Simple Harmonic Motion and Damping & Uncertainty Analysis Review

Lecture # 5

Physics 2BL

Winter 2011

Outline

- Significant figures
- Gaussian distribution and probabilities
- Experiment 3 intro
- Physics of damping and SHM
- Experiment 3 objectives

Significant Figures

What is the correct way to report the following numbers: (Justify your answer)

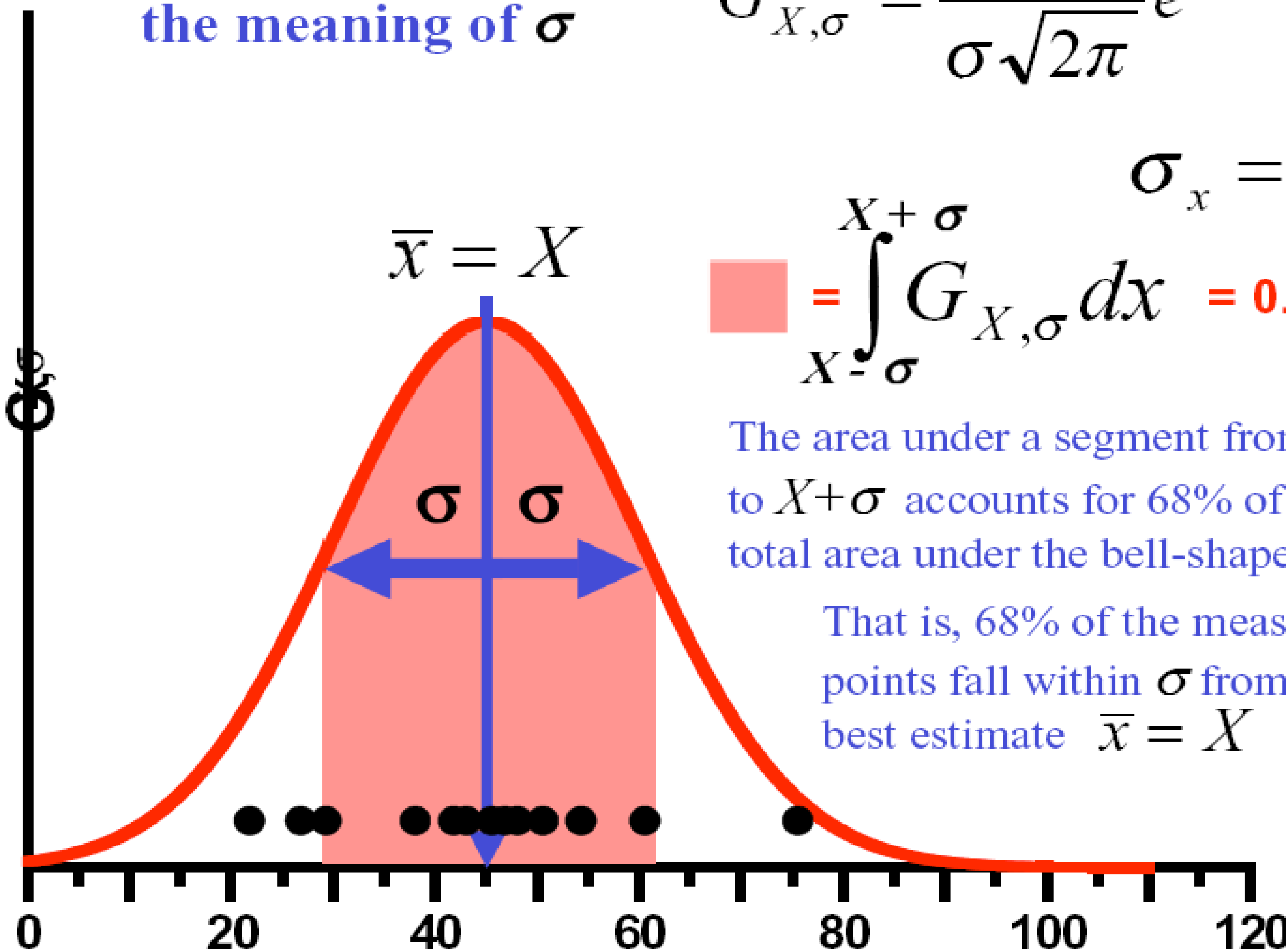
(a) 653 ± 55.4 m

(b) 256.55 ± 27 kg

Gauss distribution: the meaning of σ

$$G_{X,\sigma} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2 / 2\sigma^2}$$

$$\sigma_x = \sigma$$

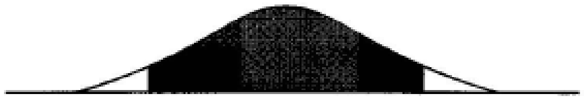


$$= \int_{X-\sigma}^{X+\sigma} G_{X,\sigma}\, dx = 0.68$$

The area under a segment from $X - \sigma$ to $X + \sigma$ accounts for 68% of the total area under the bell-shaped curve.

