Physics 4E — Quiz 2 Solutions — Spring 2011

Problem 1

(a) $E_n - E_m = E_0 Z^2\left(\frac{1}{m^2}-\frac{1}{n^2}\right) = \frac{hc}{\lambda_{mn}}$; $240 Å \leq \lambda_{mn} \leq 480 Å$

with $E_0 = 13.6\,eV$, $Z=2$ in He^+, $hc = 12{,}400\,eVÅ$ $\Rightarrow$

$$\lambda_{mn} = \frac{hc}{E_0 Z^2}\,\frac{1}{\frac{1}{m^2}-\frac{1}{n^2}} = \frac{227.94\,Å}{\frac{1}{m^2}-\frac{1}{n^2}}$$

Absorption: initial state is $m=1$. So

$m=1, n=2$: $\frac{1}{m^2}-\frac{1}{n^2} = \frac{3}{4} \Rightarrow \lambda = 303.92\,Å$

$m=1, n=3$: $\frac{1}{m^2}-\frac{1}{n^2} = \frac{8}{9} \Rightarrow \lambda = 256.43\,Å$

$m=1, n=4$: $\frac{1}{m^2}-\frac{1}{n^2} = \frac{15}{16} \Rightarrow \lambda = 243.14\,Å$

3 absorption lines

$m=1, n=5$: $\frac{1}{m^2}-\frac{1}{n^2} = \frac{24}{25} \Rightarrow \lambda = 237.44$ outside range

(b)

Emission: initial state will be $n=2$ or $n=3$ or $n=4$.

$n=4$ to $m=3$: $\frac{1}{3^2}-\frac{1}{4^2} = \frac{7}{144} \Rightarrow \lambda = 4689\,Å$

$n=4$ to $m=2$: $\frac{1}{2^2}-\frac{1}{4^2} = \frac{3}{16} \Rightarrow \lambda = 1216\,Å$

$n=3$ to $m=2$: $\frac{1}{2^2}-\frac{1}{3^2} = \frac{5}{36} \Rightarrow \lambda = 1641\,Å$

plus the same 3 lines as in absorption: 4 to 1, 3 to 1 and 2 to 1

~~6~~ emission lines

(c) Longest wavelength absorbed was $\lambda = 303.92\,Å = \frac{hc}{E_0 Z^2}\cdot\frac{4}{3}$

$E_0 = C\cdot m \rightarrow E_0 . C . \mu = C . m . \frac{1}{1+m/M}$

$\Rightarrow \lambda \rightarrow \lambda(1+m/M) = \lambda(1+1/(4\times 1836)) = \lambda(1+0.00014) \equiv \lambda'$

$\Rightarrow \boxed{\lambda' = 303.96\,Å}$, $\boxed{\lambda' - \lambda = 0.04\,Å}$

Problem 2

(a) $L = m_e v r = n\hbar$, $n = 1$ (ground state)

$$r = \frac{a_0}{Z} \Rightarrow m_e \cdot v \cdot \frac{a_0}{Z} = \hbar \Rightarrow Z = \frac{m_e v a_0}{\hbar} \Rightarrow$$

$$Z = \frac{\cancel{m_e} \cdot \hbar^2}{\cancel{m_e} k e^2 \cancel{\hbar}} \cdot v = \frac{\hbar}{ke^2} v = \frac{\hbar c}{ke^2} \frac{v}{c} = 137.05 \times 0.3 = 41.1$$

$$\Rightarrow \boxed{Z = 41} \qquad \boxed{r = \frac{a_0}{Z} = 0.0129\,\text{Å}}$$

(b) $\Delta x \Delta p \sim \hbar \Rightarrow \Delta p \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{r}$

$$E_{kin} = \frac{\overline{p^2}}{2m_e} = \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2} = \frac{3.81\,\text{eV}\,\text{Å}^2}{(0.0129\,\text{Å})^2} = 22,887\,\text{eV}$$

$$\boxed{E_{kin} = 22,887\,\text{eV}}$$

(c) $E_{pot} = -\frac{ke^2 Z}{r} = \frac{-14.4\,\text{eV}\,\text{Å} \times 41}{0.0129\,\text{Å}} = \boxed{-45,767\,\text{eV}}$

