# Uncertainty, Measurement, and Models
# Overview Exp #1

Lecture # 2

Physics 2BL

Spring Quarter 2012

# Laboratory TAs

| | | |
|---|---|---|
| Leandra | Boucheron | lsbouche@ucsd.edu |
| Alvin | Cabrera | amcabrer@ucsd.edu |
| Casey | Conger | cconger@ucsd.edu |
| L. Stevie | Green | lsgreen@ucsd.edu |
| Han | Lin | hwl005@ucsd.edu |
| Devon | Murphy | dwmurphy@ucsd.edu |
| Andreas | Stergiou | anstergi@ucsd.edu |
| Po-Chun | Wu | pow004@ucsd.edu |
| Jingxin | Ye | j9ye@ucsd.edu |

| | | |
|---|---|---|
| A03 | **Boucheron**, **Lin** | 744463 |
| A01 | **Conger**, **Green** | 744461 |
| A02 | **Stergiou**, **Ye** | 744462 |
| A04 | **Boucheron**, **Conger** | 744464 |
| A05 | **Green**, **Wu** | 744465 |
| A06 | **Stergiou**, **Ye** | 744466 |
| A07 | **Lin**, **Green** | 744467 |
| A08 | **Green**, **Ye** | 744468 |
| A09 | **Murphy** | |
| A10 | **Murphy**, **Cabrera** | 744470 |

# Laboratory LTAC

Tera Austrum tbell@physics.ucsd.edu

| | | | | |
|---|---|---|---|---|
| | A03 | | A07 | |
| | Boucheron | A04 | Lin | A10 |
| | Lin | Boucheron | Green | Murphy |
| | 744463 | Conger | 744467 | Cabrera |
| | A01 | 744464 | | 744470 |
| | Conger | A05 | | |
| | Green | Green | | |
| | 744461 | Wu | | |
| | A02 | 744465 | A08 | |
| | Stergiou | A06 | Green | |
| | Ye | Stergiou | Ye | |
| | 744462 | Ye | 744468 | |
| | | 744466 | A09 | |
| | | | Murphy | |

# Outline

- What uncertainty (error) analysis can for you
- Issues with measurement and observation
- What does a model do?
- General error propagation formula with example
- Overview of Experiment # 1
- Homework

# What is uncertainty (error)?

- Uncertainty (or error) in a measurement is not the same as a mistake
- Uncertainty results from:
  - Limits of instruments
    - finite spacing of markings on ruler
  - Design of measurement
    - using stopwatch instead of photogate
  - Less-well defined quantities
    - composition of materials

# Understanding uncertainty is important

- for comparing values
- for distinguishing between models
- for designing to specifications/planning

Measurements are less useful (often useless) without a statement of their uncertainty

# An example

- Batteries
  - rated for 1.5 V potential difference across terminals
  - in reality…

# Utility of uncertainty analysis

- Evaluating uncertainty in a measurement
- Propagating errors – ability to extend results through calculations or to other measurements
- Analyzing a distribution of values
- Quantifying relationships between measured values

# Evaluating error in measurements

- To measure height of building, drop rock and measure time to fall: $d = \frac{1}{2} g t^2$
- Measure times

  2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s
- What is the "best" value
- How certain are we of it?

# Calculate "best" value of the time

- Calculate average value (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
  - $\bar{t} = \sum_{i=1}^{n} t_i/n$
  - $\bar{t} = 2.516666666666666666666666 \text{ s}$
- Is this reasonable?

Significant figures

# Uncertainty in time

- Measured values - (2.6s, 2.4s, 2.5s, 2.4s, 2.3s, 2.9s)
- By inspection can say uncertainty < 0.4 s
- Calculate **standard deviation**

$$\sigma = \sqrt{\sum (t_i - \overline{t})^2/(n-1)}$$

$\sigma = 0.2137288$ s

$\sigma = 0.2$ s (But what does this mean???)