That is, 68% of the measured points fall within σ from the best estimate $\bar{x} = X$

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx$, as a function of t.



$t = 1$

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.80	1.60	2.39	3.19	3.99	4.78	5.58	6.38	7.17
0.1	7.97	8.76	9.55	10.34	11.13	11.92	12.71	13.50	14.28	15.07
0.2	15.85	16.63	17.41	18.19	18.97	19.74	20.51	21.28	22.05	22.82
0.3	23.58	24.34	25.10	25.86	26.61	27.37	28.12	28.86	29.61	30.35
0.4	31.08	31.82	32.55	33.28	34.01	34.73	35.45	36.16	36.88	37.59
0.5	38.29	38.99	39.69	40.39	41.08	41.77	42.45	43.13	43.81	44.48
0.6	45.15	45.81	46.47	47.13	47.78	48.43	49.07	49.71	50.35	50.98
0.7	51.61	52.23	52.85	53.46	54.07	54.67	55.27	55.87	56.46	57.05
0.8	57.63	58.21	58.78	59.35	59.91	60.47	61.02	61.57	62.11	62.65
0.9	63.19	63.72	64.24	64.76	65.28	65.79	66.29	66.80	67.29	67.78
1.0	68.27	68.75	69.23	69.70	70.17	70.63	71.09	71.54	71.99	72.43
1.1	72.87	73.30	73.73	74.15	74.57	74.99	75.40	75.80	76.20	76.60
1.2	76.99	77.37	77.75	78.13	78.50	78.87	79.23	79.59	79.95	80.29
1.3	80.64	80.98	81.32	81.65	81.98	82.30	82.62	82.93	83.24	83.55
1.4	83.85	84.15	84.44	84.73	85.01	85.29	85.57	85.84	86.11	86.38
1.5	86.64	86.90	87.15	87.40	87.64	87.89	88.12	88.36	88.59	88.82
1.6	89.04	89.26	89.48	89.69	89.90	90.11	90.31	90.51	90.70	90.90
1.7	91.09	91.27	91.46	91.64	91.81	91.99	92.16	92.33	92.49	92.65
1.8	92.81	92.97	93.12	93.28	93.42	93.57	93.71	93.85	93.99	94.12
1.9	94.26	94.39	94.51	94.64	94.76	94.88	95.00	95.12	95.23	95.34
2.0	95.45	95.56	95.66	95.76	95.86	95.96	96.06	96.15	96.25	96.34
2.1	96.43	96.51	96.60	96.68	96.76	96.84	96.92	97.00	97.07	97.15
2.2	97.22	97.29	97.36	97.43	97.49	97.56	97.62	97.68	97.74	97.80

p. 287 Taylor

Compatibility of a measured result(s): t-score

- Best estimate of x: $x_{best} \pm \sigma_{\bar{X}}$

- Compare with expected answer x_{exp} and compute t-score:

$$t \equiv \frac{\left| x_{best} - x_{expected} \right|}{\sigma_X}$$

- This is the number of standard deviations that x_{best} differs from x_{exp}.
- Therefore,the probability of obtaining an answer that differs from x_{exp} by t or more standard deviations is:

Prob(outside $t\sigma$) = 1-Prob(within $t\sigma$))

Example problem

Measure wavelength λ four times:

503 ± 10 nm

491 ± 8 nm

525 ± 20 nm

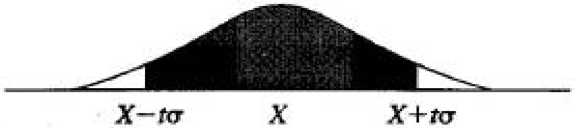
570 ± 40 nm

Should we reject the last data point?

$$t_{sus} = \Delta\lambda = \frac{|570 - 500| \text{ nm}}{\sqrt{6^2 + 40^2} \text{ nm}} = 1.73\,\sigma$$

Prob of λ outside Δλ =

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx$, as a function of t.



