Physics 4E Quiz 1 Solutions Spring 2011

Problem 1

a) Wien's law: $\lambda_m = \dfrac{hc}{4.96\,kT}$; $k = \dfrac{1}{11{,}600}\dfrac{eV}{K} \Rightarrow \boxed{kT = 1\,eV} \Rightarrow$

$$\lambda_m = \frac{12{,}400\,eV\text{Å}}{4.96\,eV} \Rightarrow \boxed{\lambda_m = 2500\,\text{Å}}$$

(b) $$\bar{E}(\lambda_m) = \frac{hc}{\lambda_m}\frac{1}{e^{hc/\lambda_m kT}-1} = kT\cdot\frac{hc}{\lambda_m kT}\frac{1}{e^{hc/\lambda_m kT}-1}$$

Using that $\dfrac{hc}{\lambda_m kT} = 4.96 \Rightarrow \bar{E}(\lambda_m) = \dfrac{4.96}{e^{4.96}-1}\cdot kT = 0.035\,kT$

$\Rightarrow \boxed{\bar{E}(\lambda_m) = 0.035\,eV}$ since $kT = 1\,eV$ here

(c) For very large λ we will have $\bar{E}(\lambda) \longrightarrow kT = 1\,eV$.

E.g. for $\lambda = 10\lambda_m$: $\bar{E}(\lambda) = \dfrac{hc}{10\lambda_m}\dfrac{1}{e^{hc/10\lambda_m kT}-1} =$

$= \dfrac{0.496}{e^{0.496}-1}kT = 0.77\,kT$, not large enough.

For $\lambda = 100\lambda_m$, $\bar{E}(\lambda) = \dfrac{0.0496}{e^{0.0496}-1} = 0.975\,kT = 0.975\,eV > 0.9\,eV$

So a possible range is $\boxed{100\lambda_m < \lambda < 101\lambda_m}$

or $\boxed{250{,}000\,\text{Å} < \lambda < 251{,}000\,\text{Å}}$

(d) Power emitted is proportional to area A. area is proportional to radius^2.
So power $(R/2) = 1/4$ power (R). Power emitted per unit area is $R = \sigma T^4$. So for smaller sphere, we need
$(T')^4 = 4T^4 \Rightarrow T' = 4^{1/4}T = \sqrt{2}T \Rightarrow \boxed{T' = 16{,}405\,°K}$

Problem 2

(a) $E_e = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$ by energy conservation. $\lambda_2 = 0.9\,\mathring{A} \Rightarrow$

$$\frac{hc}{\lambda_1} = E_e + \frac{hc}{\lambda_2} \Rightarrow \frac{1}{\lambda_1} = \frac{E_e}{hc} + \frac{1}{\lambda_2} = \frac{188\,eV}{12,400\,eV\,\mathring{A}} + \frac{1}{0.9\,\mathring{A}} = \frac{1.1263}{\mathring{A}}$$

$$\Rightarrow \boxed{\lambda_1 = 0.8879\,\mathring{A}}$$

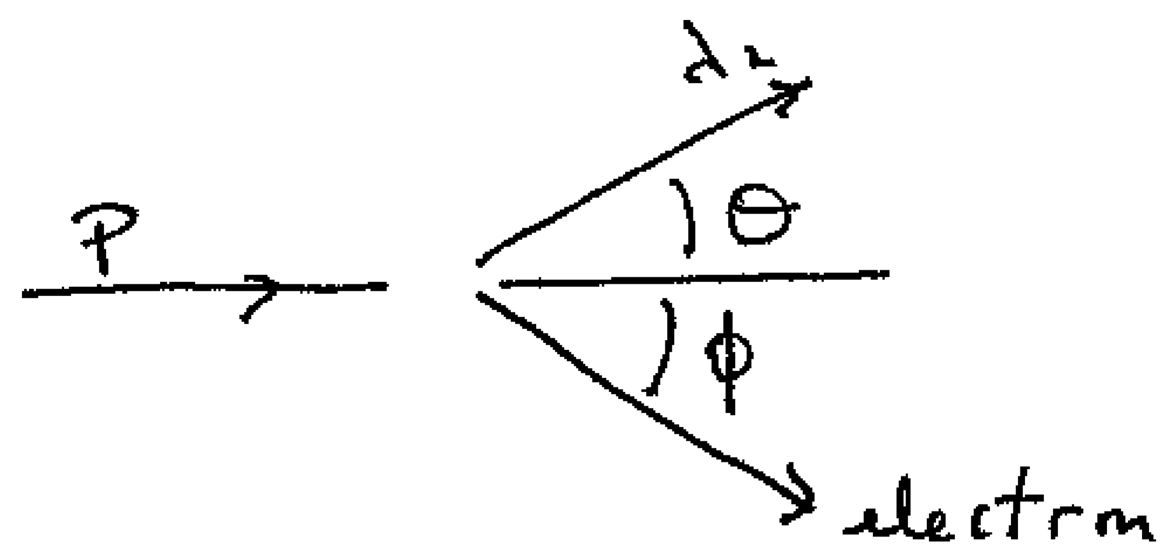
(b) $\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1-\cos\theta) \Rightarrow 1-\cos\theta = \frac{\lambda_2-\lambda_1}{h/m_e c} \Rightarrow$