# How to quote best value

- What is uncertainty in average value?
  - Introduce **standard deviation of the mean**

    $\sigma_{\bar{t}} = \sigma/\sqrt{n} = 0.08725\ s = 0.09\ s$
- Now what is best quote of average value
  - $\bar{t} = 2.516666666666666666666666\ s$
  - $\bar{t} = 2.52\ s$
- Best value is
  - $\bar{t} = 2.52 \pm 0.09\ s$

# Propagation of error

- Same experiment, continued…
- From best estimate of time, get best estimate of distance: 31 meters
- Know uncertainty in time, what about uncertainty in distance?
- From error analysis tells us how errors propagate through mathematical functions

(2 meters)

# Expected uncertainty in a calculated sum $a = b + c$

- Each value has an uncertainty
  - $b = \bar{b} \pm \delta b$
  - $c = \bar{c} \pm \delta c$
- Uncertainty for $a$ ($\delta a$) is **at most** the sum of the uncertainties

  $\delta a = \delta b + \delta c$
- Better value for $\delta a$ is

  $\delta a = \sqrt{(\delta b^2 + \delta c^2)}$
- Best value is
  - $a = \bar{a} \pm \delta a$

# Expected uncertainty in a calculated product $a = b*c$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
- Relative uncertainty for a ($\varepsilon a$) is **at most** the sum of the RELATIVE uncertainties

  $\varepsilon a = \delta a/a = \varepsilon b + \varepsilon c$
- Better value for $\delta a$ is

  $\varepsilon a = \sqrt{(\varepsilon b^2 + \varepsilon c^2)}$
- Best value is
  - $a = a \pm \varepsilon a$ (fractional uncertainty)

# What about powers in a product

## $a = b*c^2$

- Each value has an uncertainty
  - $b = b \pm \delta b$
  - $c = c \pm \delta c$
  - $\varepsilon a = \delta a/a$ (relative uncertainty)
- powers become a prefactor (weighting) in the error propagation
  - $\varepsilon a^2 = (\varepsilon b^2 + (2*\varepsilon c)^2)$