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Prob of λ outside $\Delta\lambda$ = 100 % - 91.6 % = 8.4 %

Total Prob = N x Prob = 4 * 8.4 % = 33.6 %

Is Total Prob < 50 % ? Yes, therefore can reject data point

The Four Experiments

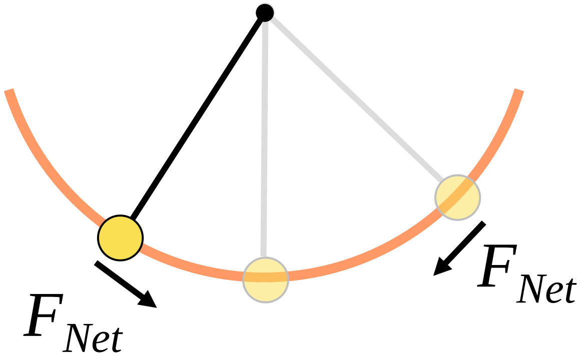
- **Determine the average density of the earth**
  - Measure simple things like lengths and times
  - Learn to estimate and propagate errors
- **Non-Destructive measurements of densities, structure**
  - Measure moments of inertia
  - Use repeated measurements to reduce random errors
- **Test model for damping; Construct and tune a shock absorber**
  - Damping model based on simple assumption
  - Adjust performance of a mechanical system
  - Demonstrate critical damping of your shock absorber
  - Does model work? Under what conditions? If needed, what more needs to be considered?
- **Measure coulomb force and calibrate a voltmeter.**
  - Reduce systematic errors in a precise measurement.

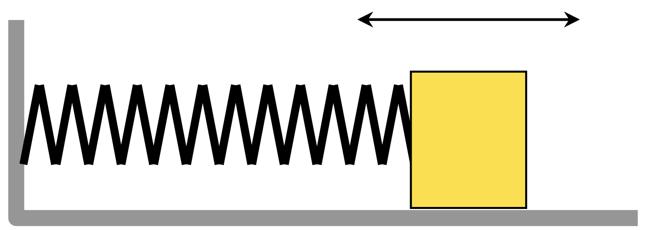
Experiment 3

- Goals: Test model for damping
- Model of a shock absorber in car
- Procedure: develop and demonstrate critically damped system
- check out setup, take data, do data make sense?
- Write up results - Does model work under all conditions, some conditions? Need modification?

Simple Harmonic Motion

- Position oscillates if force is always directed towards equilibrium position (restoring force).
- If restoring force is ~ position, motion is easy to analyze.





Springs

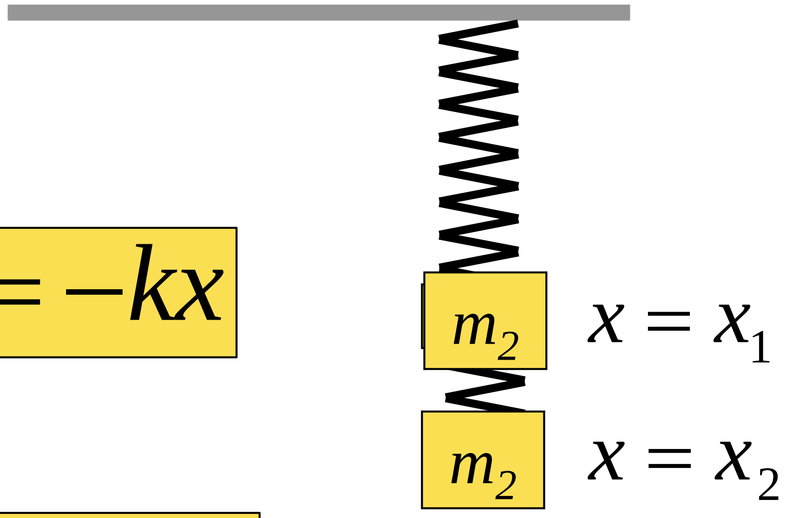
- Mag. of force from spring ~ extension (compression) of spring
- Mass hanging on spring: forces due to gravity, spring
- Stationary when forces balance

