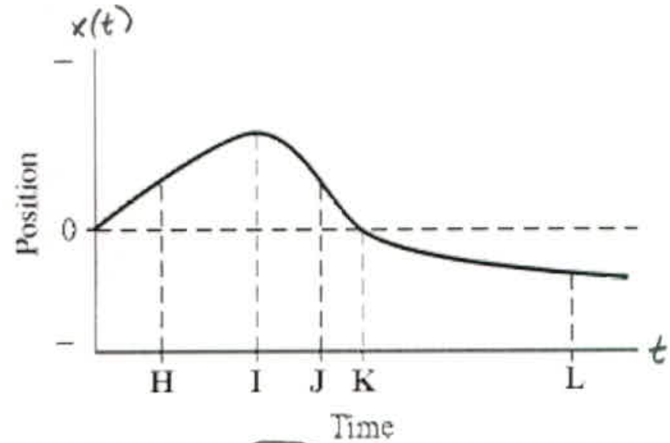
INSTRUCTIONS: Use a pencil #2 to fill your scantron. Write your code number and bubble it in under "EXAM NUMBER;" an entry in error will result in an automatic 10% deduction. Bubble in the quiz form (see letter A--D at bottom of page) in your scantron under "TEST FORM;" an error entering the "test form" will result in automatic 20% deductions, and may lead to disqualification. Write your name and 3-digit ID at the bottom of this page and turn it in with your scantron when you are finished working on the exam.

1) The plot below shows the position of an object as a function of time. The letters H-L represent particular moments of time. At which moment in time is the speed of the object the highest?



A) L B) I C) J D) K E) H

Speed is the magnitude of velocity, so we can read off speed on a position vs. time graph as the "steepness" or the absolute value of the slope. The graph appears steepest at time J (despite $\frac{d}{dt}x(t=J)<0$, $|\frac{d}{dt}x(t=J)|$ is large).

Solutions by Brian Shotwell (bshotwell@ucsd.edu)

Name KEY Student 3-digit ID_______

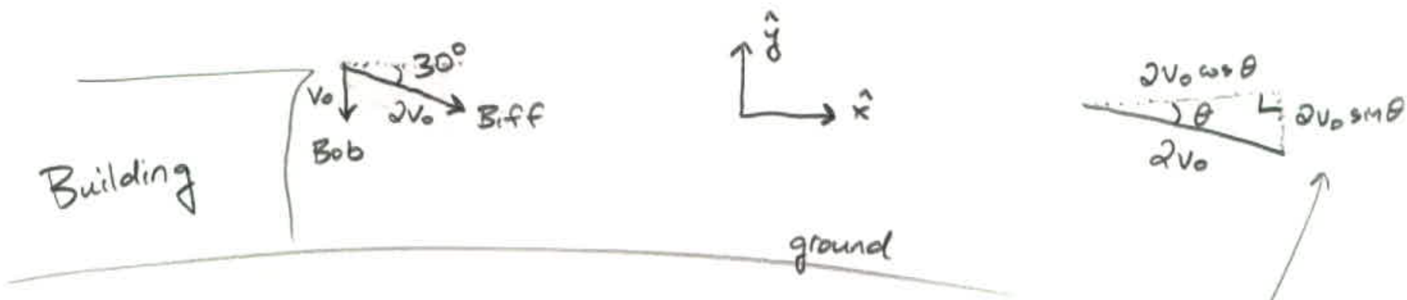
2) Bob and Biff throw identical rocks off a tall building at the same time. Bob throws his rock straight downward. Biff throws his rock downward and outward such that the angle between the initial velocity of the rock and the horizontal is 30 degrees. Biff throws the rock with a speed twice that of Bob's rock. Which rock hits the ground first (assume the ground near the building is flat)?

A) Bob's rock

B) Biff's rock

C) They hit at the same time.

D) Impossible to determine



The y-component of Bob's rock's (initial) velocity vector is $-V_0$

The y-component of Biff's rock's (initial) velocity vector is $-2V_0 \sin 30° = -V_0$

Both rocks have the same y-component of their initial velocity vectors, and both start from the same location at the same time. Therefore, both rocks hit the ground at the same time.

3) A ball is tossed vertically upward. When it reaches its highest point (before falling back downward),

A) the velocity is zero, the acceleration is zero, and the force of gravity acting on the ball is directed downward.

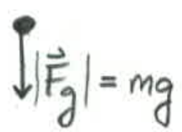
B) the velocity is zero, the acceleration is directed downward, and the force of gravity acting on the ball is directed downward.

C) the velocity is zero, the acceleration is zero, and the force of gravity acting on the ball is zero.