# How does uncertainty in t effect the calculated parameter d ?

- $d = \frac{1}{2} g t^2$

$$\varepsilon d = \sqrt{(2*\varepsilon t)^2} = 2*\varepsilon t$$

$$\varepsilon d = 2*(.09/2.52) = 0.071$$

$$\delta d = .071*31 \text{ m} = 2.2 \text{ m} = 2 \text{ m}$$

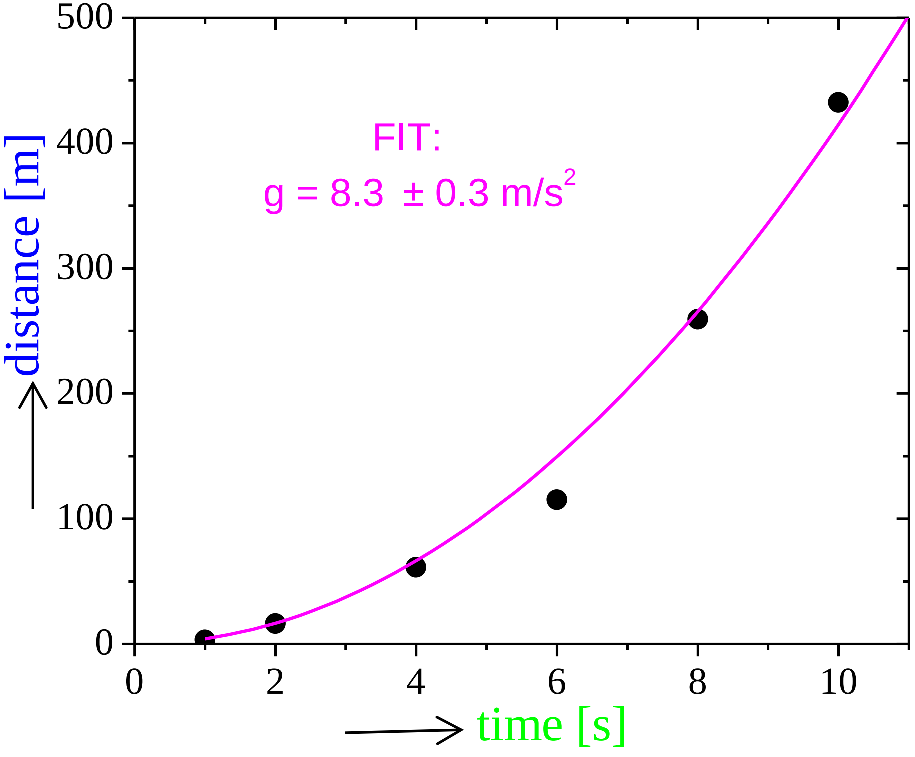
Statistical error

# Relationships

- Know there is a functional relation between d and t $\qquad d = \frac{1}{2} g t^2$
- d is directly proportional to $t^2$
- Related through a constant ½ g
- Can measure time of drop (t) at different heights (d)
- plot d versus t to obtain constant

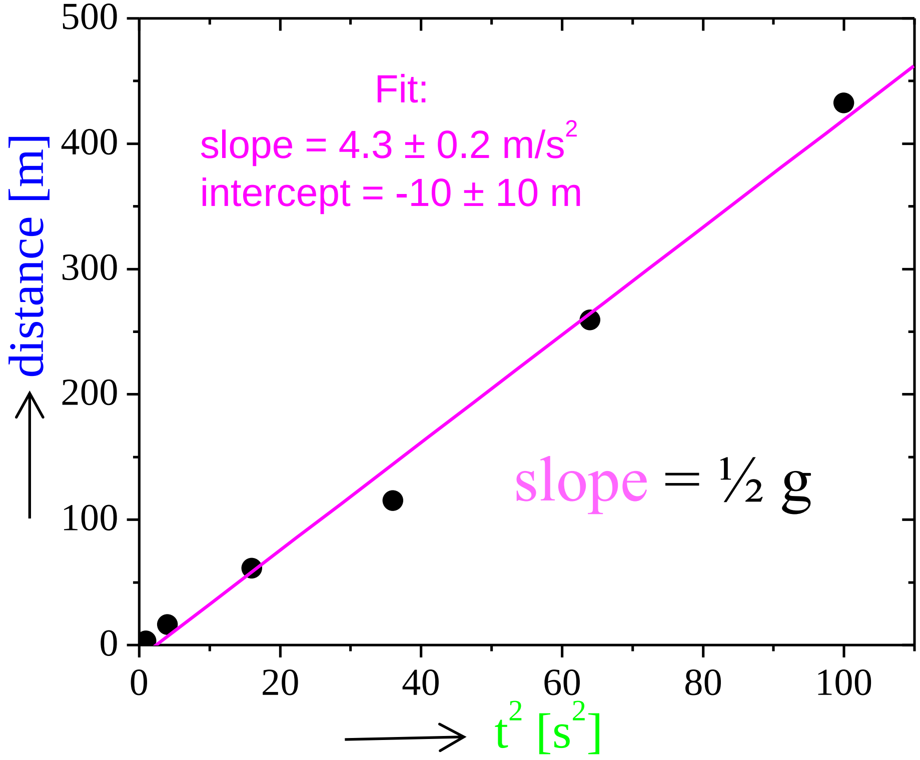
# Quantifying relationships

$d = \frac{1}{2} g t^2$




$g = 8.3 \pm 0.3\ m/s^2$

$d = \frac{1}{2} g (t^2)$




$g = 8.6 \pm 0.4\ m/s^2$

# General Formula for error propagation

For independent, random errors

$$\delta q = \left| \frac{dq}{dx} \right| \delta x$$

$$\delta q = \sqrt{\left( \frac{\partial q}{\partial x} \delta x \right)^2 + \left( \frac{\partial q}{\partial y} \delta y \right)^2}$$

