

5-2 The issue is: Can we use the simpler classical expression $p = (2mK)^{1/2}$ instead of the exact

relativistic expression $p = \frac{K\left(1 + \frac{2mc^2}{K}\right)^{1/2}}{c}$? As the relativistic expression reduces to $p = (2mK)^{1/2}$ for $K \ll 2mc^2$, we can use the classical expression whenever $K \ll 1$ MeV because mc^2 for the electron is 0.511 MeV.

(a) Here 50 eV \ll 1 MeV, so $p = (2mK)^{1/2}$

$$\lambda = \frac{h}{p} = \frac{h}{\left[(2)\left(\frac{0.511 \text{ MeV}}{c^2}\right)(50 \text{ eV})\right]^{1/2}} = \frac{hc}{\left[(2)(0.511 \text{ MeV})(50 \text{ eV})\right]^{1/2}}$$

$$= \frac{1240 \text{ eV nm}}{\left[(2)(0.511 \times 10^6)(50)(\text{eV})^2\right]^{1/2}} = 0.173 \text{ nm}$$

(b) As 50 eV \ll 1 MeV, $p = (2mK)^{1/2}$

$$\lambda = \frac{hc}{\left[(2)\left(\frac{0.511 \text{ MeV}}{c^2}\right)(50 \times 10^3 \text{ eV})\right]^{1/2}} = 5.49 \times 10^{-3} \text{ nm}.$$

As this is clearly a worse approximation than in (a) to be on the safe side use the

relativistic expression for p : $p = K \frac{\left(1 + \frac{2mc^2}{K}\right)^{1/2}}{c}$ so

$$\lambda = \frac{h}{p} = \frac{hc}{\left(K^2 + 2Kmc^2\right)^{1/2}} = \frac{1240 \text{ eV nm}}{\left[\left(50 \times 10^3\right)^2 + (2)(50 \times 10^3)(0.511 \times 10^6 \text{ eV})\right]^{1/2}}$$

$$= 5.36 \times 10^{-3} \text{ nm} = 0.00536 \text{ nm}$$

5-7 A 10 MeV proton has $K = 10$ MeV \ll $2mc^2 = 1877$ MeV so we can use the classical expression $p = (2mK)^{1/2}$. (See Problem 5-2)

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(2)(938.3 \text{ MeV})(10 \text{ MeV})\right]^{1/2}} = \frac{1240 \text{ MeV fm}}{\left[(2)(938.3)(10)(\text{MeV})^2\right]^{1/2}} = 9.05 \text{ fm} = 9.05 \times 10^{-15} \text{ m}$$

5-8 $\lambda = \frac{h}{p} = \frac{h}{(2mK)^{1/2}} = \frac{h}{(2meV)^{1/2}} = \left[\frac{h}{(2me)^{1/2}}\right] V^{-1/2}$

$$\lambda = \left[\frac{6.626 \times 10^{-34} \text{ Js}}{(2 \times 9.105 \times 10^{-31} \text{ kg} \times 1.602 \times 10^{-19} \text{ C})^{1/2}}\right] V^{-1/2}$$

$$\lambda = \left[\frac{1.226 \times 10^{-9} \text{ kg}^{1/2} \text{ m}^2}{\text{sC}^{1/2}}\right] V^{-1/2}$$

5-10 As $\lambda = 2a_0 = 2(0.0529) \text{ nm} = 0.1058 \text{ nm}$ the energy of the electron is nonrelativistic, so we can use

$$p = \frac{h}{\lambda} \text{ with } K = \frac{p^2}{2m};$$

$$K = \frac{h^2}{2m\lambda^2} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(9.11 \times 10^{-31} \text{ kg})(1.058 \times 10^{-10} \text{ m})^2} = 21.5 \times 10^{-18} \text{ J} = 134 \text{ eV}$$

- This is about ten times as large as the ground-state energy of hydrogen, which is 13.6 eV.
- 5-11 (a) In this problem, the electron must be treated relativistically because we must use relativity when $pc \approx mc^2$. (See problem 5-5). the momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{10^{-14} \text{ m}} = 6.626 \times 10^{-20} \text{ kg} \cdot \text{m/s}$$

and $pc = 124 \text{ MeV} \gg mc^2 = 0.511 \text{ MeV}$. The energy of the electron is

$$E = (p^2 c^2 + m^2 c^4)^{1/2}$$

$$= \left[(6.626 \times 10^{-20} \text{ kg} \cdot \text{m/s})^2 (3 \times 10^8 \text{ m/s})^2 + (0.511 \times 10^6 \text{ eV})^2 (1.602 \times 10^{-19} \text{ J/eV})^2 \right]^{1/2}$$

$$= 1.99 \times 10^{-11} \text{ J} = 1.24 \times 10^8 \text{ eV}$$

so that $K = E - mc^2 \approx 124 \text{ MeV}$.

