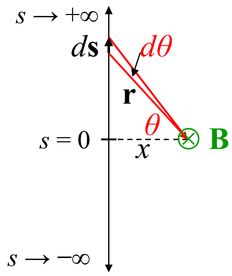
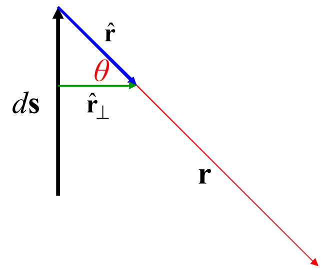
To find the B-field from an infinite straight wire, we integrate over Biot-Savart current elements.





We set up the integral, taking **B** into the page from the right hand rule. Notice that for our choice of θ, the perpendicular component of $\hat{\mathbf{r}}$, $\hat{\mathbf{r}}_\perp$, is cos θ, rather than sin θ:

$$d\mathbf{B}(x) = k_m I \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}, \qquad \cos\theta = \frac{x}{\left(s^2 + x^2\right)^{1/2}}$$

$$B(x) = k_m I \int_{-\infty}^{\infty} \frac{ds \cos\theta}{s^2 + x^2} = k_m I \int_{-\infty}^{\infty} \frac{ds}{s^2 + x^2} \frac{x}{\left(s^2 + x^2\right)^{1/2}} = k_m I \int_{-\infty}^{\infty} \frac{x\, ds}{\left(s^2 + x^2\right)^{3/2}}$$

The integral can be evaluated with a trigonometric substitution, but that is equivalent to setting up the problem in terms of θ in the first place:

$$\tan\theta = \frac{s}{x} \quad \Rightarrow \quad s = x \tan\theta, \qquad ds = x \sec^2\theta\, d\theta$$

$$\cos\theta = \frac{x}{r} \quad \Rightarrow \quad r = x \sec\theta$$

$$B(x) = k_m I \int_{-\infty}^{\infty} \frac{ds \cos\theta}{r^2} = k_m I \int_{-\pi/2}^{+\pi/2} \frac{x \sec^2\theta\, d\theta \cos\theta}{x^2 \sec^2\theta} = k_m I \int_{-\pi/2}^{+\pi/2} \frac{d\theta \cos\theta}{x}$$

$$= \frac{k_m I}{x} \left[\sin\theta\right]_{-\pi/2}^{+\pi/2} = \frac{2 k_m I}{x} \quad or \quad \frac{\mu_0 I}{2\pi x}, \qquad \text{since} \quad k_m = \frac{\mu_0}{4\pi}$$