$$F_S = -kx$$

$$F_G = -mg$$

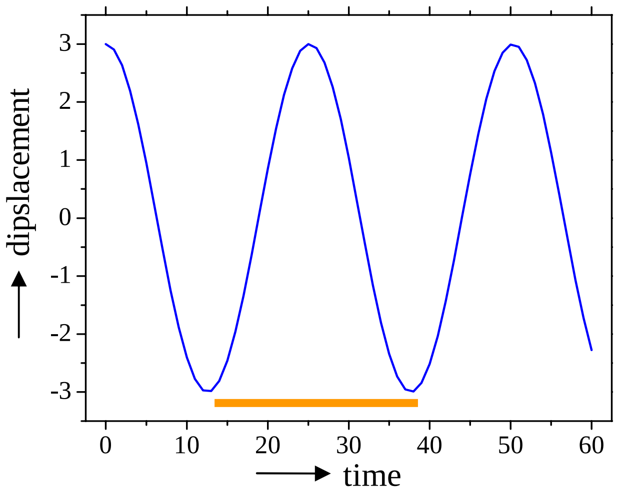
$$F_G = F_S$$

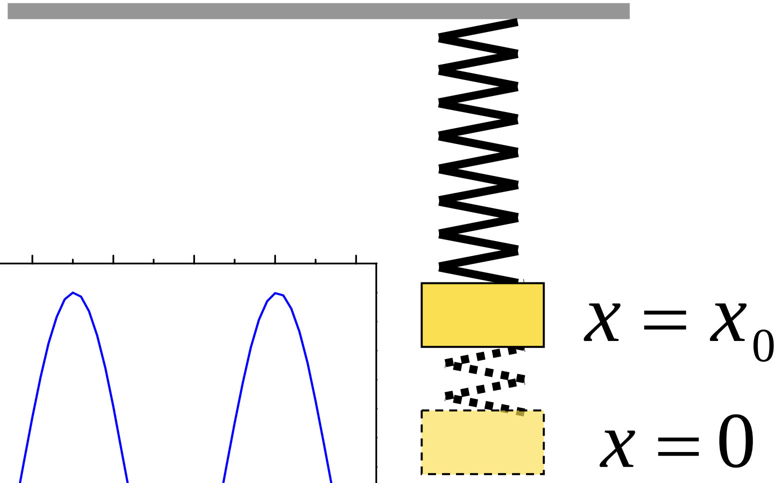
$$mg = kx$$



Simple Harmonic Motion

- Spring provides linear restoring force $\Rightarrow$ Mass on a spring is a harmonic oscillator





$$F = -kx$$

$$m\frac{d^2x}{dt^2} = -kx$$

$$x(t) = x_0 \cos \omega t$$

$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Damping

- Damping force opposes motion, magnitude depends on speed
- For falling object, constant gravitational force
- Damping force increases as velocity increases until damping force equals gravitational force
- Then no net force so no acceleration (constant velocity)

$$\vec{F}_{damping} = -b\vec{v}$$

$$F_{gravity} = -mg$$

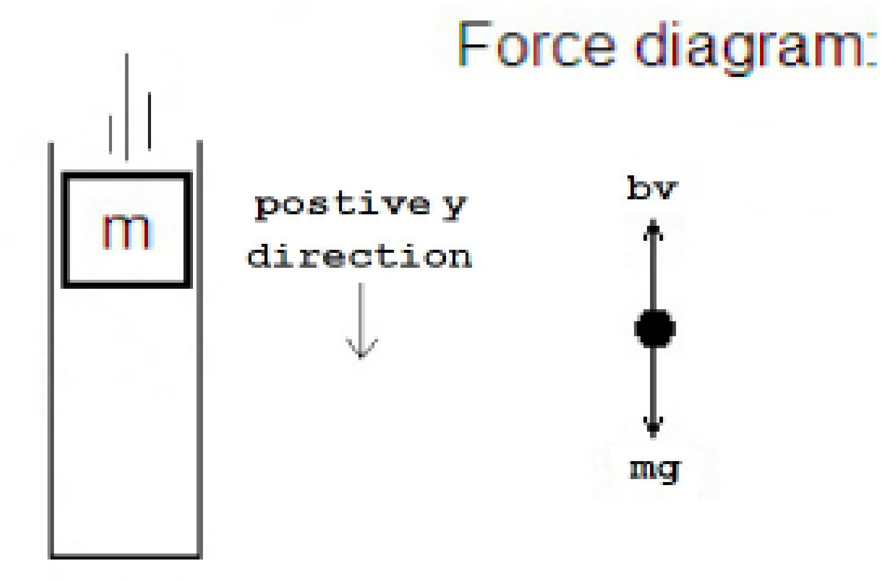
$$bv = mg$$

$$v_{terminal} = (mg)/b$$

Terminal velocity

- What is terminal velocity?
- How can it be calculated?