- (b) The kinetic energy is too large to expect that the electron could be confined to a region the size of the nucleus.

- 5-12 Using $p = \frac{h}{\lambda} = mv$, we find that $v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(1 \times 10^{-10} \text{ m})} = 7.27 \times 10^6 \text{ m/s}$.

From the principle of conservation of energy, we get

$$eV = \frac{mv^2}{2} = \frac{(9.11 \times 10^{-31} \text{ kg})(7.27 \times 10^6 \text{ m/s})^2}{2} = 2.41 \times 10^{-17} \text{ J} = 151 \text{ eV}.$$

Therefore $V = 151 \text{ V}$.

- 5-15 For a free, non-relativistic electron $E = \frac{m_e v_0^2}{2} = \frac{p^2}{2m_e}$. As the wavenumber and angular frequency of the electron's de Broglie wave are given by $p = \hbar k$ and $E = \hbar \omega$, substituting these results gives the dispersion relation $\omega = \frac{\hbar k^2}{2m_e}$. So $v_g = \frac{d\omega}{dk} = \frac{\hbar k}{m_e} = \frac{p}{m_e} = v_0$.

- 5-17 $E^2 = p^2 c^2 + (m_e c^2)^2$
 $E = \left[p^2 c^2 + (m_e c^2)^2 \right]^{1/2}$. As $E = \hbar \omega$ and $p = \hbar k$

$$\hbar\omega = \left[\hbar^2 k^2 c^2 + (m_e c^2)^2 \right]^{1/2} \text{ or}$$

$$\omega(k) = \left[k^2 c^2 + \frac{(m_e c^2)^2}{\hbar^2} \right]^{1/2}$$

$$v_p = \frac{\omega}{k} = \frac{\left[k^2 c^2 + (m_e c^2 / \hbar)^2 \right]^{1/2}}{k} = \left[c^2 + \left(\frac{m_e c^2}{\hbar k} \right)^2 \right]^{1/2}$$

$$v_g = \left. \frac{d\omega}{dk} \right|_{k_0} = \frac{1}{2} \left[k^2 c^2 + \left(\frac{m_e c^2}{\hbar} \right)^2 \right]^{-1/2} 2kc^2 = \frac{kc^2}{\left[k^2 c^2 + (m_e c^2 / \hbar)^2 \right]^{1/2}}$$

$$v_p v_g = \left\{ \frac{\left[k^2 c^2 + (m_e c^2 / \hbar)^2 \right]^{1/2}}{k} \right\} \left\{ \left[k^2 c^2 + (m_e c^2 / \hbar)^2 \right]^{1/2} \right\} = c^2$$

Therefore, $v_g < c$ if $v_p > c$.

5-23 (a) $\Delta p \Delta x = m \Delta v \Delta x \geq \frac{\hbar}{2}$
 $\Delta v \geq \frac{\hbar}{4\pi m \Delta x} = \frac{2\pi \text{ J} \cdot \text{s}}{4\pi(2 \text{ kg})(1 \text{ m})} = 0.25 \text{ m/s}$

(b) The duck might move by $(0.25 \text{ m/s})(5 \text{ s}) = 1.25 \text{ m}$. With original position uncertainty of 1m, we can think of Δx growing to $1 \text{ m} + 1.25 \text{ m} = 2.25 \text{ m}$.

5-24 (a) $\Delta x \Delta p = \hbar$ so if $\Delta x = r$, $\Delta p \approx \frac{\hbar}{r}$

(b) $K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2}$

$$U = -\frac{ke^2}{r}$$

$$E = \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$$

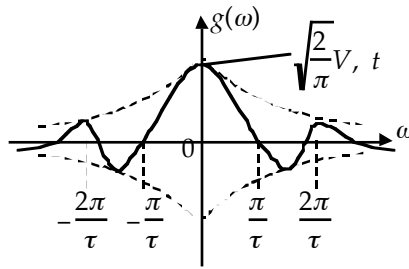
(c) To minimize E take $\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{ke^2}{r^2} = 0 \Rightarrow r = \frac{\hbar^2}{m_e ke^2} = \text{Bohr radius} = a_0$. Then

$$E = \left(\frac{\hbar}{2m_e} \right) \left(\frac{m_e ke^2}{\hbar^2} \right)^2 - ke^2 \left(\frac{m_e ke^2}{\hbar^2} \right) = \frac{m_e k^2 e^4}{2\hbar^2} = -13.6 \text{ eV}.$$

5-34 (a) $g(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} V(t)(\cos \omega t - i \sin \omega t) dt$, $V(t) \sin \omega t$ is an odd function so this

integral vanishes leaving $g(\omega) = 2(2\pi)^{-1/2} \int_0^{\tau} V_0 \cos \omega t dt = \left(\frac{2}{\pi} \right)^{1/2} V_0 \frac{\sin \omega \tau}{\omega}$. A

sketch of $g(\omega)$ is given below.