From Yagil

# Measurement and Observation

- Measurement: deciding the amount of a given property by observation
- Empirical
- Not logical deduction
- Not all measurements are created equal…

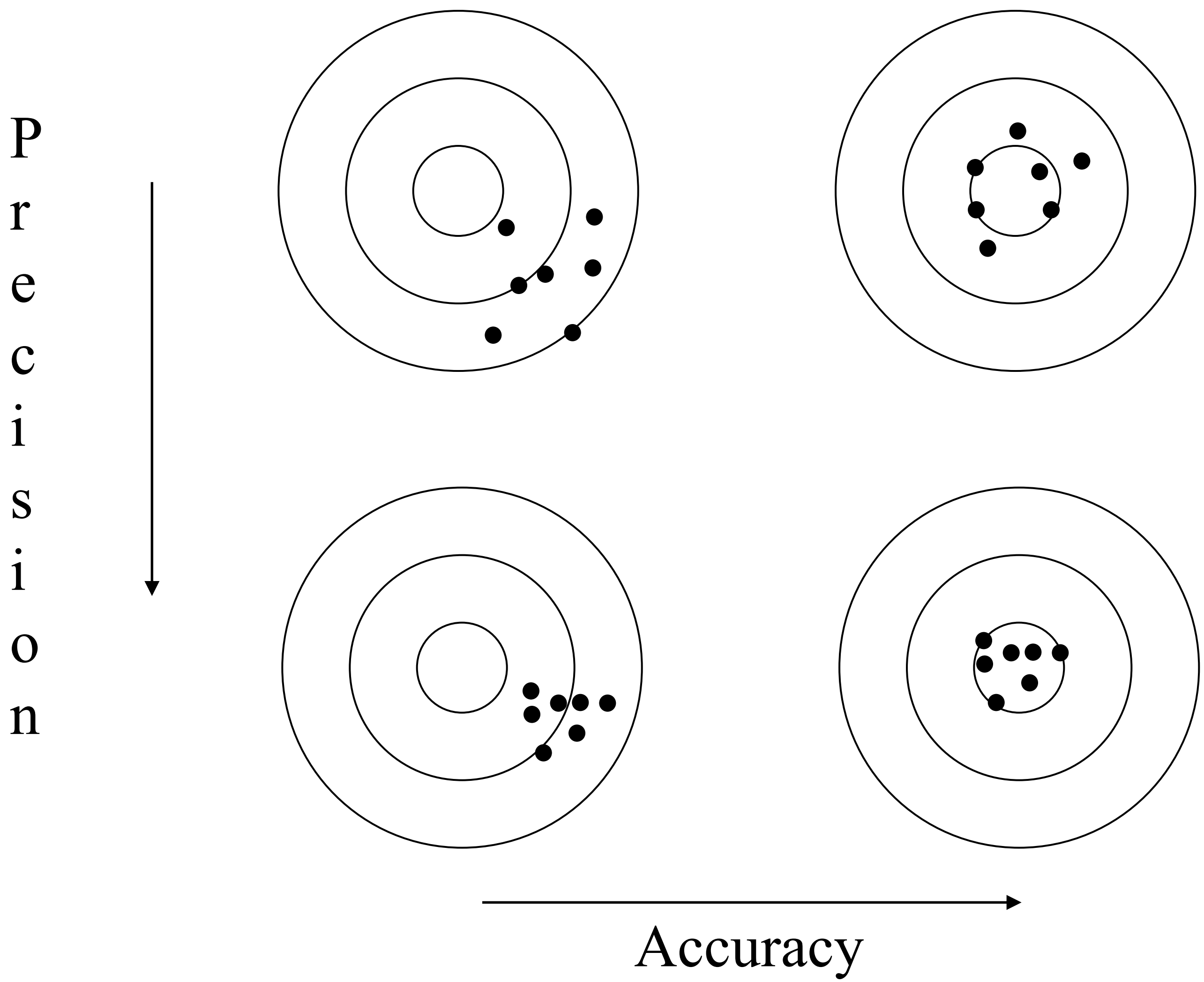
# Reproducibility

- Same results under similar circumstances
  - Reliable/precise
- ‘Similar’ - a slippery thing
  - Measure resistance of metal
    - need same sample purity for repeatable measurement
    - need same people in room?
    - same potential difference?
  - Measure outcome of treatment on patients
    - Can’t repeat on same patient
    - Patients not the same

# Precision and Accuracy

- Precise - reproducible
- Accurate - close to true value
- Example - temperature measurement
  - thermometer with
    - fine divisions
    - or with coarse divisions
  - and that reads
    - 0 C in ice water
    - or 5 C in ice water

# Accuracy vs. Precision



# Random and Systematic Errors

- Accuracy and precision are related to types of errors