$$\cos\theta = 1 - \frac{\lambda_2-\lambda_1}{h/m_e c} = 1 - \frac{0.9-0.8879}{0.0243} = 0.5 \Rightarrow \boxed{\theta = 60^\circ}$$

$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

(c)



$$\begin{aligned} P_1 &= P_e\cos\phi + P_2\cos\theta \\ 0 &= P_e\sin\phi - P_2\sin\theta \end{aligned} \Rightarrow \begin{aligned} P_e\cos\phi &= P_1 - P_2\cos\theta \\ P_e\sin\phi &= P_2\sin\theta \end{aligned} \Rightarrow$$

$$\Rightarrow \tan\phi = \frac{P_2\sin\theta}{P_1 - P_2\cos\theta} = \frac{\frac{h}{\lambda_2}\sin\theta}{\frac{h}{\lambda_1} - \frac{h}{\lambda_2}\cos\theta} = \frac{\frac{hc}{\lambda_2}\sin\theta}{\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}\cos\theta} =$$

$$= \frac{\frac{\cancel{12,400}}{0.9}\cdot\frac{\sqrt{3}}{2}}{\frac{\cancel{12,400}}{0.8879} - \frac{\cancel{12,400}}{0.9}\cdot\frac{1}{2}} = \frac{\frac{1}{0.9}\frac{\sqrt{3}}{2}}{\frac{1}{0.8879} - \frac{1}{1.8}} = 1.686$$

$$\Rightarrow \boxed{\phi = 59.3^\circ}$$

Problem 3

$r_d = \frac{k q_\alpha Q}{E_{\alpha,in}} \Rightarrow E_{\alpha,in} = \frac{k q_\alpha Q}{r_d}$; for $r_d = R$, $E_{\alpha,in} = \frac{k q_\alpha Q}{R} \Rightarrow$

$$E_{\alpha,in} = \frac{14.4\,\text{eV}\,\text{Å} \times 2 \times 13}{3.6 \times 10^{-5}\,\text{Å}} = 1.04 \times 10^{7}\,\text{eV} \Rightarrow \boxed{E_{\alpha,in} = 10.4\,\text{MeV}}$$

(b) Use: $b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot\frac{\theta}{2} = \frac{k q_\alpha Q}{2 E_{\alpha,in}} \cot\frac{\theta}{2}$

Using that for this $E_{\alpha,in}$, $\frac{k q_\alpha Q}{E_{\alpha,in}} = R \Rightarrow$

$\Rightarrow b = \frac{R}{2} \cot\frac{\theta}{2}$, for $\theta = 90°$, $\cot\frac{\theta}{2} = \frac{\cos 45°}{\sin 45°} = 1 \Rightarrow$

$$\boxed{b = R/2}$$

(c) From Rutherford's law, $\Delta N(45°) = \Delta N(90°) \cdot \frac{\sin^4 90/2}{\sin^4 \frac{45}{2}} =$

$$= \Delta N(90°) \cdot \frac{\sin^4 45°}{\sin^4 22.5°} = \Delta N(90°) \cdot \frac{0.25}{0.0214} = \Delta N(90°) \cdot 11.66$$

So if 100 particles per second scatter at 90°, $\boxed{1{,}166 \text{ scatter at } 45°.}$

$$b(45°) = b(90°) \cdot \frac{\cot 22.5°}{\cot 45°} = b(90°) \times 2.414 = \frac{R}{2} \times 2.414$$

$$\Rightarrow \boxed{b(45°) = 1.21\,R}$$

(d) There will be deviations from Rutherford's law at large θ for $\boxed{E_{\alpha,in} > 10.4\,\text{MeV}}$

since some α-particles will penetrate the nucleus.