- (b) As the major contribution to this pulse comes from ω 's between $-\frac{\pi}{\tau}$ and $\frac{\pi}{\tau}$, let $\Delta\omega \approx \frac{\pi}{\tau}$ and since $\Delta t = \tau$.

$$\Delta\omega\Delta t = \left(\frac{\pi}{\tau}\right)\tau = \pi$$

- (c) Substituting $\Delta t = 0.5 \mu\text{s}$ in $\Delta\omega = \frac{\pi}{\Delta t}$ we find $\frac{\Delta f}{2\Delta t} = \frac{1}{2(0.5 \times 10^{-6} \text{ s})} = 1 \times 10^6 \text{ Hz}$.

As the range is $2\Delta f$, the range is $2 \times 10^6 \text{ Hz}$. For $\Delta t = 0.5 \text{ ns}$, the range is $2\Delta f = 2 \times 10^9 \text{ Hz}$.

5-35 (a)

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k)e^{ikx} dk = \frac{A}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\alpha^2(k-k_0)^2} e^{ikx} dk = \frac{A}{\sqrt{2\pi}} e^{-\alpha^2 k_0^2} \int_{-\infty}^{+\infty} e^{-\alpha^2(k^2 - (2k_0 + ix/\alpha^2)k)} dk$$

. Now complete the square in order to get the integral into the standard form

$$\int_{-\infty}^{+\infty} e^{-az^2} dz:$$

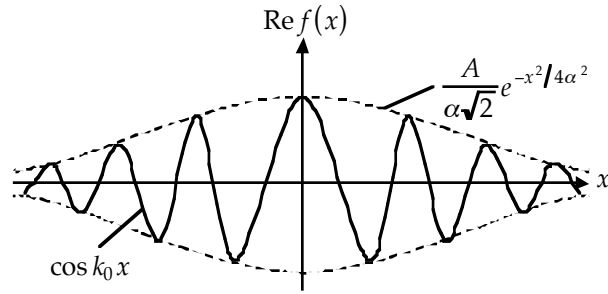
$$e^{-\alpha^2(k^2 - (2k_0 + ix/\alpha^2)k)} = e^{+\alpha^2(k_0 + ix/2\alpha^2)^2} e^{-\alpha^2(k - (k_0 + ix/2\alpha^2))^2}$$

$$\begin{aligned} f(x) &= \frac{A}{\sqrt{2\pi}} e^{-\alpha^2 k_0^2} e^{+\alpha^2(k_0 + ix/2\alpha^2)^2} \int_{k=-\infty}^{+\infty} e^{-\alpha^2(k - (k_0 + ix/2\alpha^2))^2} dk \\ &= \frac{A}{\sqrt{2\pi}} e^{-x^2/4\alpha^2} e^{ik_0 x} \int_{z=-\infty}^{+\infty} e^{-\alpha^2 z^2} dz \end{aligned}$$

where $z = k - \left(k_0 + \frac{ix}{2\alpha^2}\right)$. Since $\int_{z=-\infty}^{+\infty} e^{-\alpha^2 z^2} dz = \frac{\pi^{1/2}}{\alpha}$, $f(x) = \frac{A}{\alpha\sqrt{2}} e^{-x^2/4\alpha^2} e^{ik_0 x}$. The

real part of $f(x)$, $\text{Re } f(x)$ is $\text{Re } f(x) = \frac{A}{\alpha\sqrt{2}} e^{-x^2/4\alpha^2} \cos k_0 x$ and is a gaussian

envelope multiplying a harmonic wave with wave number k_0 . A plot of $\text{Re } f(x)$ is shown below:



Comparing $\frac{A}{\alpha\sqrt{2}} e^{-x^2/4\alpha^2}$ to $A e^{-(x/2\Delta x)^2}$ implies $\Delta x = \alpha$.

(c) By same reasoning because $\alpha^2 = \frac{1}{4\Delta k^2}$, $\Delta k = \frac{1}{2\alpha}$. Finally $\Delta x \Delta k = \alpha \left(\frac{1}{2\alpha} \right) = \frac{1}{2}$.

5-36 $E = K = \frac{1}{2} m u^2 = h f$ and $\lambda = \frac{h}{m u}$. $v_{\text{phase}} = f \lambda = \frac{m u^2}{2 h} \frac{h}{m u} = \frac{u}{2} = v_{\text{phase}}$. This is different from the speed u at which the particle transports mass, energy, and momentum.