  - random (thermometer with coarse scale)
    - can be reduced with repeated measurements, careful design
  - systematic (calibration error)
    - difficult to detect with error analysis
    - compare to independent measurement

# Observations in Practice

- Does a measurement measure what you think it does? Validity
- Are scope of observations appropriate?
  - Incidental circumstances
  - Sample selection bias
- Depends on model

# Models

- Model is a construction that represents a subject or imitates a system
- Used to predict other behaviors (extrapolation)
- Provides context for measurements and design of experiments
  - guide to features of significance during observation

# Testing model

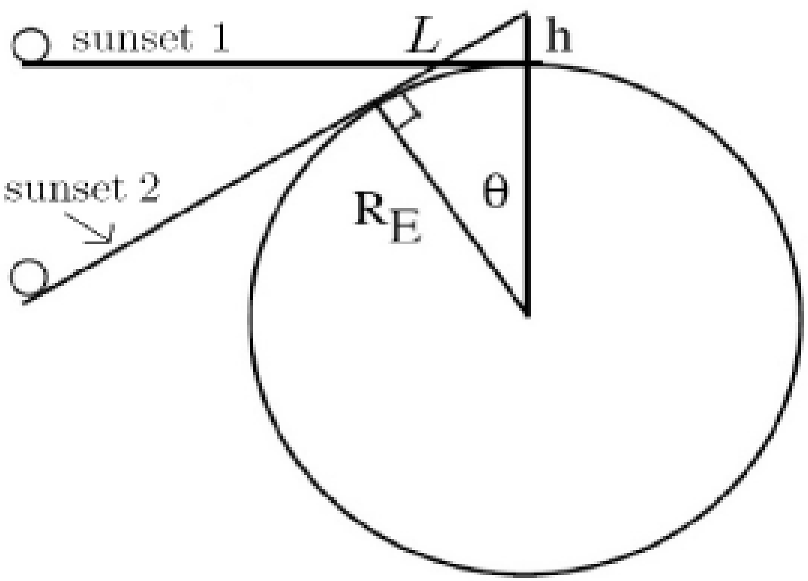
- Models must be consistent with data
- Decide between competing models
  - elaboration: extend model to region of disagreement
  - precision: prefer model that is more precise
  - simplicity: Ockham's razor