D) None of the above

When the ball reaches its apex, it isn't moving, and so the velocity is zero. A free-body diagram of the ball at this point would only have one force acting on the ball: the gravitational force acting downward. From $\sum \vec{F}_{ext} = m\vec{a}$, we know that this one force will provide an acceleration in the same direction as the force, so the acceleration at this point is directed downwards.

Ball's FBD



4) Bill and Susan are both standing on identical skateboards (with really good ball bearings, so you may neglect friction), initially at rest. Bill weighs three times as much as Susan. Bill pushes horizontally on Susan's back, causing Susan to start moving away from Bill. Immediately after Bill stops pushing,

A) Susan is moving away from Bill, and Bill is stationary.

B) Susan and Bill are moving away from each other, with equal speeds.

C) Susan and Bill are moving away from each other, and Susan's speed is three times that of Bill.

D) Susan and Bill are moving away from each other, and Susan's speed is a third that of Bill.

Newton's 3rd Law says that whatever force Susan feels from Bill, Bill receives a force from Susan of equal magnitude and opposite direction. Thus, both people are moving away from each other, and neither one is stationary.

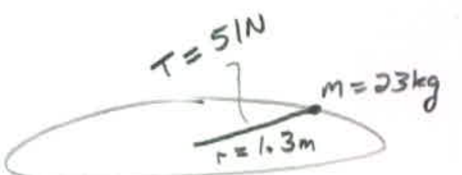
Also, $\Sigma \vec{F}_{ext} = m\vec{a} = m\,\frac{d\vec{v}}{dt} \rightarrow \int_{t_i}^{t_f} (\Sigma \vec{F}_{ext})\,dt = m\int_{t_i}^{t_f} d\vec{v} = m\Delta\vec{V}$

For any time $t \in [t_i, t_f]$, the force that Susan feels from Bill is the same magnitude as the force that Bill receives from susan.

Thus $|m_{Bill}\,\Delta\vec{V}_{Bill}| = |m_{Susan}\,\Delta\vec{V}_{Susan}|$, or $|\Delta\vec{V}_{Susan}| = 3\,|\Delta\vec{V}_{Bill}|$

5) A 23 kg mass is connected to a nail on a frictionless table by a (massless) string of length 1.3 m. If the tension in the string is 51 N while the mass moves uniformly on a circle on the table, how long does it take for the mass to make one complete revolution?

A) 4.5 s B) 5.2 s C) 3.8 s D) 4.8 s



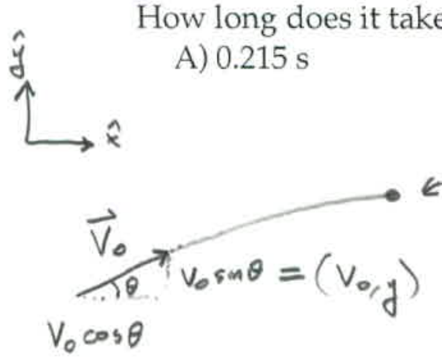
$\frac{mv^2}{r} = T$ (the tension is the force providing the centripital acc.)

$V = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{t}$ let t be the period, or, the time for 1 revolution

$T = \frac{mv^2}{r} = \frac{(2\pi)^2 mr}{t^2} \longrightarrow t = 2\pi\sqrt{\frac{mr}{T}} = 2\pi\sqrt{\frac{(23kg)(1.3m)}{(51\,N)}} = \underline{4.8\,sec}$

6) A boy throws a rock with an initial velocity of 2.15 m/s at 30.0° above the horizontal. How long does it take for the rock to reach the maximum height of its trajectory?

A) 0.215 s B) 0.303 s C) 0.110 s D) 0.194 s

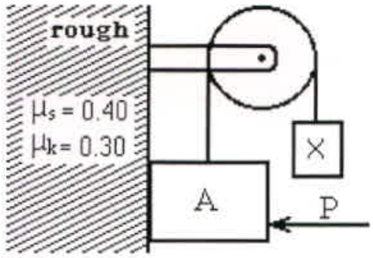


← max. height of trajectory, where $V_y = 0$

$$V_{f,y} = V_{0,y} + at$$

$$0 = V_0 \sin\theta - gt$$

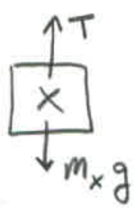
$$t = \frac{V_0 \sin\theta}{g} = \frac{(2.15\text{m}) \sin 30^\circ}{(9.8\ \text{m/s}^2)} = 0.11\ \text{sec}$$



7) Block A of mass 6 kg and block X are attached to a rope which passes over a pulley,as indicated in the figure above. A 50 N force is applied horizontally to block A, keeping it in contact with a rough vertical face. The coefficients of static and kinetic friction are $\mu_s = 0.40$ and $\mu_k = 0.30$. The pulley is light and frictionless. The mass of block X is set so that block A is on the verge of slipping upward. The mass of block X is closest to:

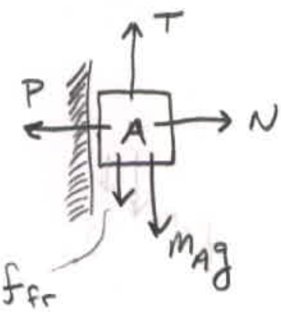
A) 7.6 kg B) 8.5 kg C) 7.2 kg D) 6.8 kg E) 8.0 kg

If block A is on the verge of slipping upward, we know that $f_{fr} = \mu_s N$ and that this frictional force is directed downwards.



Thus,

$T = m_x g$



$$N = P$$

$$T = f_{fr} + m_A g$$

$$m_x g = \mu_s P + m_A g$$

$$\boxed{m_x = m_A + \frac{\mu_s P}{g}}$$

$$m_x = 6\,kg + \frac{(0.4)(50\,N)}{(9.8\,m/s^2)} = \underline{8.0\,kg}$$

Note all forces cancel for each object because the system is (just barely) in equilibrium.

8) A particle is moving clockwise in a circle of radius 1 m about the origin of an x-y frame. If it completes 3 revolutions in 2π seconds, starting at $t = 0$ from the positive y-axis, the functional time dependence, with t in seconds, of the y-component of the velocity vector (in m/s) is given by

A) 3 cos(3t) B) sin(3t) C) -3 sin(3t) D) - cos(3t) E) cos(3t)

$$y = (1\,m)\cos[3t]$$

(assuming no phase)

Why? Amplitude must be 1 meter, since that is the radius.

Must use $\cos[\cdots]$ because we want $y = y_{max} = 1m$ at $t=0$

We want 3 revolutions to take (2π) seconds. Thus, if we have $\cos[\omega t]$, we want $\omega(2\pi)$ to equal 6π (the angle for 3 revs.) $\Rightarrow \omega = 3$

$$V_y = \frac{dy}{dt} = (3\ m/s)(-\sin[3t])$$, or choice C

9) A particle is moving clockwise in a circle of radius 1 m about the origin of an x-y frame. If it completes 3 revolutions in 2π seconds, starting at $t = 0$ from the positive y-axis, the functional time dependence, with t in seconds, of the y-component of the acceleration vector (in m/s^2) is given by

A) - 9 sin(3t)
B) - cos(3t)
C) - 9 cos(3t)
D) cos(3t)
E) sin(3t)

For the same reasons as above,

$$y = (1\,m)\cos[3t]$$

$$V_y = \frac{dy}{dt} = (-3\ m/s)\sin[3t]$$

$$a_y = \frac{dv_y}{dt} = (-9\ m/s^2)\cos[3t]$$

10) A particle is moving clockwise in a circle of radius 1 m about the origin of an x–y frame. If it completes 3 revolutions in 2π seconds, starting at $t = 0$ from the positive y–axis, the functional time dependence, with t in seconds, of the x–component of the acceleration vector (in m/s^2) is given by

A) – 9 sin(3t)
B) – cos(3t)
C) cos(3t)
D) – 9 cos(3t)
E) sin(3t)

$$x = (1m)(\sin[3t])$$

Why? Reasons for (1m) and [3t] are given in prob. #8 soln.

Must use sin[...] because we want $x = 0$ at $t = 0$. Also, when $t = \varepsilon$ (some arbitrarily small, positive number), we want x to be positive, since the particle is moving clockwise from $(x,y) = (0, 1m)$.

$\sin[3\varepsilon] \overset{\varepsilon \to 0}{\cong} 3\varepsilon > 0$, so the signs are okay and $+\sin[...]$ describes the x-motion of the particle

Therefore,

$$x = +(1m)\sin[3t]$$

$$V_x = (+3\,m/s)\cos[3t]$$

$$a_x = (-9\,m/s^2)\sin[3t]$$

11) The x– and y–coordinates of a particle in motion, as functions of time t, are given by:

$$x = 4t^2 - 3t + 6 \qquad y = 2t^3 - 3t^2 - 12t - 8$$

At the instant the x–component of velocity is equal to zero, the y–component of the acceleration is closest to:

A) $-3.7\ \mathrm{m/s^2}$

B) $3.0\ \mathrm{m/s^2}$

C) $-15\ \mathrm{m/s^2}$

D) $-10\ \mathrm{m/s^2}$

E) $-1.5\ \mathrm{m/s^2}$

$$x = 4t^2 - 3t + 6$$

$$V_x = 8t - 3 = 0$$

$$\longrightarrow V_x = 0 \text{ when } t = 3/8$$

$$y = 2t^3 - 3t^2 - 12t - 8$$

$$V_y = 6t^2 - 6t - 12$$

$$a_y = 12t - 6$$

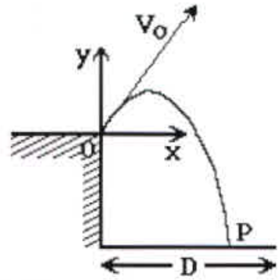
$$a_y(t = 3/8) = 12\left[\frac{3}{8}\right] - 6 = (-1.5)$$

All quantities are in SI units, so $\underline{a_y = -1.5\ \mathrm{m/s^2}}$ when $V_x = 0\ \mathrm{m/s}$

12) A projectile is fired at time $t = 0.0$s, from point 0 at the edge of a cliff, with initial velocity components of $v_{ox} = 80$ m/s and $v_{oy} = 600$ m/s. The projectile rises, then falls into the sea at point P. The time of flight of the projectile is 150.0 s.

$\mathbf{v_{ox} = 80}$ **m/s**

$\mathbf{v_{oy} = 600}$ **m/s**



The magnitude of the velocity at time $t = 15.0$ s is closest to:

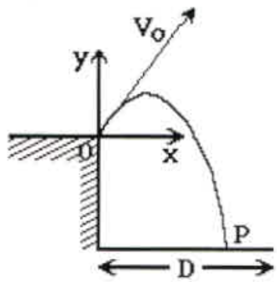
A) 747 m/s B) 460 m/s C) 455 m/s D) 751 m/s E) 453 m/s

$V_x = V_{o,x} = 80 \, m/s$ (no acceleration in $\hat{x}$-direction).

$V_y = V_{o,y} + at = 600 \, m/s - g \, (15.0 \text{ sec}) = 453 \, m/s$

$|\vec{v}_f| = \sqrt{(80 \, m/s)^2 + (453 \, m/s)^2} = \underline{460 \, m/s}$

13) A projectile is fired at time $t = 0.0$s, from point 0 at the edge of a cliff, with initial velocity components of $v_{OX} = 40$ m/s and $v_{OY} = 800$ m/s. The projectile rises, then falls into the sea at point P. The time of flight of the projectile is 200.0 s.



The x-coordinate of the projectile when its y-component of velocity equals 640 m/s is closest to:

A) 560 m B) 650 m C) 690 m D) 620 m E) 590 m

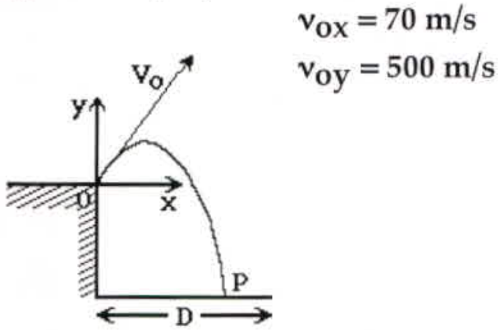
First find the time when $V_y = 640$ m/s, then use this to find x:

$$V_y = 640\,\text{m/s} = V_{o,y} + at = 800\,\text{m/s} - gt$$

$$t = \frac{160\,\text{m/s}}{9.8\,\text{m/s}^2} = 16.3\,\text{sec}$$

$$D = (V_{o,x})(t) = (40\,\text{m/s})(16.3\,\text{sec}) = 653\,\text{m}$$

14) A projectile is fired at time $t = 0.0$s, from point 0 at the edge of a cliff, with initial velocity components of $v_{OX} = 70$ m/s and $v_{OY} = 500$ m/s. The projectile rises, then falls into the sea at point P. The time of flight of the projectile is 125.0 s.



The y-coordinate of the projectile when its x-coordinate is 3500 m is closest to:

A) –430 m
B) +12,750 m
C) +3750 m
D) +7750 m
E) 430 m

First, find the time it takes to get to $x = 3500$m, then use this to find y.

$$3500\text{m} = (V_{o,x})t = (70\,\text{m/s})t \longrightarrow t = 50\,\text{sec}$$

$$y = (V_{o,y})t + \frac{1}{2}at^2$$

$$y = (500\,\text{m/s})(50\,\text{sec}) - \frac{1}{2}(9.8\,\text{m/s}^2)(50\,\text{sec})^2$$

$$y = 25000\text{m} - 12250\text{m} = 12750\text{m}$$

Answer Key
Testname: MIDTERM1

1) C
2) C
3) B
4) C
5) D
6) C
7) E
8) C
9) C
10) A
11) E
12) B
13) B
14) B

Name______________________________ Student 3-digit ID_______