PHYS 4D
Solution to Quiz 1

January 14, 2011

Problem 1 Electromagnetic waves and sound waves can have the same frequency.

(a) What is the wavelength of a $1KHz$ electromagnetic wave?
Solution:

$$\lambda = c/f = (3.0 \times 10^8 m/s)/(1 \times 10^3/s) = 3 \times 10^5 m.$$

(b) What is the wavelength of a $1KHz$ sound wave? ($v_{sound} = 341m/s$)
Solution:

$$\lambda = v_{sound}/f = (341m/s)/(1 \times 10^3/s) = 0.341m.$$

Problem 2 An electromagnetic wave has a magnetic field given by

$$\mathbf{B} = \mathbf{i}B_0 \cos(kz - \omega t),$$

where $B_0 = 1.5 \times 10^{-4}T$, $k = 10^{-3}m^{-1}$ and $\omega = 3 \times 10^5 rad/s$.

(a) Write down an expression for the electric field.

Solution: The directions of $\mathbf{E}, \mathbf{B}, \mathbf{k}$ obey $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{k}}$. Now, $\hat{\mathbf{B}} = \mathbf{i}$, $\hat{\mathbf{k}} = \mathbf{z} \Rightarrow \hat{\mathbf{E}} = -\mathbf{j}$.

$$E_0 = cB_0 = 4.5 \times 10^4 V/m.$$

$$\mathbf{E} = -\mathbf{j}E_0 \cos(kz - \omega t),$$

(b) What are the wave length and frequency of the wave?
Solution:

$$\lambda = \frac{2\pi}{k} = 6.3 \times 10^3 m, f = \frac{\omega}{2\pi} = 4.8 \times 10^4 Hz.$$

Problem 3 An air-gap capacitor has circular plates of radius $k = 2.5cm$ and separation $d = 1.6mm$. A $76.0Hz$ emf $V = V_0 \cos \omega t$ is applied to the capacitor. The maximum displacement current is $35\mu A$. Determine

(a) the maximum conduction current I;

Solution: The footnote on page 816 indicates that Kirchhoff's junction rule is valid at a capacitor plate, and so the conduction current is the same as the displacement current. Thus $I_{\max} = 35\mu A$.

(b) the value of V_0;

Solution:

$$\omega = 2\pi f = 4.78 \times 10^2 rad/s$$

$$\begin{aligned} I &= \frac{dQ}{dt} = C\frac{dV}{dt} = -C\omega V_0 \sin\omega t, \\ I_{\max} &= C\omega V_0, C = \frac{\varepsilon_0 A}{d}. \\ &\Rightarrow V_0 = \frac{I_{\max}}{C\omega} = \frac{I_{\max} d}{\omega\varepsilon_0 A} = 6.75 \times 10^3 V \end{aligned}$$

(c) the maximum value of $d\Phi_E/dt$ between plates.
Solution:

$$\begin{aligned} I_D &= \varepsilon_0 \frac{d\Phi_E}{dt} \Rightarrow \\ \left(\frac{d\Phi_E}{dt}\right)_{max} &= \frac{I_{\max}}{\varepsilon_0} = 3.95 \times 10^6 V \cdot m/s. \end{aligned}$$

Bonus Problem A 1500-nF capacitor with circular parallel plates $2cm$ in diameter is accumulating charge at the rate $38mC/s$ at some instant in time.
(a) What will be the induced magnetic field strength $10cm$ radially outward from the center of the plates?
Solution:

$$\begin{aligned} B(2\pi r) &= \mu_0 I. \\ B &= \frac{\mu_0 I}{2\pi r} = 7.6 \times 10^{-8} T \end{aligned}$$

(b) What will be the value of the field strength after the capacitor is fully charged?
Solution: Since $I = 0$, $B = 0$.