# The Earth

Volume – radius
Mass
Density

# Experiment 1 Overview: Density of Earth

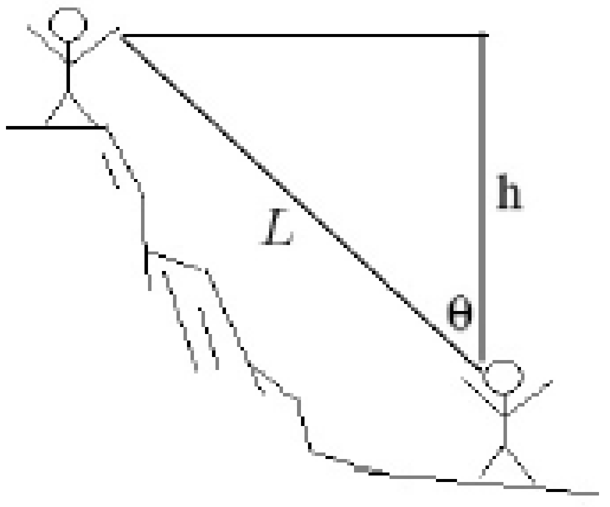
density $$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{3g}{4\pi G R_E}$$ $$= \frac{G M_E m}{R_E^2} = mg$$

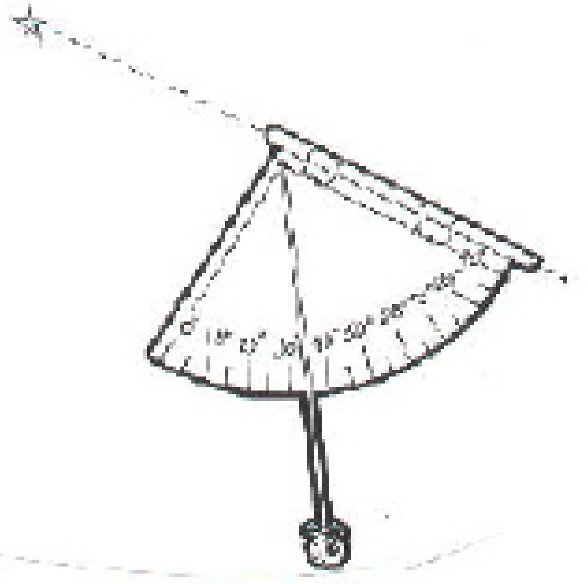



$$R_E = \frac{2h}{\omega^2(\Delta t)^2}$$

measure Δt between sunset on cliff and at sea level

# Experiment 1: Height of Cliff



rangefinder to get *L*

Sextant to get θ

Wear comfortable shoes

Make sure you use θ and not (90 – θ)

# Measure Earth's Radius using Δt Sunset

Now, is this time delay measurable?