Falling Mass and Drag

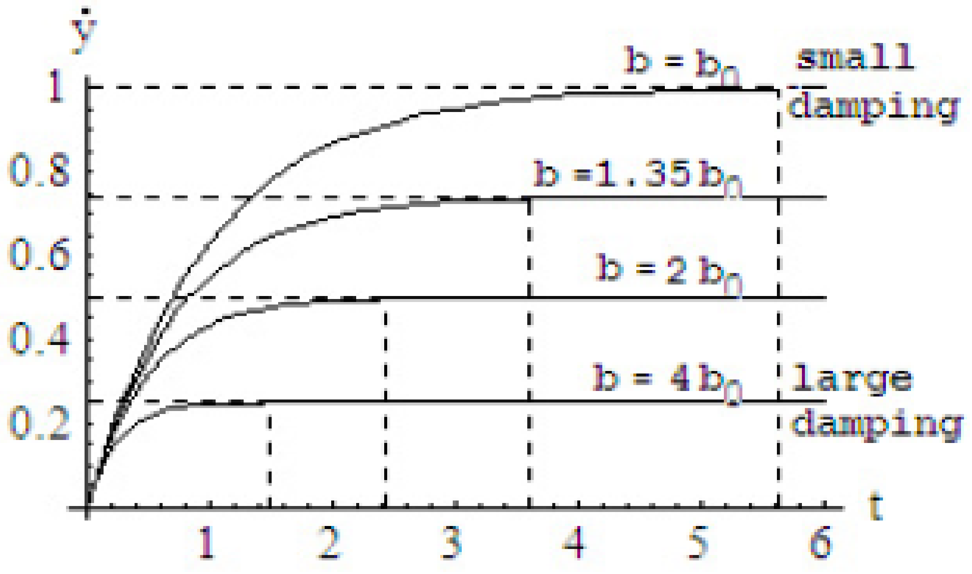


At steady state: $F_{drag} = F_{gravity}$

$$bv_t = mg$$

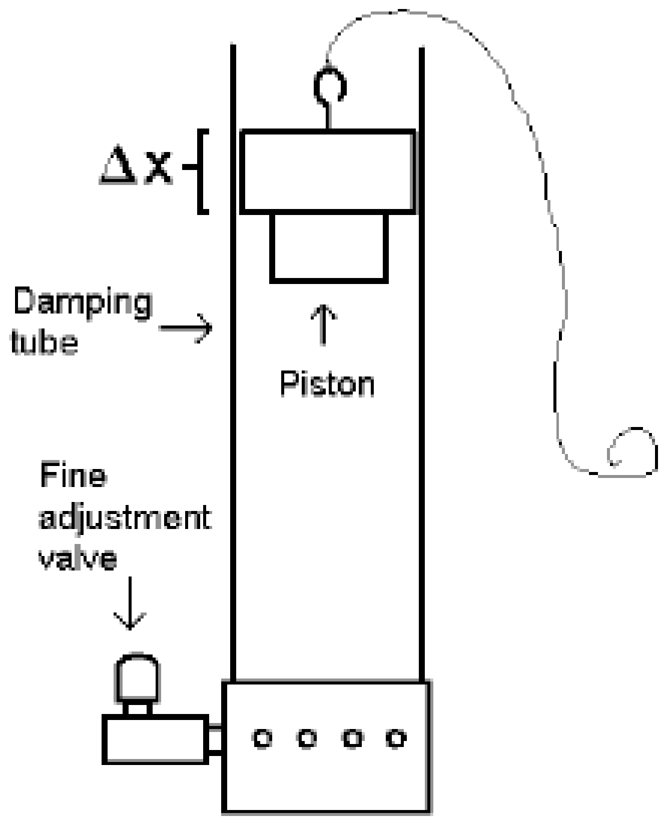
From rest: $y(t) = v_t[(m/b)(e^{-(b/m)t} - 1) + t]$

Terminal Velocity



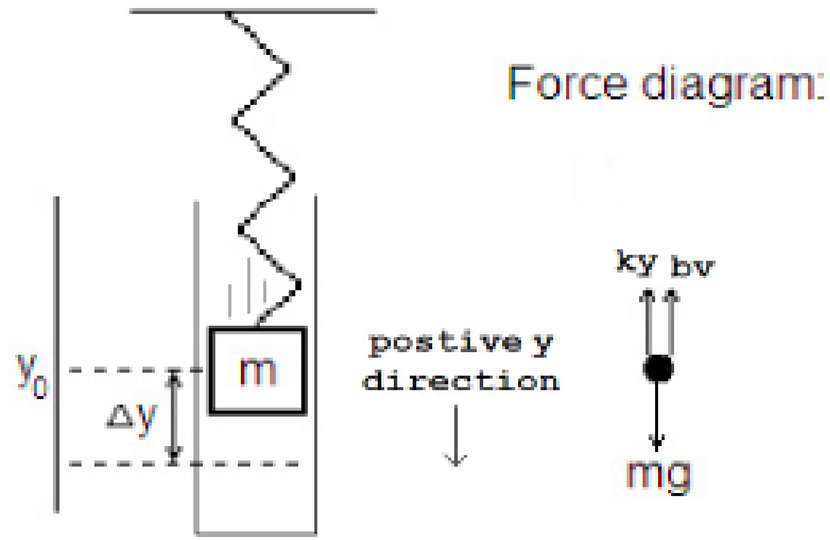
For velocity: $\dot{y}(t) = v_t[1 - e^{-(b/m)t}]$

Experimental Setup for Falling Mass and Drag



How do you measure velocity?

