\underset{\sim}{A} = \sum_k (\underset{\sim}{a}_k + a^*_{-k}) e^{i\underline{k}\cdot\underline{r}} = \sum_{\underline{k}} \underbrace{\underset{\sim}{a}_k e^{i\underline{k}\cdot\underline{r}} + a^*_k e^{-i\underline{k}\cdot\underline{r}}}_{\text{real traveling wave in direction } +\underline{k}}$$

$$\underset{\sim}{E} = -\frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} = -\frac{1}{c}\sum_{\underset{\sim}{k}} -ikc\,\underset{\sim}{a}_k e^{i\underline{k}\cdot\underline{r}} + ikc\, a^*_k e^{-i\underline{k}\cdot\underline{r}}$$

$$\underset{\sim}{E} = \sum_k ik\,\underline{a}_k e^{i\underline{k}\cdot\underline{r}} - ik\,\underset{\sim}{a}^*_k e^{-i\underline{k}\cdot\underline{r}}$$

$$\underline{B} = \underline{\nabla}\times\underline{A} = \sum_{\underline{k}} i\underline{k}\times\underline{a}_k e^{i\underline{k}\cdot\underline{r}} - i\underline{k}\times a^*_k e^{-i\underline{k}\cdot\underline{r}}$$

$$W = \iiint_{V = L^3} d^3\underset{\sim}{r} \left(\frac{E^2 + B^2}{8\pi} \right) = \sum_{\underset{\sim}{k}} \frac{V}{8\pi} \left(2k^2 |\underset{\sim}{a}_k|^2 + 2|\underset{\sim}{k} \times \underset{\sim}{a}_k|^2 \right)$$

$$= \sum_{\underset{\sim}{k}} \frac{V}{2\pi} k^2 |\underset{\sim}{a}_k|^2 = \sum_{\underset{\sim}{k}} \mathcal{E}_{\underset{\sim}{k}}$$

$$\underset{\sim}{P} = \iiint_V d^3\underset{\sim}{r} \, \frac{\underset{\sim}{E} \times \underset{\sim}{B}}{4\pi c} = \sum_{k} \underbrace{\frac{\underset{\sim}{k}}{k} \frac{\mathcal{E}_k}{c}}_{\underset{\sim}{k} \frac{\mathcal{E}_k}{\omega_k}}$$

$$\underset{\sim}{q}_k \equiv \sqrt{\frac{V}{8\pi c^2}} \left(\underset{\sim}{a}_k + \underset{\sim}{a}_k^* \right) \leftarrow \text{real}$$

$$\underset{\sim}{P}_k \equiv -i\omega_k \sqrt{\frac{V}{8\pi c^2}} \left(\underset{\sim}{a}_k - \underset{\sim}{a}_k^* \right) = \dot{\underset{\sim}{q}}_k$$

$$\dot{\underset{\sim}{P}}_k = \ddot{\underset{\sim}{q}}_k = -\omega_k^2 \underset{\sim}{q}_k$$

$$H = \sum_{\underset{\sim}{k}} \underbrace{\frac{\underset{\sim}{p}_k^2}{2} + \frac{\omega_k^2}{2} \underset{\sim}{q}_k^2}_{\mathcal{E}_k}$$

$\underset{\sim}{q}_k, \underset{\sim}{p}_k \perp \underset{\sim}{k}$

for each $\underset{\sim}{k}$ there are two polarizations

$$\underset{\sim}{p}_k^2 = \sum_{\alpha=1,2} p_{k\alpha}^2 \quad , \quad \underset{\sim}{q}_k^2 = \sum_{\alpha=1,2} q_{k,\alpha}^2$$

$$\therefore H = \sum_{\underset{\sim}{k},\alpha} \frac{p_{k\alpha}^2}{2} + \frac{\omega_k^2}{2} q_{\underset{\sim}{k},\alpha}^2 = \sum_{\underset{\sim}{k},\alpha} H_{\underset{\sim}{k},\alpha}$$

Discuss

1. Quantization as harmonic oscillators $[q_{\underset{\sim}{k},\alpha}, p_{\underset{\sim}{k}\alpha}] = i\hbar$
   mention state vectors $|N_1, N_2 \text{---} \rangle$
2. Interpretation of photons through 2nd quantization
3. Interaction with wall

