2-2 (a) Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma mv = 1.90mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1-(v/c)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1-\left(\frac{1}{1.90}\right)^2\right]^{1/2} c$$

$$= 0.85c$$

(b) No change, because the masses cancel each other

2-3 As **F** is parallel to **v**, scalar equations are used. Relativistic momentum is given by $p = \gamma mv = \frac{mv}{\left[1-(v/c)^2\right]^{1/2}}$, and relativistic force is given by

$$F = \frac{dp}{dt} = \frac{d}{dt}\left\{\frac{mv}{\left[1-(v/c)^2\right]^{1/2}}\right\}$$

$$F = \frac{dp}{dt} = \frac{m}{\left[1-(v^2/c^2)\right]^{3/2}}\left(\frac{dv}{dt}\right)$$

2-7 $E = \gamma mc^2,\ p = \gamma mu;\ E^2 = (\gamma mc^2)^2;\ p^2 = (\gamma mu)^2;$

$$E^2 - p^2c^2 = (\gamma mc^2)^2 - (\gamma mu)^2 c^2 = \gamma^2\left\{(mc^2)^2 - (mc)^2u^2\right\}$$

$$= (mc^2)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = (mc^2)^2 \text{ Q.E.D.}$$

$$E^2 = p^2c^2 + (mc^2)^2$$

2-8 (a) $E_R = mc^2 = (1.67\times10^{-27}\text{ kg})(3\times10^8\text{ m/s})^2 = 1.503\times10^{-10}\text{ J} = 939.4\text{ MeV}$
(Numerical round off gives a slightly larger value for the proton mass)

(b) $E = \gamma mc^2 = \frac{1.503\times10^{-10}\text{ J}}{\left(1-(0.95c/c)^2\right)^{1/2}} = 4.813\times10^{-10}\text{ J} \approx 3.01\times10^3\text{ MeV}$

(c) $K = E - mc^2 = 4.813\times10^{-10}\text{ J} - 1.503\times10^{-10}\text{ J} = 3.31\times10^{-10}\text{ J} = 2.07\times10^3\text{ MeV}$

2-9 (a) When $K = (\gamma-1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma mc^2 = 6(0.511\ 0\text{ MeV}) = 3.07\text{ MeV}$.

(b) $\frac{1}{\gamma} = \left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}$ and $v = c\left[1-\left(\frac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1-\left(\frac{1}{6}\right)^2\right]^{1/2} = 0.986c$

2-12 (a) When $K_e = K_p$, $m_e c^2(\gamma_e - 1) = m_p c^2(\gamma_p - 1)$. In this case $m_e c^2 = 0.511\,0$ MeV and $m_p c^2 = 938$ MeV, $\gamma_e = \left[1-(0.75)^2\right]^{1/2} = 1.511\,9$. Substituting these values into the first equation, we find $\gamma_p = 1 + \frac{m_e c^2(\gamma - 1)}{m_p c^2} = 1 + \frac{(0.511\,0)(1.511\,9-1)}{939} = 1.000\,279$.

But $\gamma_p = \frac{1}{\left[1-(u_p/c)^2\right]^{1/2}}$; therefore $u_p = c\left(1-\gamma_p^{-2}\right)^{1/2} = 0.023\,6c$.

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\frac{\gamma_e}{\gamma_p}\right)\left(\frac{m_e}{m_p}\right)u_e$,

$$u_p = \left(\frac{1.511\,9}{1.000\,279}\right)\left[\frac{0.511\,0/c^2}{939/c^2}\right](0.75c) = 6.17\times 10^{-4}\,c.$$

2-13 (a)

$$E = 400mc^2 = \gamma\, mc^2$$
$$\gamma = \left(1-\frac{v^2}{c^2}\right)^{-1/2} = 400$$
$$\left(1-\frac{v^2}{c^2}\right) = \left(\frac{1}{400}\right)^2$$
$$\frac{v}{c} = \left[1-\frac{1}{400^2}\right]^{1/2}$$
$$v = 0.999\,997c$$

(b) $K = E - mc^2 = (400-1)mc^2 = 399mc^2 = (399)(938.3\text{ MeV}) = 3.744\times 10^5$ MeV

2-14 (a)

$$E = mc^2$$
$$m = \frac{E}{c^2} = \frac{4\times 10^{26}\text{ J}}{\left(3.0\times 10^8\text{ m/s}\right)^2} = 4.4\times 10^9\text{ kg}$$

(b) $t = \frac{\left(2.0\times 10^{30}\right)\text{ kg}}{4.4\times 10^9\text{ kg/s}} = 4.5\times 10^{20}\text{ s} = 1.4\times 10^{13}$ years

2-15 (a) $K = \gamma\, mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1+\frac{Vq}{mc^2}\right)^2$ and $\frac{v}{c} = \left\{1-\left(1+\frac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{1-\frac{1}{1+\left(5.0\times 10^4\text{ eV}/0.511\text{ MeV}\right)^2}\right\}^{1/2} = 0.412\,7$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0\times10^4\ \text{eV}\right)}{0.511\ \text{MeV}/c^2}\right\}^{1/2} = 0.442\,c$$

(c) The error in using the classical expression is approximately $\frac{3}{40}\times100\%$ or about 7.5% in speed.

