Serway and Jewett, *Principles of Physics*, Physics I: Volume II

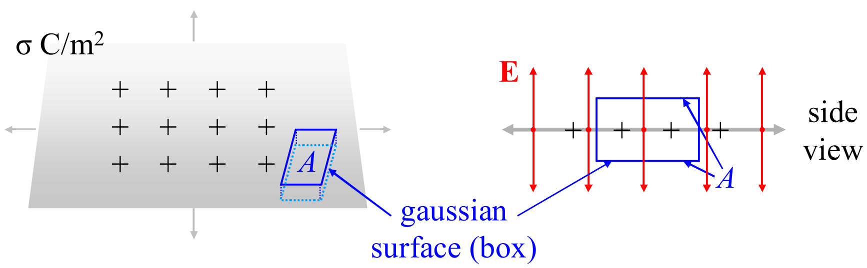
**Chapter 19, Problem 19.49:**

The book's solution is correct, but done the hard way. I will show the easy way first, then discuss why the book's way is also correct.

The question states a "thin" conductor. "Thin" means "negligibly thick." We can take the sheet charge as infinitesimally thin, so the areal density is simply

$$\sigma = \frac{Q}{A_{plate}} \quad \text{in C/m}^2$$

As on the slide in class, we take our Gaussian surface to be a box with vertical sides. The only flux is through the top and bottom surfaces, each of area $A$:

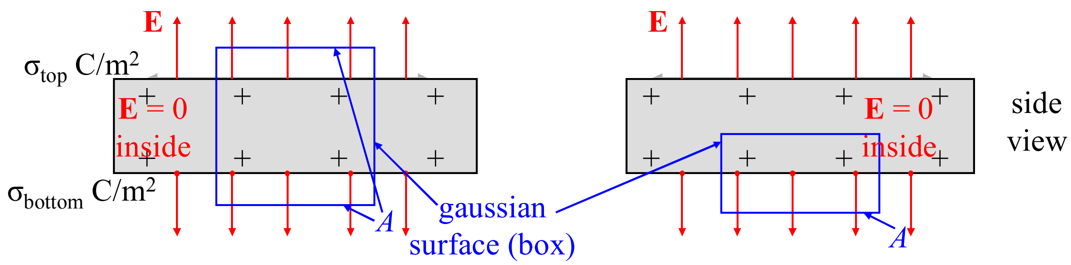



Again, as in class, we find **E** from:

$$\Phi_E = 4\pi k_e q_{in} \quad \Rightarrow \quad 2EA = 4\pi k_e \sigma A, \quad E = 2\pi k_e \sigma = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A_{plate}}$$

Just below the plate, the E-field points down, so $\boxed{\mathbf{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{z}}}$.

The book treats the plate as "thick", and explicitly distributes the charge equally over the top and bottom surfaces. While this is not necessary, it also does not hurt:




In this case, the sheet charge density on each face is ½ the total charge density, because the total charge is split between the top and bottom surfaces:

$$\sigma_{top} = \sigma_{bottom} = \frac{Q}{2A_{plate}}$$

Even in this model, we can still choose a gaussian surface as above left, and the charge enclosed is exactly as in the thin-plate model. Gauss' Law then yields exactly the same result. The book chose to put the Gaussian surface with one face inside the plate, as above right. Conductors always have zero E-field inside them, so the E-field is zero inside the plate. Thus there is only flux out the bottom of the Gaussian surface. Gauss' Law is then:

$$\Phi_E = \frac{q_{in}}{\varepsilon_0} \qquad \Rightarrow \qquad EA = \frac{\sigma_{bottom} A}{\varepsilon_0}, \quad E = \frac{\sigma_{bottom}}{\varepsilon_0} = \frac{Q}{2A_{plate}\varepsilon_0},$$

which is exactly the same answer as in the thin-plate model, as it must be.