Damped Harmonic Motion



Damped SHM

- Consider both position and velocity dependant forces
- Behavior depends on how much damping occurs during one ‘oscillation’

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

$$x = x_0 \exp\left(-\frac{b}{2m}t\right)\exp\left(it\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}\right)$$

$$x = x_0 \exp\left(-\frac{b}{2m}t\right)\cos\left(t\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}\right).$$

or

$$x = x_0 \exp\left(-\frac{b}{2m}+\sqrt{\frac{b^2}{4m^2}-\frac{k}{m}}\right)t$$

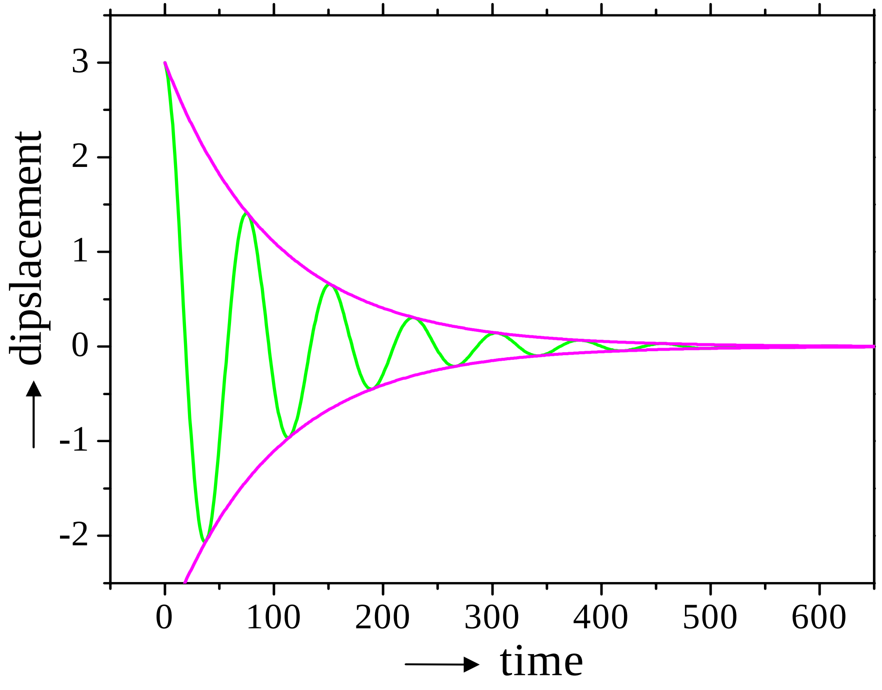
Relative Damping Strength: Weak damping

$$x = x_0 \exp\left(-\frac{b}{2m}t\right)\cos\left(t\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}\right).$$

$$\frac{b^2}{4m^2} << \frac{k}{m}$$

weak damping

(underdamped)



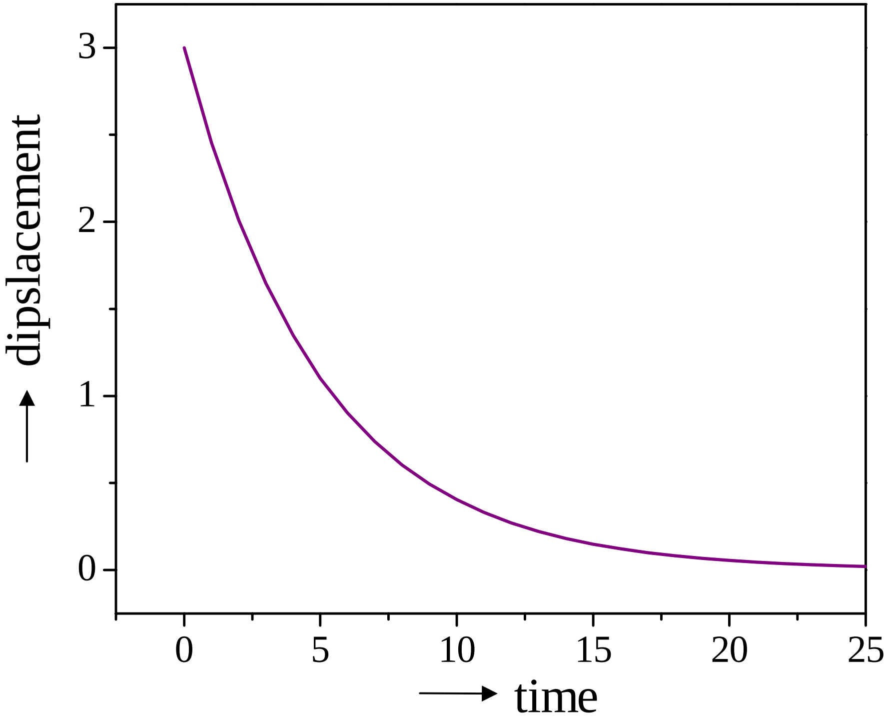
Relative Damping Strength: Strong damping

$$x = x_0 \exp\left(-\frac{b}{2m} + \sqrt{\frac{b^2}{4m^2} - \frac{k}{m}}\right)t$$

$$\frac{b^2}{4m^2} >> \frac{k}{m}$$

strong damping

(overdamped)