$$\boxed{E_{tot} = E_{kin} + E_{pot} = -22,880\,\text{eV}}$$

(d) If $r = 0.0129\,\text{Å}/2 = 0.00645\,\text{Å}$

$E'_{kin} = 4 \times 22,887\,\text{eV} = 91,548\,\text{eV}$

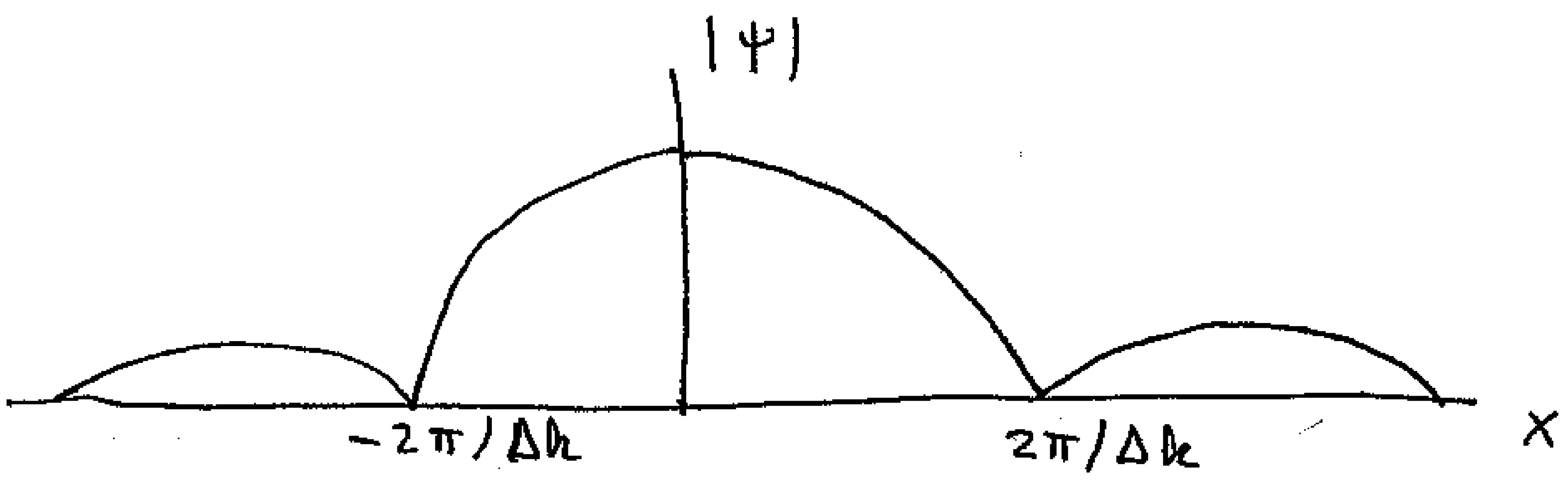
$E'_{pot} = E_{pot} \times 2 = -91,534\,\text{eV}$

$\boxed{E_{tot} = E_{kin} + E_{pot} \sim 0, \text{ much larger than for } r = 0.0129\,\text{Å}}$

Problem 3

$$\psi(x,t=0) = C\int_{k_0-\frac{\Delta k}{2}}^{k_0+\Delta k/2} dk\, e^{ikx} = \frac{C}{ix} e^{ik_0x}\left(e^{i\frac{\Delta k}{2}x} - e^{-i\frac{\Delta k}{2}x}\right) \Rightarrow$$

$$\Rightarrow \psi(x,t=0) = 2Ce^{ik_0x}\frac{\sin\frac{\Delta k}{2}x}{x} \Rightarrow \quad |\psi| = 2C\frac{|\sin\frac{\Delta k}{2}x|}{|x|}$$



First zero: $\frac{\Delta k}{2}x = \pi \Rightarrow x = \frac{2\pi}{\Delta k}$

$\Delta x = \frac{4\pi}{\Delta k}$, for $\Delta k = 1\,\mathring{A}^{-1} \Rightarrow \boxed{\Delta x = 12.6\,\mathring{A}}$

(c)

Is this electron relativistic? $p = \hbar k_0 = \hbar \cdot 10\mathring{A}^{-1} = \frac{\hbar c}{c}\cdot 10\mathring{A}^{-1} = \frac{19730\,eV}{c}$

$\Rightarrow pc = 19{,}730\,eV$. Since $pc \ll m_ec^2 = 511{,}000\,eV$, $\boxed{\text{it's not relativistic}}$

then $E = \hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2k_0^2}{2m} \Rightarrow \frac{\omega}{k_0} = \frac{\hbar k_0}{2m} = v_p \Rightarrow$

(d) $\frac{v_p}{c} = \frac{\hbar k_0}{2mc} = \frac{\hbar c}{2mc^2}k_0 = \frac{1973}{2\times 0.511\times 10^6}\times 10 = \boxed{0.019}$

Group velocity: $v_g = \frac{d\omega}{dk} = \frac{\hbar k_0}{m} = 2v_p \Rightarrow \boxed{\frac{v_g}{c} = 0.038}$