2-18 (a) The mass difference of the two nuclei is

$$\Delta m = 54.927\ 9\ \text{u} - 54.924\ 4\ \text{u} = 0.003\ 5\ \text{u}$$
$$\Delta E = \left(931\ \text{MeV/u}\right)\left(0.003\ 5\ \text{u}\right) = 3.26\ \text{MeV}.$$

(b) The rest energy for an electron is 0.511 MeV. Therefore,

$$K = 3.26\ \text{MeV} - 0.511\ \text{MeV} = 2.75\ \text{MeV}.$$

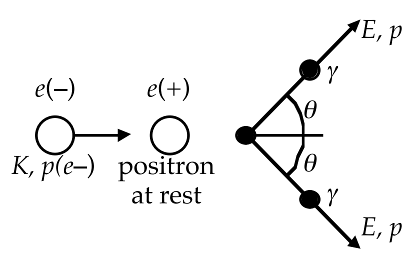
2-19 $\Delta m = 6m_p + 6m_n - m_C = \left[6\left(1.007\ 276\right) + 6\left(1.008\ 665\right) - 12\right]\ \text{u} = 0.095\ 646\ \text{u}$,

$$\Delta E = \left(931.49\ \text{MeV/u}\right)\left(0.095\ 646\ \text{u}\right) = 89.09\ \text{MeV}.$$

Therefore the energy per nucleon $= \frac{89.09\ \text{MeV}}{12} = 7.42\ \text{MeV}$.

2-20 $\Delta m = m\ \ - m_p - m_e = 1.008\ 665\ \text{u} - 1.007\ 276\ \text{u} - 0.000\ 548\ 5\ \text{u} = 8.404\times10^{-4}\ \text{u}$
$E = c^2\left(8.404\times10^{-4}\ \text{u}\right) = \left(8.404\times10^{-4}\ \text{u}\right)\left(931.5\ \text{MeV/u}\right) = 0.783\ \text{MeV}$.

2-21



Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = \left(0.500 + 0.511\right)\ \text{MeV} = 1.011\ \text{MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\frac{E}{c} = 1.011\ \text{MeV}/c$.

Using $E_{e^-}^2 = p_{e^-}^2c^2 + \left(mc^2\right)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)}\text{ MeV}}{c} = 1.422\text{ MeV}/c.$$

Finally, $\cos\theta = \dfrac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-23 In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1 - v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1 - v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987\,c)(6.22) = (m_h)(0.868\,c)(2.01)$$

$$m_l = \frac{(m_h)(0.868\,c)(2.01)}{(0.987)(6.22)} = 0.284 m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34 \times 10^{-27}\text{ kg} = 6.22 m_l + 2.01 m_h$ which in turn gives $3.34 \times 10^{-27}\text{ kg} = (6.22)(0.284) m_l + 2.01 m_h$ or $m_h = \dfrac{3.34 \times 10^{-27}\text{ kg}}{3.78} = 8.84 \times 10^{-28}\text{ kg}$ and $m_l = (0.284) m_h = 2.51 \times 10^{-28}\text{ kg}$.

2-31 Conservation of momentum γmu:

$$\frac{mu}{\sqrt{1 - u^2/c^2}} + \frac{m(-u)}{3\sqrt{1 - u^2/c^2}} = \frac{Mv_\text{f}}{\sqrt{1 - v_\text{f}^2/c^2}} = \frac{2mu}{3\sqrt{1 - u^2/c^2}}.$$

Conservation of energy γmc^2:

$$\frac{mc^2}{\sqrt{1 - u^2/c^2}} + \frac{mc^2}{3\sqrt{1 - u^2/c^2}} = \frac{Mc^2}{\sqrt{1 - v_\text{f}^2/c^2}} = \frac{4mc^2}{3\sqrt{1 - u^2/c^2}}.$$

To start solving we can divide: $v_\text{f} = \dfrac{2u}{4} = \dfrac{u}{2}$. Then

$$\frac{M}{\sqrt{1 - u^2/4c^2}} = \frac{4m}{3\sqrt{1 - u^2/c^2}} = \frac{M}{(1/2)\sqrt{4 - u^2/c^2}}$$

$$M = \frac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}}$$

Note that when $v \ll c$, this reduces to $M = \dfrac{4m}{3}$, in agreement with the classical result.

2-33 The energy that arrives in one year is

$$E = \mathscr{P}\,\Delta t = \left(1.79 \times 10^{17}\ \text{J/s}\right)\left(3.16 \times 10^{7}\ \text{s}\right) = 5.66 \times 10^{24}\ \text{J}.$$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66 \times 10^{24}\ \text{J}}{\left(3.00 \times 10^{8}\ \text{m/s}\right)^2} = 6.28 \times 10^{7}\ \text{kg}$