

Formulas:

Time dilation; Length contraction : $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation : $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse : $v \rightarrow -v$

Spacetime interval : $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation : $u_x' = \frac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse : $v \rightarrow -v$

Relativistic Doppler shift : $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$ (approaching)

Momentum : $\vec{p} = \gamma m \vec{u}$; Energy : $E = \gamma mc^2$; Kinetic energy : $K = (\gamma - 1)mc^2$

Rest energy : $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron : $m_e = 0.511 \text{ MeV}/c^2$; Proton : $m_p = 938.26 \text{ MeV}/c^2$; Neutron : $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit : $1 u = 931.5 \text{ MeV}/c^2$; electron volt : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law : $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution : $P(E) = C e^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons : $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV/K}$

Photoelectric effect : $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation : $n\lambda = 2d \sin \vartheta$

Compton scattering : $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$; Coulomb constant : $ke^2 = 14.4 \text{ eV \AA}$

Coulomb force : $F = \frac{kq_1 q_2}{r^2}$; Coulomb potential : $V = \frac{kq}{r}$; Coulomb energy : $U = \frac{kq_1 q_2}{r}$

Force in electric and magnetic fields (Lorentz force) : $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering : $\Delta n = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\phi/2)}$; $\hbar c = 1,973 \text{ eV \AA}$

Hydrogen spectrum : $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{911.3 \text{ \AA}}$

Bohr atom : $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$; $L = m_e v r = n\hbar$ angular momentum

Justify all your answers to all problems. Write clearly.

Problem 1 (10 points)

Rutherford scattering experiments are performed with an Al foil and with an Au foil as targets. The radius of the Al nucleus ($Z=13$) is approximately $4.9 \times 10^{-5} \text{A}$, the radius of the Au nucleus ($Z=79$) is approximately $7.3 \times 10^{-5} \text{A}$.

(a) With incident α -particle kinetic energy 5 MeV, how many α -particles will be scattered at 45° angle for every 100 particles scattered at 180° angle for (i) Al and (ii) Au?

(b) For what range of α -particle kinetic energies do you expect deviation from the result found in (a) for the case of Al? Will there be more or fewer α -particles scattered at 45° angle for every 100 particles scattered at 180° angle in that range?

(c) Same as (b) for the case of Au.

Problem 2 (10 points)

(a) An electron in H and an electron in He^+ have the same total energy (obviously they can't both be in their ground states). How do their angular momenta compare? Find their ratio.

(b) Light of wavelengths in the range 1000A to 5000A is incident on a mixture of H atoms and He^+ ions, where the electrons are in their lowest energy state (ground state) in both the H atoms and the He^+ ions. What wavelengths will be absorbed?

(c) Following (b), what wavelengths will be emitted?

Problem 3 (10 points)

The distance between plates in a Thomson tube is 1cm, the plates length is 5cm, and the applied voltage between plates is 50 V. When a perpendicular magnetic field of magnitude $7.5 \times 10^{-4} \text{T}$ is applied, the beam of electrons does not deflect.

(a) Find the speed of the electrons, in m/s.

(b) Find the angle at which the electrons are deflected when the magnetic field is turned off, in degrees.

Use $e/m_e = 1.76 \times 10^{11} \text{C/kg}$.