See Vol 4 of L+L series (Berestetskii, Lifshitz, Pitaevskii)
& Quantum Theory of Radiation by Heitler

2/27/03

Radiation J. Chapter 9
L&L Vol 2 chapters 7 & 8

$$\underbrace{\nabla \times \nabla \times A}_{\nabla \nabla \cdot A - \nabla^2 A} = \nabla \times B = \frac{4\pi}{c}\underbrace{J}_{\text{total current}} + \underbrace{\frac{1}{c}\frac{\partial E}{\partial t}}_{\frac{1}{c}\frac{\partial}{\partial t}\left[-\frac{1}{c}\frac{\partial A}{\partial t} - \nabla \phi\right]}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A - \nabla\underbrace{\left(\nabla \cdot A + \frac{1}{c}\frac{\partial \phi}{\partial t}\right)}_{=0 \text{ for Lorentz gauge}} = -\frac{4\pi}{c}J$$

likewise

$$0 = \frac{1}{c}\frac{\partial}{\partial t}\left(\nabla \cdot A + \frac{1}{c}\frac{\partial \phi}{\partial t}\right) = \nabla \cdot (+E + \nabla \phi) + \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = -\nabla \cdot E = -4\pi\rho$$

Green's function for wave equation in free space (see J. page 245)

discuss

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]G(r,t|r',t') = -4\pi\delta(r-r')\delta(t-t')$$

$$G(r,t|r',t') = \frac{\delta\left[t - t' - \frac{|r-r'|}{c}\right]}{|r-r'|}$$ retardation

$$\therefore \underline{A}(\underline{r},t) = \int d^3\underline{r}' \int dt' \, \frac{\underline{J}(\underline{r}',t')}{c} \, \frac{\delta\left[t-t'-\frac{|\underline{r}-\underline{r}'|}{c}\right]}{|\underline{r}-\underline{r}'|}$$

$$\varphi(\underline{r},t) = \int d^3\underline{r}' \int dt' \, \rho(\underline{r}',t') \, \frac{\delta\left[t-t'-\frac{|\underline{r}-\underline{r}'|}{c}\right]}{|\underline{r}-\underline{r}'|}$$

quasi-static limit when J and ρ change by only small amount during time $|\underline{r}-\underline{r}'|/c$.

Oscillating fields (all fields $\sim e^{-i\omega t}$)

$\underline{J}(\underline{r},t) = \mathrm{Re}\, \underline{J}(\underline{r}) e^{-i\omega t}$

$\underline{A}(\underline{r},t) = \mathrm{Re}\, \underline{A}(\underline{r}) e^{-i\omega t}$

etc. (all fields $\sim e^{-i\omega t}$)

$$\therefore \underline{A}(\underline{r})\cancel{e^{-i\omega t}} = \int d^3\underline{r}' \, \frac{\underline{J}(\underline{r}')}{c} \, \frac{e^{i\frac{\omega}{c}|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \cancel{e^{-i\omega t}}$$

($\frac{\omega}{c} = k$)

$$\varphi(\underline{r}) = \int d^3\underline{r}' \, \rho(\underline{r}') \, \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}$$

$k = \omega/c$

Multipole expansion for small source

let d = characteristic length scale of source

assume $d \ll c/\omega$, r

(1) near (static) zone

$$d \ll r \ll c/\omega$$

$$\frac{1}{|\underline{r}-\underline{r}'|} \simeq \frac{1}{r} - \underline{r}'\cdot\nabla\frac{1}{r} + \frac{1}{2}x'_\alpha x'_\beta \frac{\partial^2}{\partial x_\alpha \partial x_\beta}\frac{1}{r}$$

$$e^{ik|\underline{r}-\underline{r}'|} \simeq 1$$

previous result

(2) far (radiation) zone

$$d \ll c/\omega \ll r$$

$$|\underline{r}-\underline{r}'| \simeq r - \hat{\underline{r}}\cdot\underline{r}' \quad \text{expansion in } \frac{r'}{r}$$

$$e^{ik|\underline{r}-\underline{r}'|} \simeq e^{ikr - ik\hat{r}\cdot\underline{r}'} \qquad \text{exp in } (kr')$$

