PHYSICS 2D QUIZ 5 WINTER QUARTER 2011
PROF. HIRSCH MARCH 4th, 2011

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Velocity transformation: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c} \,/\, \sqrt{1 - v/c}$ (approaching)

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Atomic mass unit: $1\, u = 931.5\, MeV/c^2$; electron volt: $1eV = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, $e_{tot} =$ power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, $U =$ energy density $= \int_0^\infty u(\lambda, T)d\lambda$; Wien's law: $\lambda_m T = \frac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda)\times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4}\times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\; eV A$; $k_B = (1/11{,}600) eV/K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d\sin\vartheta$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$; $\frac{h}{m_e c} = 0.0243A$; Coulomb constant: $ke^2 = 14.4\; eV A$

Coulomb force: $F = \frac{kq_1q_2}{r^2}$; Coulomb potential: $V = \frac{kq}{r}$; Coulomb energy: $U = \frac{kq_1q_2}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n = C\frac{Z^2}{K_\alpha^2}\frac{1}{\sin^4(\phi/2)}$; $\hbar c = 1{,}973\; eV A$

Hydrogen spectrum: $\frac{1}{\lambda_{mn}} = R(\frac{1}{m^2} - \frac{1}{n^2})$; $R = 1.097\times 10^7\; m^{-1} = \frac{1}{911.3A}$

Bohr atom: $E_n = -\frac{ke^2Z}{2r_n} = -E_0\frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity: $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

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Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+U(x)\psi(x)=E\psi(x)$; $\int_{-\infty}^{\infty} dx\ |\psi(x)|^2=1$

∞ square well: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n=\frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e}=3.81 eVA^2$ (electron)

Harmonic oscillator: $\Psi_n(x)=H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n=(n+\frac{1}{2})\hbar\omega$; $E=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\omega^2A^2$; $\Delta n=\pm 1$

Expectation value of[Q]: $<Q>=\int\psi^*(x)[Q]\psi(x)\,dx$; Momentum operator: $p=\frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: [Q] $\Psi = q\,\Psi$ (q is a constant) ; uncertainty: $\Delta Q=\sqrt{<Q^2>-<Q>^2}$

Step potential: reflection coef: $R=\frac{(k_1-k_2)^2}{(k_1+k_2)^2}$, $T=1-R$; $k=\sqrt{\frac{2m}{\hbar^2}(E-U)}$

Tunneling: $\psi(x)\sim e^{-\alpha x}$; $T=e^{-2\alpha\Delta x}$; $T=e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x)=\sqrt{\frac{2m[U(x)-E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2m}\nabla^2\Psi+U(\vec{r})\Psi(\vec{r},t)=i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\vec{r},t)=\psi(\vec{r})e^{-i\frac{E}{\hbar}t}$

3D square well: $\Psi(x,y,z)=\Psi_1(x)\Psi_2(y)\Psi_3(z)$; $E=\frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2})$

Justify all your answers to all problems. Write clearly.

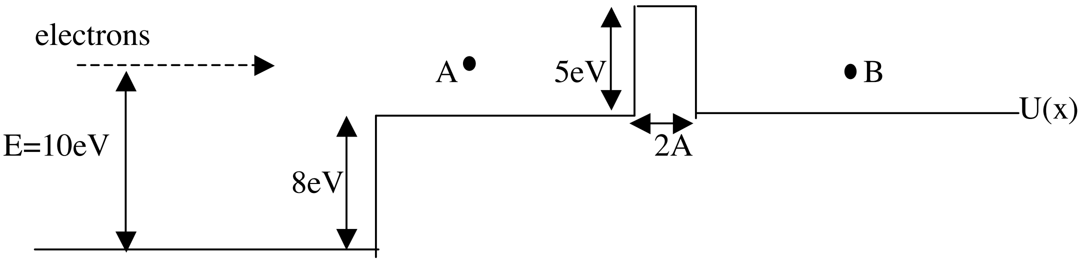
Problem 1 (10 points)

An electron in the ground state of a quantum oscillator has uncertainty in momentum $\Delta p=500eV/c$.

(a) Find the uncertainty in the position, Δx, in A, and find $<x^2>$ in A^2.
(b) Find the average kinetic energy <K>, with $K=p^2/2m_e$, in eV.
(c) Find the average potential energy <U>, with $U=\frac{1}{2}m_e\omega^2x^2$, in eV. Justify your answer.
(d) What is the wavelength of the photons that this system will absorb or emit? Give your answer in A (Angstrom).

You may use that $\Delta x\Delta p=\hbar/2$ and that $<x^2>=\frac{\hbar}{2m_e\omega}$ for the ground state of the quantum oscillator potential without proving these relations.

Problem 2 (10 points)



1000 electrons of kinetic energy 10eV are incident from the left and encounter the potential U(x) shown in the figure.

(a) How many electrons will reach point A?

(b) How many electrons will reach point B?

(c) According to classical mechanics, how many electrons would reach point A? Point B?

Use $\hbar^2/2m_e = 3.81 eV\,A^2$.

Problem 3 (10 points)

A three-dimensional box has side lengths $L_1=L_2=1A$, $L_3=0.6667A$. For an electron in this box,

(a) Find the four lowest distinct energy values, in eV.

(b) Draw a schematic energy level diagram showing these energy levels and how many states correspond to each energy level (degeneracy). Give the quantum numbers of all the states corresponding to each energy level.

(c) Assuming the dimensions L_1, L_2, L_3 correspond to the x, y and z directions, give the quantum numbers and the energies of the two lowest energy states for which the wavefunction is zero on the plane defined by the equation $y=L_2/2$.

Use $\hbar^2\pi^2/2m_e = 37.6\ eV\,A^2$

Justify all your answers to all problems. Write clearly.