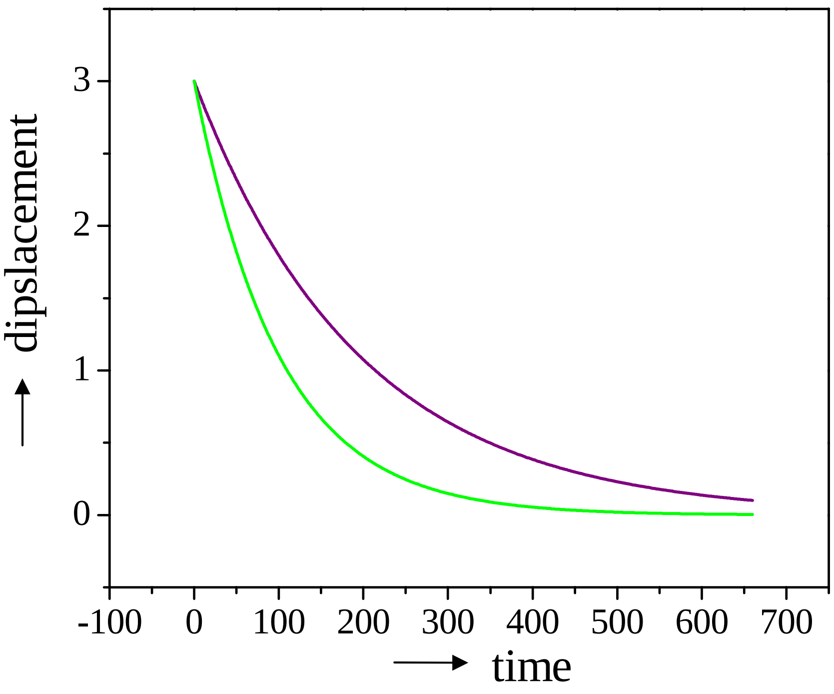
Relative Damping Strength: Critical damping

$$x = x_0 \exp\left(\left(-\frac{b}{2m} + \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right)t\right)$$