$$\simeq e^{ikr}\sum_{n=0}^{\infty}\frac{1}{n!}(ik\hat{r}\cdot\underline{r}')^n$$

$$\underset{\sim}{A}(\underline{r}) \simeq \frac{e^{ikr}}{cr} \sum_{n=0}^{\infty} \int d^3\underline{r}'\, \underset{\sim}{J}(\underline{r}') \frac{(-ik\hat{r}\cdot\underline{r}')^n}{n!}$$

using $kr >> 1$

$$\underset{\sim}{B}(\underline{r}) = \underset{\sim}{\nabla} \times \underset{\sim}{A}(\underline{r}) \simeq ik\hat{r} \times \underset{\sim}{A}(\underline{r})$$

outside of source

$$-ik\underset{\sim}{E} = \underset{\sim}{\nabla} \times \underset{\sim}{B} \simeq (ik)^2 \hat{r} \times \hat{r} \times \underset{\sim}{A}(\underline{r})$$

$$\underset{\sim}{E}(\underline{r}) \simeq -ik\, \hat{r} \times \hat{r} \times \underset{\sim}{A}(\underline{r})$$

start

electric dipole radiation $(n=0)$

n = 0 term vanishes in expansion of ... $\int \rho(\underline{r})\, d^3\underline{r} = 0$ for localized oscillating charge dist

$$\underset{\sim}{A}(\underline{r}) \simeq \frac{e^{ikr}}{cr} \int d^3\underline{r}'\, \underset{\sim}{J}(\underline{r}')$$

$$-i\omega \rho(\underline{r}') + \underset{\sim}{\nabla}' \cdot \underset{\sim}{J}(\underline{r}') = 0$$

$$+i\omega \underbrace{\int d^3\underline{r}'\, \underline{r}' \rho(\underline{r}')}_{\underset{\sim}{p}} = \int d^3\underline{r}'\, \underline{r}'\, \underset{\sim}{\nabla}' \cdot \underset{\sim}{J} = -\int d^3\underline{r}'\, \underset{\sim}{J}(\underline{r}')$$

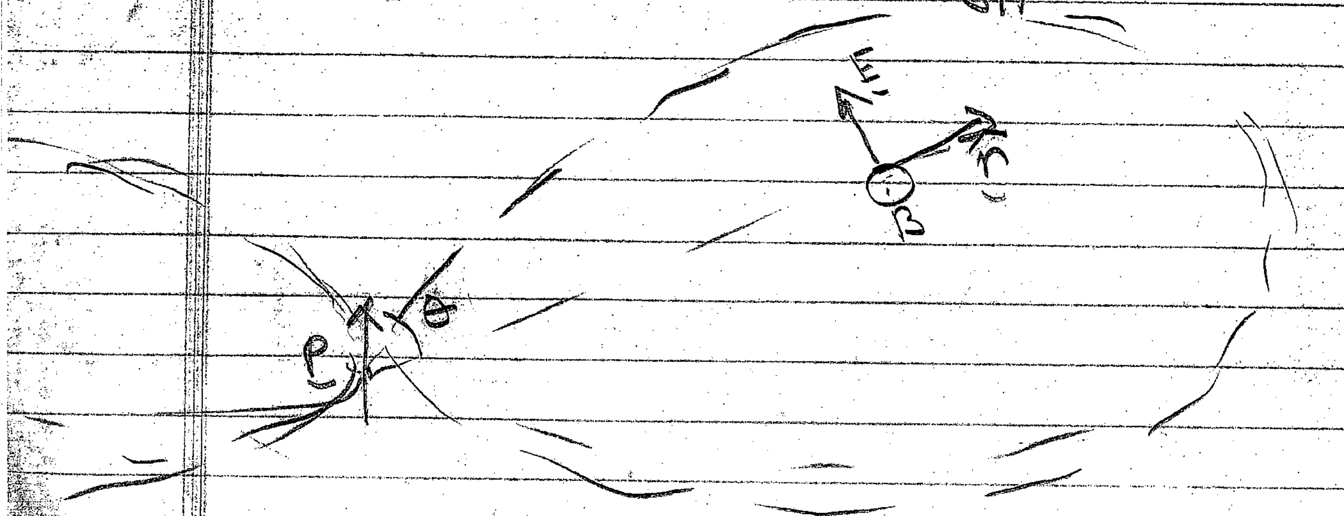
$$\therefore\ \underset{\sim}{A}(\underset{\sim}{r}) = -ik\,\underset{\sim}{p}\,\frac{e^{ikr}}{r}$$

