PHYS 4D
Solution to HW 1

January 9, 2011

Problem Giancoli 31-1 (I) Determine the rate at which the electric field changes between the round plates of a capacitor, $6.0cm$ in diameter, if the plates are spaced $1.1mm$ apart and the voltage across them is changing at a rate of $120V/s$.

Solution:

The electric field between the plates depends on the voltage:

$$E = V/d,$$

so

$$\frac{dE}{dt} = \frac{1}{d}\frac{dV}{dt} = \frac{1}{1.1 \times 10^{-3}m} 120V/s = 1.1 \times 10^5 V/(m \cdot s)$$

Problem Giancoli 31-3 (II) At a given instant, a $2.8A$ current flows in the wires connected to a parallel-plate capacitor. What is the rate at which the electric field is changing between the plates if the square plates are $1.60cm$ on a side?

Solution:

The current in the wires must also be the displacement current in the capacitor. We find the rate at which the electric field is changing from

$$I_D = \varepsilon_0 A \frac{dE}{dt}$$

$\Rightarrow$

$$2.8A = (8.85 \times 10^{-12} C^2/N \cdot\ m^2)(0.0160m)^2 \frac{dE}{dt}$$

$\Rightarrow$

$$\frac{dE}{dt} = 1.2 \times 10^{15} V/m \cdot\ s.$$

Problem Giancoli 31-5 (II) Show that the displacement current through a parallel-plate capacitor can be written $I_D = CdV/dt$, where V is the voltage across the capacitor at any instant.

Solution:

The electric field between the plates depends on the voltage:

$$E = V/d,$$

so

$$dE/dt = (1/d)dV/dt.$$

Thus the displacement current is

$$I_D = \varepsilon_0 A \frac{dE}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = CdV/dt$$

Problem Giancoli 31-7 (III) Suppose that a circular parallel-plate capacitor has radius $R_0 = 3.0cm$ and plate separation $d = 5.0mm$. A sinusoidal potential difference $V = V_0 sin(2\pi ft)$ is applied across the plates, where $V_0 = 150V$ and $f = 60Hz$.

(a) In the region between the plates, show that the magnitude of the induced magnetic field is given by $B = B_0(R)cos(2\pi ft)$, where R is the radial distance from the capacitor's central axis.

(b) Determine the expression for the amplitude $B_0(R)$ of this time-dependent (sinusoidal) field when $R \leq R_0$, and when $R > R_0$.

(c) Plot $B_0(R)$ in tesla for the range $0 \leq R \leq 10cm$.

Solution:

(a) (b) By Ampere's law, we have

For $R \leq R_0$,

$$B(2\pi r) = \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt}.$$

$\Rightarrow$

$$B = (1/2)\mu_0 \varepsilon_0 r \frac{dE}{dt} = V_0 \pi f/(dc^2) r cos(2\pi f t) = B_0(R) cos(2\pi f t),$$

$\Rightarrow$

$$\begin{aligned} B_0(R) &= rV_0 \pi f/(dc^2) \\ &= r \times 150V \times 3.14 \times 60Hz/(\times 5.0 \times 10^{-3}m \times \left(3.0 \times 10^8 m/s\right)^2) \\ &= r \times 6.28 \times 10^{-11} T/m. \end{aligned}$$

For $R > R_0$,

$$B(2\pi r) = \mu_0 \varepsilon_0 \pi R_0^2 \frac{dE}{dt}.$$

$\Rightarrow$

$$B = (1/2)\mu_0 \varepsilon_0 r \frac{dE}{dt} = \frac{1}{r} R_0^2 V_0 \pi f/(dc^2) cos(2\pi f t) = B_0(R) cos(2\pi f t),$$

$\Rightarrow$

$$\begin{aligned} B_0(R) &= \frac{1}{r} R_0^2 V_0 \pi f/(dc^2) \\ &= \frac{1}{r} \times (3.0 \times 10^{-2})^2 \times 6.28 \times 10^{-11} Tm \\ &= \frac{1}{r} \times 5.62 \times 10^{-14} Tm. \end{aligned}$$

Problem Giancoli 31-8 (I) If the electric field in an EM wave has a peak magnitude of $0.57 \times 10^{-4} V/m$, what is the peak magnitude of the magnetic field strength?