$h$ - height above the sea level

$L$ - distance to the horizon line

$$t = \frac{L}{2\pi R_e} T = \frac{T}{2\pi}\sqrt{\frac{2h}{R_e}}$$

$$T = 24\,\text{hr} = 24 \cdot 60 \cdot 60\ \text{s}$$

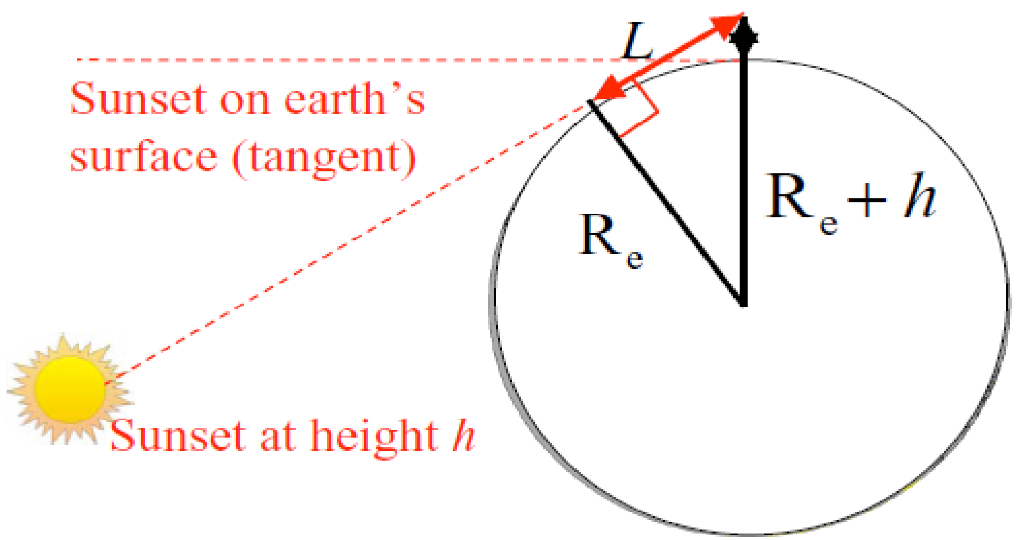
$$= 86400\ \text{s}$$

$$R_e = 6{,}000{,}000\ \text{m}$$

$h \sim 100$ m - our cliff

$$t = \frac{86400\ \text{s}}{2\pi}\sqrt{\frac{200}{6\times 10^6}} \approx 80\ \text{s}$$

Looks doable!




Have we forgotten something?

From Yagil

# "The Equation" for Experiment 1a

$$t = \frac{T}{2\pi}\sqrt{\frac{2Ch}{R_e}} = \frac{1}{\omega}\sqrt{\frac{2Ch}{R_e}}$$

from previous page.

$$\omega = \frac{2\pi}{24\ \text{hr}}$$

Which are the variables that contribute to the error significantly?

$$\Delta t = t_1 - t_2 = \frac{1}{\omega}\sqrt{\frac{2\overline{C}}{\mathrm{R_e}}}\left(\sqrt{h_1} - \sqrt{h_2}\right)$$

Time difference between the two sunset observers.

$$C \equiv \frac{1}{\cos^2(\lambda)\cos^2(\lambda_s) - \sin^2(\lambda)\sin^2(\lambda_s)}$$

Season dependant factor slightly greater than 1.

The formula for your error analysis.

$$\mathrm{R_e} = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1} - \sqrt{h_2}}{\Delta t}\right)^2$$

What other methods could we use to measure the radius of the earth?

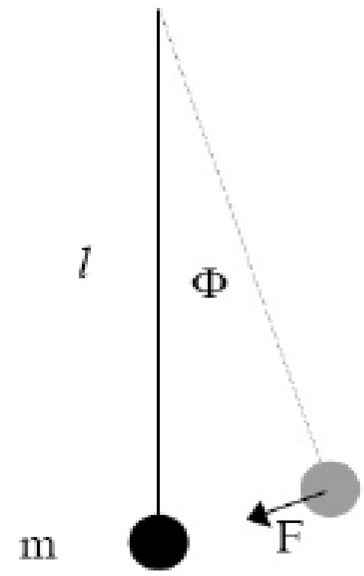
Eratosthenes

angular deviation = angle subtended

From Yagil

# Experiment 1: Determine g

pendulum




$$F = -mg\sin(\phi) = -mg\phi$$

$$F = m\alpha = ml\ddot{\phi}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

period

# Experiment 1: Pendulum

- For release angle $\theta_i$, you should have a set of time data $(t_1^p, t_2^p, t_3^p, ..., t_N^p)$.

- Calculate the average, $\bar{t^p}$, and the the standard deviation, $\sigma_{t^p}$, of this data.

- Divide $\bar{t^p}$ and $\sigma_{t^p}$ by p to get average time of a *single* period, $\bar{T}$ and standard deviation of a single period $\sigma_T$.

- Calculate SDOM, $\sigma_{\bar{T}} = \frac{\sigma_T}{\sqrt{N}}$.

- Now you should have $\bar{T} \pm \sigma_{\bar{T}}$ for you data at $\theta_i$.

- Repeat these calculations for data at each release angle.

Grading rubric uploaded on website

# Error Propagation - example

Ws saw earlier how to determine the acceleration of gravity, g.