$$\underset{\sim}{B}(\underset{\sim}{r}) = k^2\,\hat{r}\times\underset{\sim}{p}\,\frac{e^{ikr}}{r}$$

$$\underset{\sim}{E}(\underset{\sim}{r}) = \cancel{-}\,\hat{r}\times\underset{\sim}{B}$$

$$\langle\underset{\sim}{S}\rangle = \frac{1}{2}\mathrm{Re}\,\frac{c}{4\pi}\,\underset{\sim}{E}\times B^{*} = \hat{r}\,\frac{ck^4}{8\pi}\,\frac{|\hat{r}\times\underset{\sim}{p}|^2}{r^2}$$

$$\frac{d\langle P\rangle}{d\Omega} = r^2\,\hat{r}\cdot\langle\underset{\sim}{S}\rangle = \frac{ck^4}{8\pi}\,|\underset{\sim}{p}|^2\sin^2\theta$$



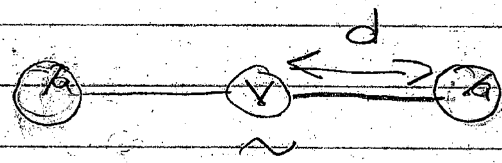
newer start

5/29/03

$$\langle P \rangle = \int d\Omega \, \frac{d\langle P\rangle}{d\Omega} = \frac{c k^4}{8\pi} |p|^2 \underbrace{\int_0^{\pi} 2\pi \sin\theta \, d\theta \, \sin^2\theta}_{2\pi \underbrace{\int_{-1}^{1} d(\underbrace{\cos\theta}_{x}) \left[1 - (\underbrace{\cos\theta}_{x^2})^2\right]}_{\underbrace{x - \frac{x^3}{3}\Big|_{-1}^{1}}_{\frac{4}{3}}}}$$

$$\langle P \rangle = \frac{c k^4 |p|^2}{3} = \frac{\omega^4 |p|^2}{3 c^3}$$

example



$$a \ll d \ll c/\omega$$

$$V(t) = R_e \, V_0 \, e^{-i\omega t}$$

static approx $\rightarrow$ $$2 \frac{Q(t)}{a} = V(t)$$

$$\underset{\sim}{P}(t) = \int d^3r\, \underset{\sim}{r}\, \mathrm{Re}\, \rho(r) e^{-i\omega t}$$

$$= \mathrm{Re}\, \cancel{2} \underset{\sim}{d}\, \frac{a v_0}{\cancel{2}} e^{-i\omega t}$$

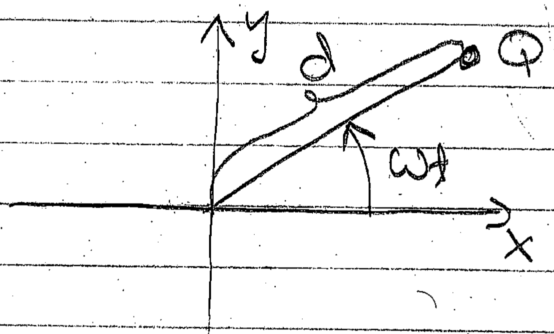
$$\langle P \rangle = \frac{\omega^4}{3} \frac{|\underset{\sim}{P}|^2}{c^3} = \frac{\omega^4 d^2 a^2 v_0^2}{3c^3}$$

$$I(t) = \mathrm{Re}\, \dot{Q} = \mathrm{Re}\, \underbrace{-i\omega \frac{a v_0}{2}}_{I_0} e^{-i\omega t}$$

$$\langle P \rangle = \underbrace{\frac{\omega^2 d^2 8}{3c^3}}_{R_{rad}} \frac{I_0^2}{2}$$