Solution:

We find the magnetic field from

$$\begin{aligned} E_0 &= cB_0, \\ 0.57 \times 10^{-4} V/m &= (3.00 \times 10^8 m/s)B. \\ B &= 1.9 \times 10^{-13} T. \end{aligned}$$

Problem Giancoli 31-10 (I) In an EM wave traveling west, the B field oscillates vertically and has a frequency of $80.0kHz$ and an rms strength of $7.75 \times 10^{-9} T$. Determine the frequency and rms strength of the electric field. What is its direction?

Solution:

The frequency of the two fields must be the same:

$$80.0kHz.$$

The rms strength of the electric field is

$$E_{rms} = cB_{rms} = (3.00 \times 10^8 m/s)(7.75 \times 10^{-9} T) = 2.33V/m.$$

The electric field is perpendicular to both the direction of travel and the magnetic field, so the electric field oscillates along the horizontal north-south line.

Problem Giancoli 31-12 (III) Consider two possible candidates $E(x,t)$ as solutions of the wave equation for an EM wave's electric field. Let A and α be constants. Show that (a) $E(x,t) = Ae^{-\alpha(x-vt)^2}$ satisfies the wave equation, and that (b) $E(x,t) = Ae^{-(\alpha x^2 - vt)}$ does not satisfy the wave equation.

Solution 1: Check by putting into the wave equation directly.

Solution 2: Noticing the wave equation has scaling invariance: when we multiply t and x by the same constant w, the equation is invariant, we find only the first solution satisfies the invariance in its form.

Problem Giancoli 31-13 (I) What is the frequency of a microwave whose wavelength is $1.50cm$?

Solution:

The frequency of the microwave is

$$f = c/\lambda = (3.00 \times 10^8 m/s)/(1.50 \times 10^{-2} m) = 2.00 \times 10^{10} Hz.$$

Problem Giancoli 31-15 (I) How long does it take light to reach us from the Sun, $1.50 \times 10^8 km$ away?

Solution:

$$t = d/c = (1.50 \times 10^{11} m)/(3.00 \times 10^8 m/s) = 5.00 \times 10^2 s.$$

Problem Giancoli 31-18 (II) Pulsed lasers used for science and medicine produce very brief bursts of electromagnetic energy. If the laser light wavelength is $1062nm$ (Neodymium- YAG laser), and the pulse lasts for $38\,picoseconds$, how many wavelengths are found within the laser pulse? How brief would the pulse need to be to fit only one wavelength?

Solution: The length of the pulse is $d = ct$, so the number of wavelengths in this length is

$$N = (ct)/\lambda = (3.00 \times 10^8 m/s)(38 \times 10^{-12} s)/(1062 \times 10^{-9} m) = 1.1 \times 10^4 wavelengths.$$

The time for the length of the pulse to be one wavelength is

$$t' = \lambda/c = (1062 \times 10^{-9} m)/(3.00 \times 10^8 m/s) = 3.54 \times 10^{-15} s = 3.54 fs.$$

Problem Giancoli 31-20 (II) An electromagnetic wave has an electric field given by $\mathbf{E} = \mathbf{i}(225V/m) sin[(0.077m^{-1})z - (2.3 \times 10^7 rad/s)t]$.

(a) What are the wavelength and frequency of the wave?

(b) Write down an expression for the magnetic field.

Solution:

(a) The wavelength is

$$\lambda = 2\pi/k = 2\pi/(0.077m^{-1}) = 81.60m$$

The frequency is

$$f = \omega/2\pi = (2.3 \times 10^7 rad/s)/(2\pi rad) = 3.66 \times 10^6 Hz.$$

(b)

$$B_0 = E_0/c = (225V/m)/(3.00 \times 10^8 m/s) = 7.5 \times 10^{-7} T.$$

$$\mathbf{B} = \mathbf{j}(7.5 \times 10^{-7} T) sin[(0.077m^{-1})z - (2.37.5 \times 10^7 rad/s)t].$$