

**Formulas:**

Time dilation; Length contraction:  $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$  ;  $L = L_p / \gamma$  ;  $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation:  $x' = \gamma(x - vt)$  ;  $y' = y$  ;  $z' = z$  ;  $t' = \gamma(t - vx/c^2)$  ; inverse:  $v \rightarrow -v$

Spacetime interval:  $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation:  $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$  ;  $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$  ; inverse:  $v \rightarrow -v$

Relativistic Doppler shift:  $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$  (approaching)

Momentum:  $\vec{p} = \gamma m \vec{u}$  ; Energy:  $E = \gamma mc^2$  ; Kinetic energy:  $K = (\gamma - 1)mc^2$

Rest energy:  $E_0 = mc^2$  ;  $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron:  $m_e = 0.511 \text{ MeV}/c^2$  ; Proton:  $m_p = 938.26 \text{ MeV}/c^2$  ; Neutron:  $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit:  $1 u = 931.5 \text{ MeV}/c^2$  ; electron volt:  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law:  $e_{tot} = \sigma T^4$  ,  $e_{tot} = \text{power/unit area}$  ;  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$  ,  $U = \text{energy density} = \int_0^\infty u(\lambda, T) d\lambda$  ; Wien's law:  $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution:  $P(E) = C e^{-E/(k_B T)}$

Planck's law:  $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$  ;  $N(f) = \frac{8\pi f^2}{c^3}$

Photons:  $E = hf = pc$  ;  $f = c/\lambda$  ;  $hc = 12,400 \text{ eV \AA}$  ;  $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect:  $eV_s = K_{max} = hf - \phi$  ,  $\phi \equiv \text{work function}$ ; Bragg equation:  $n\lambda = 2d \sin \theta$

Compton scattering:  $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$ ;  $\frac{h}{m_e c} = 0.0243 \text{ \AA}$  ; Coulomb constant:  $ke^2 = 14.4 \text{ eV \AA}$

Coulomb force:  $F = \frac{kq_1 q_2}{r^2}$  ; Coulomb potential:  $V = \frac{kq}{r}$  ; Coulomb energy:  $U = \frac{kq_1 q_2}{r}$

Force in electric and magnetic fields (Lorentz force):  $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering:  $\Delta n = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\phi/2)}$  ;  $\hbar c = 1,973 \text{ eV \AA}$

Hydrogen spectrum:  $\frac{1}{\lambda_{mn}} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right)$  ;  $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Bohr atom:  $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$  ;  $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$  ;  $K = \frac{m_e v^2}{2}$  ;  $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$  ;  $r_n = r_0 n^2$  ;  $r_0 = \frac{a_0}{Z}$  ;  $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$  ;  $L = m_e v r = n\hbar$  angular momentum

de Broglie:  $\lambda = \frac{h}{p}$  ;  $f = \frac{E}{h}$  ;  $\omega = 2\pi f$  ;  $k = \frac{2\pi}{\lambda}$  ;  $E = \hbar \omega$  ;  $p = \hbar k$  ;  $E = \frac{p^2}{2m}$

Wave packets:  $y(x, t) = \sum_j a_j \cos(k_j x - \omega_j t)$ , or  $y(x, t) = \int dk a(k) e^{i(kx - \omega(k)t)}$ ;  $\Delta k \Delta x \sim 1$  ;  $\Delta \omega \Delta t \sim 1$

group and phase velocity :  $v_g = \frac{d\omega}{dk}$  ;  $v_p = \frac{\omega}{k}$  ; Heisenberg:  $\Delta x \Delta p \sim \hbar$  ;  $\Delta t \Delta E \sim \hbar$

Schrodinger equation:  $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x, t) = i\hbar \frac{\partial \Psi}{\partial t}$  ;  $\Psi(x, t) = \psi(x) e^{-i\frac{E}{\hbar} t}$

Time – independent Schrodinger equation:  $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$ ;  $\int_{-\infty}^{\infty} dx |\psi(x)|^2 = 1$

$\infty$  square well:  $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ ;  $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$ ;  $\frac{\hbar^2}{2m_e} = 3.81 \text{ eV} \text{ \AA}^2$  (electron)

Harmonic oscillator:  $\Psi_n(x) = H_n(x) e^{-\frac{m\omega}{2\hbar} x^2}$ ;  $E_n = (n + \frac{1}{2})\hbar\omega$ ;  $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$ ;  $\Delta n = \pm 1$

Expectation value of  $[Q]$ :  $\langle Q \rangle = \int \psi^*(x)[Q]\psi(x) dx$ ; Momentum operator:  $p = \frac{\hbar}{i} \frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions:  $[Q]\Psi = q\Psi$  ( $q$  is a constant); uncertainty:  $\Delta Q = \sqrt{\langle Q^2 \rangle - \langle Q \rangle^2}$

Step potential: reflection coef:  $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$ ,  $T = 1 - R$ ;  $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling:  $\psi(x) \sim e^{-\alpha x}$ ;  $T = e^{-2\alpha \Delta x}$ ;  $T = e^{-2 \int_{x_1}^{x_2} \alpha(x) dx}$ ;  $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Schrodinger equation in 3D:  $-\frac{\hbar^2}{2m} \nabla^2 \Psi + U(\vec{r})\Psi(\vec{r}, t) = i\hbar \frac{\partial \Psi}{\partial t}$ ;  $\Psi(\vec{r}, t) = \psi(\vec{r}) e^{-i\frac{E}{\hbar} t}$

3D square well:  $\Psi(x, y, z) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$ ;  $E = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2} \right)$

Spherically symmetric potential:  $\Psi_{n, \ell, m_\ell}(r, \theta, \phi) = R_{n\ell}(r) Y_\ell^{m_\ell}(\theta, \phi)$ ;  $Y_\ell^{m_\ell}(\theta, \phi) = P_\ell^{m_\ell}(\theta) e^{im_\ell \phi}$

Angular momentum:  $\vec{L} = \vec{r} \times \vec{p}$ ;  $[L_z] = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$ ;  $[L^2] Y_\ell^{m_\ell} = \ell(\ell + 1) \hbar^2 Y_\ell^{m_\ell}$ ;  $[L_z] = m\hbar$

Radial probability density:  $P(r) = r^2 |R_{n\ell}(r)|^2$ ; Energy:  $E_n = -\frac{ke^2 Z^2}{2a_0 n^2}$

Ground state of hydrogen and hydrogen-like ions:  $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}} \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$

Orbital magnetic moment:  $\vec{\mu} = \frac{-e}{2m_e} \vec{L}$ ;  $\mu_z = -\mu_B m_\ell$ ;  $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} \text{ eV/T}$

Spin 1/2:  $s = \frac{1}{2}$ ,  $|S| = \sqrt{s(s+1)}\hbar$ ;  $S_z = m_s \hbar$ ;  $m_s = \pm 1/2$ ;  $\vec{\mu}_s = \frac{-e}{2m_e} g\vec{S}$

Orbital + spin mag moment:  $\vec{\mu} = \frac{-e}{2m} (\vec{L} + g\vec{S})$ ; Energy in mag. field:  $U = -\vec{\mu} \cdot \vec{B}$

Two particles:  $\Psi(\vec{r}_1, \vec{r}_2) = +/ - \Psi(\vec{r}_2, \vec{r}_1)$ ; symmetric/antisymmetric

Screening in multielectron atoms:  $Z \rightarrow Z_{\text{eff}}$ ,  $1 < Z_{\text{eff}} < Z$

Orbital ordering:

$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d \sim 5f$

**Justify all your answers to all problems. Write clearly.**