radiation resistance $\rightarrow R_{rad}$

example J. 9.1



note that $\rho(\underset{\sim}{r},t) \neq \mathrm{Re}\, \rho(\underset{\sim}{r}) e^{-i\omega t}$

first calculate $\underset{\sim}{P}(t)$ then Fourier analyze

$$P_x(t) = dQ\cos\omega t \quad , \quad P_y(t) = dQ\sin\omega t$$

$$\underset{\sim}{P}(t) = \mathrm{Re}\, \underbrace{dQ(\hat{x}+i\hat{y})}_{\underset{\sim}{P}} e^{-i\omega t}$$

$$\therefore \langle P \rangle = \frac{\omega^4}{3c^3}|\underset{\sim}{P}|^2 = \frac{\omega^4}{3c^3} Q^2 d^2 2$$

$$= \frac{2}{3}\frac{Q^2}{c^3}\underbrace{(\omega^2 d)^2}_{a^2}$$ compare to Larmor formula

note that $d \ll \frac{c}{\omega} \iff v = \omega d \ll c$

magnetic dipole & electric quadrupole ($n=1$)

$$\underset{\sim}{A}(\underset{\sim}{r}) = -\frac{e^{ikr}}{cr} ik \int d^3\underset{\sim}{r}' \, \hat{r}\cdot\underset{\sim}{r}' \, \underset{\sim}{J}(\underset{\sim}{r}')$$

let

$$(\hat{r}\cdot\underset{\sim}{r}')\underset{\sim}{J} = -\frac{1}{2}\hat{r}\times(\underset{\sim}{r}'\times\underset{\sim}{J}) + \frac{1}{2}\Big[(\hat{r}\cdot\underset{\sim}{r}')\underset{\sim}{J} + (\hat{r}\cdot\underset{\sim}{J})\underset{\sim}{r}'\Big]$$

magnetic dipole

$$\underset{\sim}{A} = \frac{ik}{r} e^{ikr} \underset{\sim}{r} \times \underbrace{\frac{1}{2c}\int d^3\underset{\sim}{r}' \, \underset{\sim}{r}'\times\underset{\sim}{J}}_{\underset{\sim}{m}}$$

$$\underset{\sim}{B} = \underset{\sim}{\nabla}\times\underset{\sim}{A} \simeq -k^2 \frac{e^{ikr}}{r} \hat{r}\times(\hat{r}\times\underset{\sim}{m})$$

$$\underset{\sim}{E} = +\frac{1}{ik}\underset{\sim}{\nabla}\times\underset{\sim}{B} \simeq -k^2 \frac{e^{ikr}}{r}(\hat{r}\times\underset{\sim}{m})$$

$$\frac{d\langle P\rangle}{d\Omega} = \frac{c}{8\pi} k^4 \underbrace{|\hat{r}\times \underline{m}|^2}_{|m|^2 \sin^2\theta}$$

electric quadrupole

$$\int d^3\underline{r}' \frac{1}{2}\left[(\hat{r}\cdot\underline{r}')\underline{J} + (\hat{r}\cdot\underline{J})\underline{r}'\right] = \frac{1}{2}\int d^3\underline{r}'\, \underline{J}\cdot\frac{\partial}{\partial \underline{r}'}\, \underline{r}'(\hat{r}\cdot\underline{r}'$$

$$\text{''} = -\frac{1}{2}\int d^3\underline{r}' \underbrace{\left(\frac{\partial}{\partial \underline{r}'}\cdot \underline{J}\right)}_{+i\omega\rho} \underline{r}'(\hat{r}\cdot\underline{r}')$$

$$\text{''} = -\frac{i\omega}{2}\int d^3\underline{r}'\, \rho(\underline{r}')\,(\hat{r}\cdot\underline{r}')\underline{r}'$$

$$\underline{A} = -\frac{k^2}{2r} e^{ikr}\int d^3\underline{r}'\, \rho(\underline{r}')\,(\hat{r}\cdot\underline{r}')\,\underline{r}'$$

