

Physics 161: Black Holes: Lecture 26/27: 4/7 Mar 2011

Professor: Kim Griest

26 Observations of Black holes: powerpoint slide presentation

27 Entropy and Black Holes

It turns out that because black holes have a temperature, they also have an entropy. In fact the entropy of the black hole is enormous even though a black hole is described by only three numbers and therefore seems to be a very simple state! Remember entropy is a measure of the disorder in a system; it is equal to k times the log of the number of quantum states of the system.

A clue to this is the Hawking area theorem: the area of the horizon of a black hole cannot decrease. That is

$$dA/dt \geq 0.$$

It turns out this is the same statement as the third law of thermodynamics, that the entropy in an isolated system cannot decrease. We won't prove this, but can get a flavor of it by fooling around with equations we already know.

Consider the area of a black hole horizon $A = 4\pi r_s^2 = 16\pi G^2 M^2 / c^4$, where I put the c 's back in. Taking the derivative of the above equation we get

$$dA = \frac{32\pi MG^2}{c^4} dM,$$

or

$$dM = \frac{c^4 dA}{32\pi G^2 M}.$$

The total energy of the black hole is $E = Mc^2$, so $dE = c^2 dM$, and the Hawking temperature of the black hole is $T = hc^3 / (8\pi kGM)$, so we can write

$$dE = \left(\frac{hc^3}{8\pi kGM} \right) \left(\frac{kc^3 dA}{4hG} \right),$$

or

$$dE = Td \left(\frac{kAc^3}{4hG} \right).$$

This all follows simply from the Hawking temperature formula, but Hawking and Bekenstein noticed that this equation is the same as a famous equation from thermodynamics.

$$dE = TdS,$$

where S is the thermodynamic entropy of the system. Thus they identified the entropy of the black hole as

$$S = \frac{A}{4} \left(\frac{kc^3}{hG} \right).$$

Thus in this identification the entropy is proportional to the surface area of the black hole. What is totally cool about this identification, is the Hawking area theorem that says the area of a black hole can never decrease becomes the **third law of thermodynamics** which says that the entropy of a system can never decrease. This thermodynamic identification also implies that black holes should radiate as blackbodies, which they do. In fact, the surface area of a black hole does decrease as it radiates, but then one needs to take into account the entropy of the radiation. Overall the entropy of the entire system, black hole plus Hawking radiation does not decrease. The thermodynamics works out great. It is a beautiful connection that Hawking and Bekenstein and others made, and fits together perfectly.

String theorists recently had one of their few successes, when they managed to calculate the entropy of a black hole using string theory and got the Hawking/Bekenstein answer. It was a post-diction not a prediction, but it gave great encouragement to the string theory community and led to several new and deep understandings of the relation between black holes and thermodynamics, including information loss in black holes.

The string theorists were able to count the quantum states of a black hole by finding a dual system, that is, a set of equations that described a black hole but using a specific quantum field theory rather than general relativity. The black hole was dual to a gas of hot gluon-like particles, in which the quantum states could be counted. It came out to be just the Hawking/Bekenstein answer. The spectrum of radiation coming from the gas of hot gluons could also be counted using the QFT and it was Hawking answer. One should note that this calculation was not for a normal spherical black hole, but for a certainly limiting case of a very particular model that does not exist in the real world, that is a maximally charged black hole with 4 supersymmetries), but still this was a great success for string theory and helps greatly in our understanding of what black holes really are.

Finally, there are interesting new developments that are very related to these issues. They go by the name “holographic” principle in string theory. The basic puzzle is why the entropy of a black hole is proportional to its surface area. Normally we think of entropy as an extensive quantity; if you double the volume of a box of gas, you double the entropy of that system. Here however, the entropy does not go as the volume. This is weird since you really think that the number of quantum states that a system has should depend upon its volume size, not the area of the bounding surface. The holographic idea is to generalize this principle to all systems: the number of degrees of freedom of any system should be limited by a bounding surface area, and scale as surface area, not volume. For a black hole it might make some kind of sense, since we saw everything kind of gets hung up at the horizon and never makes it in (as observed from far away). But why would this be true in general? There are several deep string theory reasons for these speculations and they are the topics of current research.

An interesting way to see some of this is consider paving the horizon of a black hole with little “Planck areas”, little squares of size Planck length. How many such squares are there on the horizon of a black hole? Well, the Planck length is $l_{pl} = \sqrt{hG/c^3}$, where of course this is just an order of magnitude quantity (there could easily be factor of π or 2, etc.). The Planck area then is $A_{pl} = hG/c^3 = 2.6 \times 10^{-70} \text{ m}^2$. The number of Planck areas on the horizon is then just the ratio of the horizon area, $A = 4\pi r_S^2$ to the Planck area. $N = A/(hG/c^3) = Ac^3/(hG)$. This is remarkably close to the entropy of the black hole $S = \frac{A}{4}(kc^3/(hG))$. If one decided that each Planck area was capable of holding one bit of information (e.g. 1 or 0) and redefined the Planck size to be twice as big as above, then the number of bits that

a BH horizon could hold would be $N_{bits} = \frac{A}{4}(c^3/(hG))$. Finally using information theory to calculate the entropy of an N bit system one finds $S = k \log_2(N_{bits}) = \frac{A}{4}(kc^3/(hG))$, exactly the entropy of the black hole. What does this mean? I'm not sure, but it could be saying something deep about the nature of spacetime and information. It is the basic observation that gave rise to the idea of the holographic principle. We will have to see how this holographic principle plays out over the next few years.