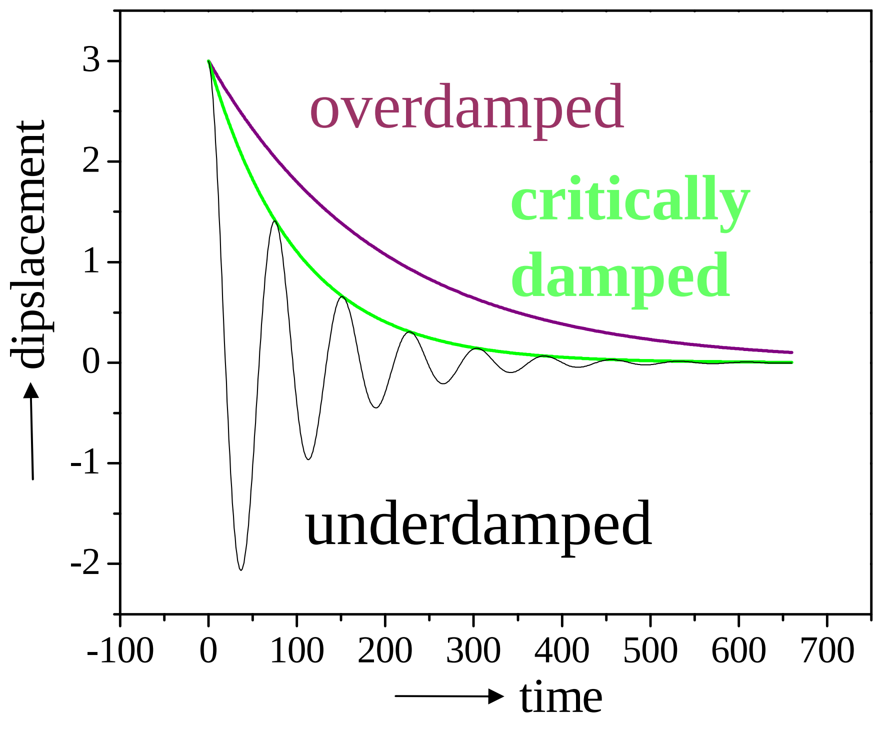
$$\frac{b^2}{4m^2} = \frac{k}{m}$$

critical damping

$$b_{crit} = 2\sqrt{mk}$$



Comparison of the various types of damping



Plotting Graphs

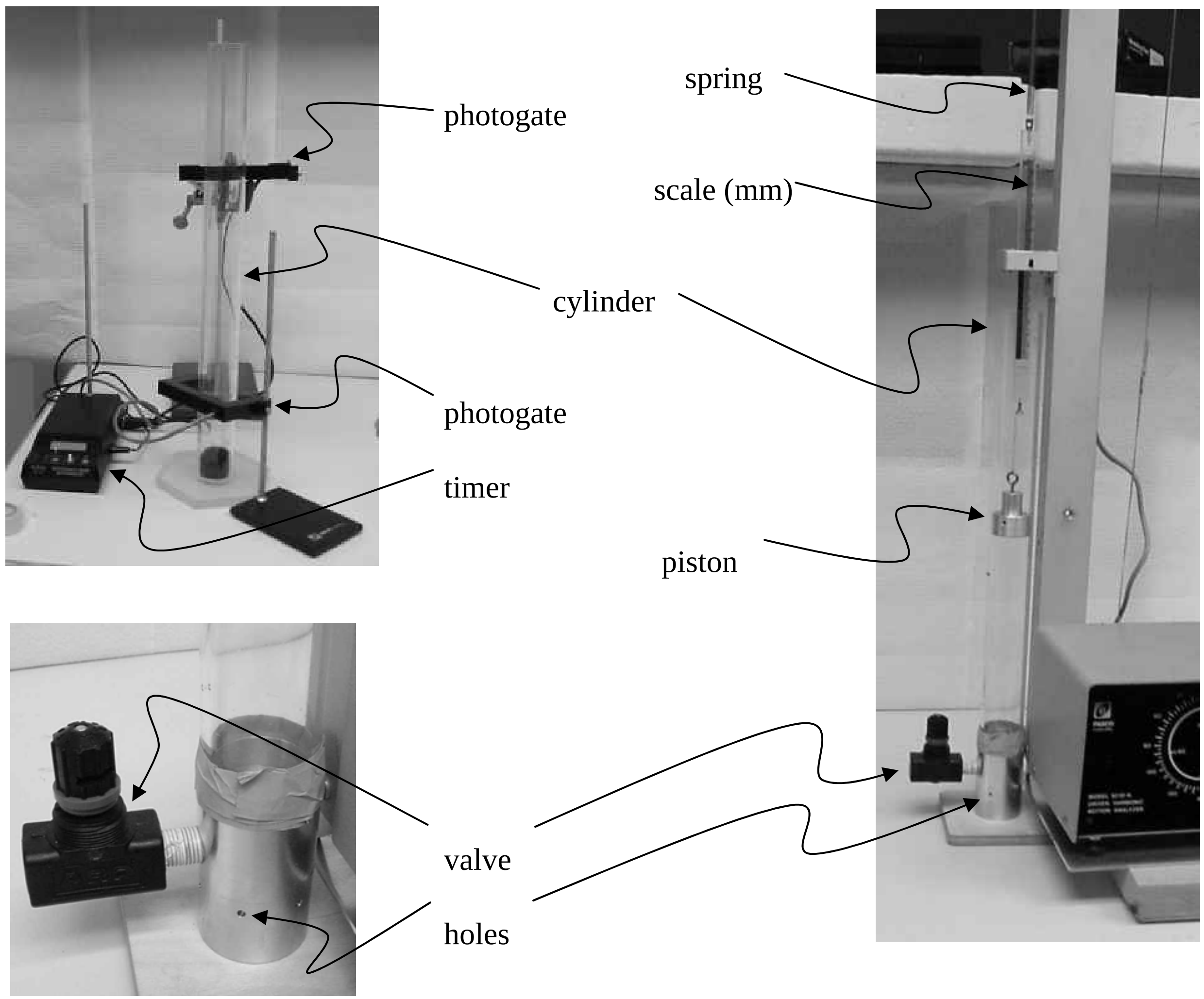
Give each graph a title

Determine independent and dependent variables

Determine boundaries

Include error bars

Experimental setup



Experiment 3: achieve critical damping

- Show/test method
  - Determine spring constant, predict critical damping coefficient
  - Determine how damping coefficient depends on air flow (valve position)
    - easy at terminal velocity
    - how do you know it's $v_{terminal}$?
  - Set damping to critical level

Demonstrate critical damping: show convincing evidence that critical damping was achieved

- Demonstrate that damping is critical
  - No oscillations (overshoot)
  - Shortest time to return to equilibrium position

Error propagation

(1) $k_{spring} = 4\pi^2 m/T^2$

$\sigma_{kspring} = \varepsilon_{kspring} * k_{spring}$

$\varepsilon_{kspring} = \sqrt{\varepsilon_m^{\,2} + (2\varepsilon_T)^2}$

(2) $k_{by\text{-}eye} = m(g\Delta t^*/2\Delta x)^2$

$\sigma_{by\text{-}eye} = \varepsilon_{by\text{-}eye} * k_{by\text{-}eye}$

$\varepsilon_{by\text{-}eye} = \sqrt{(2\varepsilon_{\Delta t^*)}{}^2 + (2\varepsilon_{\Delta x})^2 + \varepsilon_m^{\,2}}$

Remember

- Prepare for Exp. 3
- Homework Taylor #8.6, 8.10
- Read Taylor through Chapter 9