

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation : $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse : $v \rightarrow -v$

Spacetime interval : $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation : $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse : $v \rightarrow -v$

Relativistic Doppler shift : $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$ (approaching)

Momentum : $\vec{p} = \gamma m \vec{u}$; Energy : $E = \gamma mc^2$; Kinetic energy : $K = (\gamma - 1)mc^2$

Rest energy : $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron : $m_e = 0.511 \text{ MeV}/c^2$ Proton : $m_p = 938.26 \text{ MeV}/c^2$ Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law : $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W}/\text{m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution : $P(E) = C e^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons : $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV}/\text{K}$

Photoelectric effect : $eV_s = K_{max} = hf - \phi$, ϕ \equiv work function; Bragg equation : $n\lambda = 2d \sin \vartheta$

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)

The tungsten filament of the light bulb in a flashlight emits radiation approximately as a black body. When it is connected to a 3 Volt battery it emits 10 W of power. Maximum power is emitted for wavelengths around 8,000 A.

(a) What is the surface area of this filament? Give your answer in mm^2 ($1 \text{ mm} = 10^{-3} \text{ m}$).

(b) When it is connected to a 6 Volt battery, by what factor does the temperature of the filament change? Does it increase or decrease by that factor?

Hint: you learned in Physics 2B that the power dissipated in a resistor R is V^2/R . Assume all the power is dissipated by radiation.

(c) Compare the power emitted by this light bulb for wavelengths 200,000A and 400,000A when it is connected to the 6 Volt battery. By what factor do they differ? Your answer may be approximate if the approximation is well justified.

Problem 2 (10 points)

A source emits light with wavelengths in the range 2,500 Å to 7,500 Å, with the same intensity for all wavelengths in that range. The light from this source is incident on a metal surface, and the maximum kinetic energy of the emitted electrons is found to be 2eV.

- (a) If a filter is inserted between this source and this metal that blocks the light with wavelengths smaller than 5,000Å, what will be the maximum kinetic energy of the emitted electrons?
- (b) If instead a filter is inserted between this source and this metal that blocks the light with wavelengths greater than 5,000Å, what will be the maximum kinetic energy of the emitted electrons?
- (c) What is the work function of this metal? Give your answer in eV.

Problem 3 (10 points)

In a Compton scattering experiment, the scattered photon has wavelength 0.5Å and the scattered electron has kinetic energy 1,033 eV.

- (a) Find the wavelength of the incident photon.
- (b) Find the angle at which the photon is scattered relative to the direction of incidence (in degrees).
- (c) Find the y-component of the momentum of the scattered electron, in units eV/c , where y is the direction perpendicular to the incidence direction in the plane where the scattering occurs.