$$\underset{\sim}{B} = \underset{\sim}{\nabla} \times \underset{\sim}{A} = \frac{-ik^3}{2r} e^{ikr} \, \hat{r} \times \underbrace{\int d^3\underset{\sim}{r}' \rho(\underline{r}') (\hat{r} \cdot \underline{r}') \underset{\sim}{r}'}_{\frac{1}{3} \hat{r} \times \underset{\approx}{Q} \cdot \hat{r}}$$

$$Q_{\alpha\beta} = \int d^3\underline{r}' \left[3 x_\alpha' x_\beta' - \delta_{\alpha\beta} r'^2 \right] \rho(\underline{r}')$$

$$\cancel{Q_\alpha = Q_{\alpha\beta} n_\beta}$$

vanishes under dot + cross product with $\hat{r}$

$$\frac{d\langle P \rangle}{d\Omega} = \frac{c}{8\pi} \frac{k^6}{36} \left| \hat{r} \times \underset{\approx}{Q} \cdot \hat{r} \right|^2$$

Jackson shows that

$$\cancel{\int d\Omega \left| \hat{r} \times \underset{\approx}{Q} \cdot \hat{r} \right|^2 = 4\pi \cdot \frac{2}{15}}$$

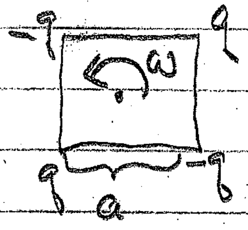
$$\langle P \rangle = \frac{c k^6}{360} \sum_{\alpha,\beta} |Q_{\alpha\beta}|^2$$

start

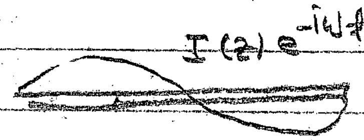
Homework

(28) J. 9.2

skip ~~(34) J. 9.5~~

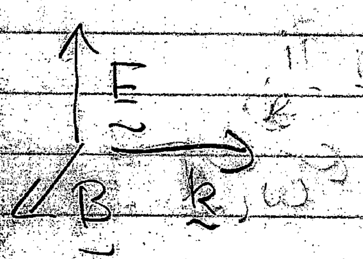


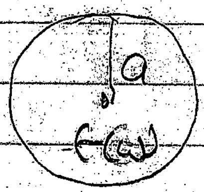
$a\omega \ll c$



not multipole approx.

Scattering by small dielectric sphere





$$a \ll k^{-1} = \frac{c}{\omega}$$

solve for polarization of sphere using static approx

dielectric sphere in uniform external field develops dipole moment

$$\underset{\sim}{p} = \frac{(\epsilon - 1)}{(\epsilon + 2)} a^3 \underline{E}$$

electric dipole radiation

$$\frac{d\langle P\rangle}{d\Omega} = \frac{c}{8\pi} k^4 |\underline{p} \times \hat{r}|^2$$

$$= \frac{c}{8\pi} k^4 \left(\frac{\epsilon-1}{\epsilon+2}\right)^2 a^6 |\underline{E} \times \hat{r}|^2$$

$|E|^2 \sin^2\theta$

differential scattering cross section

$$\frac{d\langle P\rangle}{d\Omega} = \frac{d\sigma}{d\Omega} \langle I \rangle$$

$$\langle I \rangle = \frac{c}{8\pi} |E|^2$$

$$\frac{d\sigma}{d\Omega} = k^4 a^6 \left(\frac{\epsilon-1}{\epsilon+2}\right)^2 |\hat{r} \times \hat{E}|^2$$

$\sin^2\theta$

Rayleigh's explanation of why sky is blue

Einstein – Smoluchowski explanation

Benford's seminar + nova program

Ramanathan colloquium

Start

Homework

(29) A plane polarized E+M wave,

$$\underline{E}(\underline{r},t) = \mathrm{Re}\,\underline{E}\, e^{i\underline{k}\cdot\underline{r} - i\omega t}, \quad \underline{B}(\underline{r},t) = \text{---}$$

is incident on a small conducting sphere (i.e., $a \ll c/\omega$ but $\delta \ll a$)

a. Calculate induced $\underline{p}$ and $\underline{m}$.
(Hint: use quasi-static approx.)

b. Calculate scattering cross section (Hint: don't forget interference between electric and magnetic dipole fields)

Raleigh's Theory of why the sky is blue and sunset red

Einstein - Smoluchowski explanation