Using a simple pendulum, measuring its length and period:

-Length l : $l = l_{best} \pm \delta l$

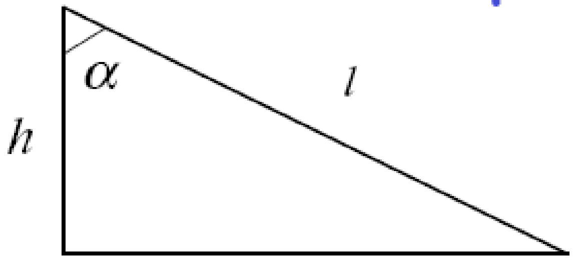
-Period T : $T = T_{best} \pm \delta T$

Determine g by solving: $g = l \cdot (2\pi / T)^2$

The question is what is the resulting uncertainty on g, **δg** ??

From Yagil

# Example




Given that: $l = 10 \pm 0.1\ \text{m}$

$\alpha = 20 \pm 3^0$

==> Find *h*.

$$h = l \cdot \cos\alpha = 10 \cdot \cos 20^0 = 10 \cdot 0.94 = 9.4\ \text{m}$$

$$\delta h = \sqrt{\left(\frac{\partial h}{\partial l}\delta l\right)^2 + \left(\frac{\partial h}{\partial \alpha}\delta\alpha\right)^2}$$

$$\frac{\partial h}{\partial l} = \cos\alpha$$

$$\frac{\partial h}{\partial \alpha} = l \cdot (-\sin\alpha)$$

always use radians when calculating the errors on trig functions

$$\delta\alpha = 3^0 = \frac{2\pi\ \text{rad}}{360^0} \cdot 3^0 = 0.05\ \text{rad}$$

$$\delta h = \sqrt{(\cos\alpha \cdot \delta l)^2 + (l \cdot (-\sin\alpha) \cdot \delta\alpha)^2} = \sqrt{(0.94 \cdot 0.1)^2 + (10 \cdot [-0.34] \cdot 0.05)^2} = 0.2\ \text{m}$$

$$h = 9.4 \pm 0.2\ \text{m}$$

From Yagil

# Propagating Errors for Experiment 1

$$\rho = \frac{3}{4\pi}\frac{g}{GR_e}$$ **Formula for density.**

$$\sigma_\rho = \frac{3}{4\pi}\frac{1}{GR_e}\sigma_g \oplus \frac{-3}{4\pi}\frac{g}{GR_e^2}\sigma_{R_e}$$ **Take partial derivatives and add errors in quadrature**

Or, in terms of relative uncertainties: $\frac{\sigma_\rho}{\rho} = \frac{\sigma_g}{g} \oplus \frac{\sigma_{R_e}}{R_e}$

shorthand notation for quadratic sum: $\sqrt{a^2 + b^2} = a \oplus b$

From Yagil

# Propagating Errors for $R_e$

$$R_e = \frac{2C}{\omega^2}\left(\frac{\sqrt{h_1}-\sqrt{h_2}}{\Delta t}\right)^2$$

basic formula

$$\sigma_{R_e} = \frac{\partial R_e}{\partial \Delta t}\sigma_{\Delta t} \oplus \frac{\partial R_e}{\partial h_1}\sigma_{h_1} \oplus \frac{\partial R_e}{\partial h_2}\sigma_{h_2}$$

Propagate errors (use shorthand for addition in quadrature)

$$\sigma_{R_e} = \frac{2R_e}{\Delta t}\sigma_{\Delta t} \oplus \frac{R_e}{\sqrt{h_1}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_1} \oplus \frac{R_e}{\sqrt{h_2}\left(\sqrt{h_1}-\sqrt{h_2}\right)}\sigma_{h_2}$$

Note that the error blows up at $h_1=h_2$ and at $h_2=0$.

From Yagil

# Reminder

- Prepare for lab
- Read Taylor chapter 4
- Homework due next meeting - Taylor 4.6, 4.14, 4.18, 4.26