Physics 161: Black Holes: Lecture 18: 14 Feb 2011

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18 Death by Black Hole

18.1 The final plunge

We discussed what it looks like for someone standing still in the shell frame as they approach a black hole. Next we ask, what does the observer who free falls into the black hole see. This is more complicated because you have to take into account special relativistic forward beaming and also abberation, that is the effect of the motion of the observer relative to the stars has on viewing angles. The basic result is that as one moves faster and closer to the black hole, everything the shell observer sees gets shifted forward in the direction of motion; that is, what the shell observer sees at 90^0, the free falling observer sees at a larger angle. Thus while the shell observer who approaches the close to the horizon sees black everywhere except directly away from the hole, the free falling observer still sees a little more than half the sky. However, almost all the stars are shifted forward in the direction of motion and form a ring very near the edge of the black hole.

For the free falling observer, the stars are visible from inside the black hole and the final view just before death is all the stars moved into a ring at 90^0, with the black hole filling exactly half the forward sky.

This all is described in more detail in a book called "Exploring Black Holes", by Taylor and Wheeler, pages 5-24 to 5-30. Andrew Hamilton of the University of Colorado has a great web page with explanations and simulated movies of orbiting and plunging into a black hole.

18.2 How do you die when you go into a black hole?

We saw before that near a black hole the gravitational acceleration can be very large. Is that true near all black holes? If someone is in free fall, then locally there is no force of gravity and so it should be very comfortable, just floating in space. That is the equivalence principle. However, this depends on the size of the spacecraft, and of the person being small enough so that locally the metric is Minkowski. We measure the amount of deviation from perfect Minkowski space using the concept of **tidal acceleration**. For someone falling feet first into a black hole this can be defined as the difference in acceleration between their feet and their head. The fact that the Moon's attraction of the oceans differ from the side of the Earth near the Moon to the middle of the Earth gives rise to the tides; which is why this is called the tidal force (or tidal acceleration in GR).

This tidal force is what will eventually kill you. If you are falling in feet first, you can see that since your feet are closer to the hole, they will be pulled in faster than your head. You will experience this as a stretching feeling. In addition since everything falls towards the center of the hole, the angle of falling on your left side will differ slightly from the angle of fall on your right side. Thus you will be squeezed side-to-side. The net result of lengthwise stretching and sideways squashing is called **spaghettification**. Things falling into small black holes get stretched and squeezed into long skinny strands, like a spaghetti noodle!

How do we calculate the tidal acceleration? First consider the Newtonian case here on Earth. The acceleration of gravity is given by

$$g_{\text{Newton}} = -\frac{GM}{r^2} = -\frac{1}{2}\frac{r_S}{r^2},$$

where we use here the Schwarzschild radius of the Earth $r_S = 3\text{km}\ (M_{\text{Earth}}/M_\odot) = 0.90$ cm! This seems to have units of meter^{-1}, so to get it into meters/second^2, we multiply by c^2. Thus $g_{\text{Earth}} = -0.5(0.009/(6.37 \times 10^6)^2)(3 \times 10^8)^2 = 9.8\ \text{m/s}^2$, as expected. Note that if you want you can just remember $r_S(\text{Earth}) = 0.90$ cm, instead of g_{Earth}. The tidal force on Earth is the difference of this acceleration between your head and feet, which we will say are seperated by a distance $\Delta r \approx 2$ meter. For small differences we can use calculus to find $\Delta g = g(r + dr) - g(r) = (dg/dr)dr \approx a_{\text{tide}}\Delta r$. Thus

$$a_{\text{tide}} = \frac{dg}{dr} = \frac{r_S}{r^3},$$

a very simple formula. The difference in acceleration is then $\Delta g = a_{\text{tide}}\Delta r$. For our 2 meter tall person and for the surface of the Earth, this is just $\Delta g = (0.009)(6.37 \times 10^6)^{-3}(2)c^2 = 3 \times 10^{-6}\text{m/s}^2$. This is only about 3×10^{-7} gee's, and you can't even feel the stretching force standing here on the Earth's surface.

Now, let's do the same thing for the Schwarzschild metric. We want to use τ rather than t since we want the experience of the free-faller. We will use r for simplicity, but perhaps we should use some other measure of distance, such as the proper length. We could convert using the formulas above, but for now we only want a rough estimate. Start from the geodesic equation for free falling from infinity, $dr/d\tau = -\sqrt{r_S/r}$, and take the derivative with respect to τ, to find the bulk acceleration, $g = d^2r/d\tau^2 = -\sqrt{r_S}(-1/2)r^{-3/2}dr/d\tau = -\frac{1}{2}r_S/r^2$. Then the tidal acceleration is the derivative of this with respect to r.

$$\Delta g = \frac{r_S}{r^3}c^2\Delta r.$$

This is the same as the Newtonian formula, except we must be careful to use coordinate variable r and proper time τ.

We can now see when someone dies when they fall into a black hole. I'm not sure what acceleration between the head and feet would kill someone, but probably 100 gee's would do it. I've heard that jet pilots black out around 7 gee's. So we can ask how close to a hole before reaching that tidal acceleration. $r_{\text{kill}} = (r_S\Delta r c^2/\Delta g)^{1/3} = r_S^{1/3}[(2\ \text{meter})c^2/((100)(9.8\text{m/s}^2))]^{1/3} = 560\text{km}(r_S/\text{km})^{1/3}$. The kill radius in units of the Schwarzschild radius can be found by dividing both sides of the above equation by r_S, giving:

$$\frac{r_{\text{kill}}}{r_S} = 560\ \text{km}\ (r_S/\text{km})^{-2/3}.$$

Thus for a $3M_\odot$ black hole, the kill radius is about 1200 km, or about 130 times the horizon radius. We see that all the discussion we had before about exploration of a small black hole and coming within 30 km of the center would have to be done by robots, if at all.

Notice that in the above equation, the kill radius in units of r_S goes like an inverse power of r_S. Thus for big enough black holes the tidal field should be managable even at the horizon. For example, consider a 3 billion solar mass black hole (such things are thought to power quasars). This would have $r_S = 3(3\times10^9)$km $= 9\times10^9$km, about twice the size of the solar system (Neptune orbits 4.5×10^9km from the Sun). Then $r_{\rm kill}/r_S = 1.3\times10^{-4}$, or $r_{\rm kill} = 1.1\times10^6$km, well inside Mercury's orbit of 58×10^6km, so you are deep within the black hole before being killed. In fact, the tidal acceleration as someone enters such a black holel is only 2.2×10^{-10} gee's, a thousand time less than we experience here on Earth. Thus someone could be falling into a very large black hole and never even feel it! They would certainly be killed at the kill radius above which would happen a few hours later.