Expectations – Review
Linear Least Squares Fitting

Lecture # 7

Physics 2BL

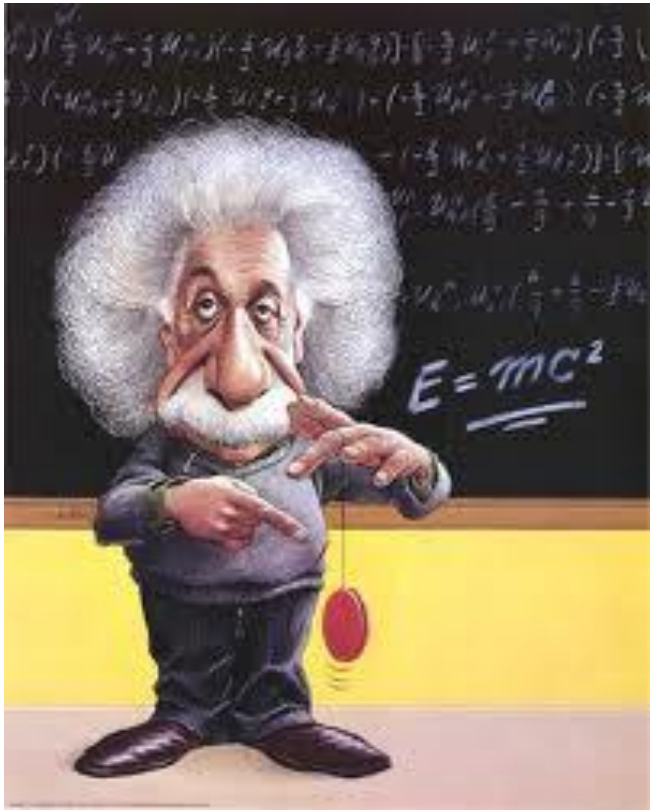
Spring 2012

Outline

- Announcements
- Least Squares Fitting
- χ^2 analysis
- Experiment # 3 analysis

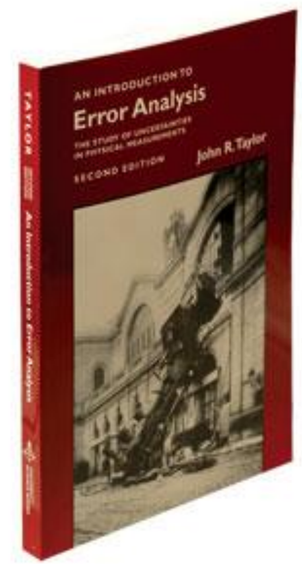
Announcements

1. **Prepare for labs, seek help if needed as resources are available**
2. **In lieu of final, will have extended quiz that may include questions not previously assigned**



Expectations - Review

1. **Understand basic concepts in error analysis**
    a. Significant figures
    b. Propagation of errors – simple forms, general form
    c. Gaussian distributions – mean, standard deviation, standard deviation of the mean
    d. Extract probabilities from t-values
    e. Rejection of data
    f. Weighted averages
    g. Linear least squares
    η. χ^2 analysis



Concepts mentioned in this brief review are not be all inclusive

Expectations - Review

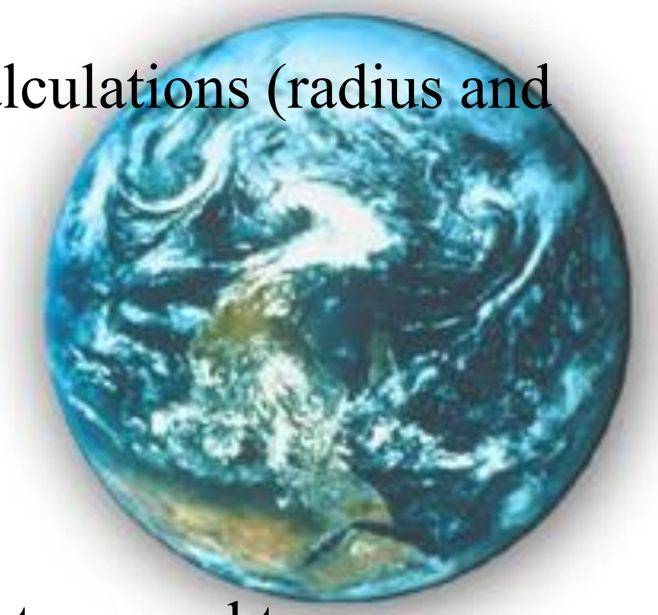
2. Apply ideas to physics lab situation

a. Presentation of measurements and errors using proper number of significant figures

b. Propagation of errors through calculations (radius and density of earth)

c. Plot of histograms

d. Gaussian fits of data – mean, standard deviation, standard deviation of the mean

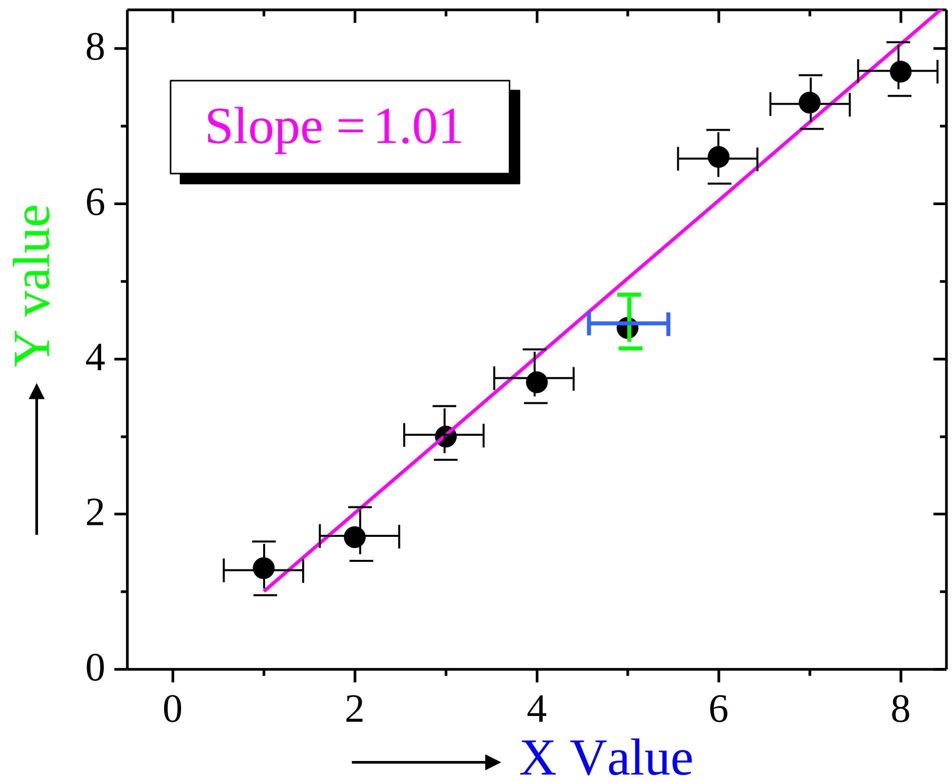


e. Extract probabilities from real data – used to determine variation in thickness of racket balls

f. Testing of a model with measurements – t-score analysis

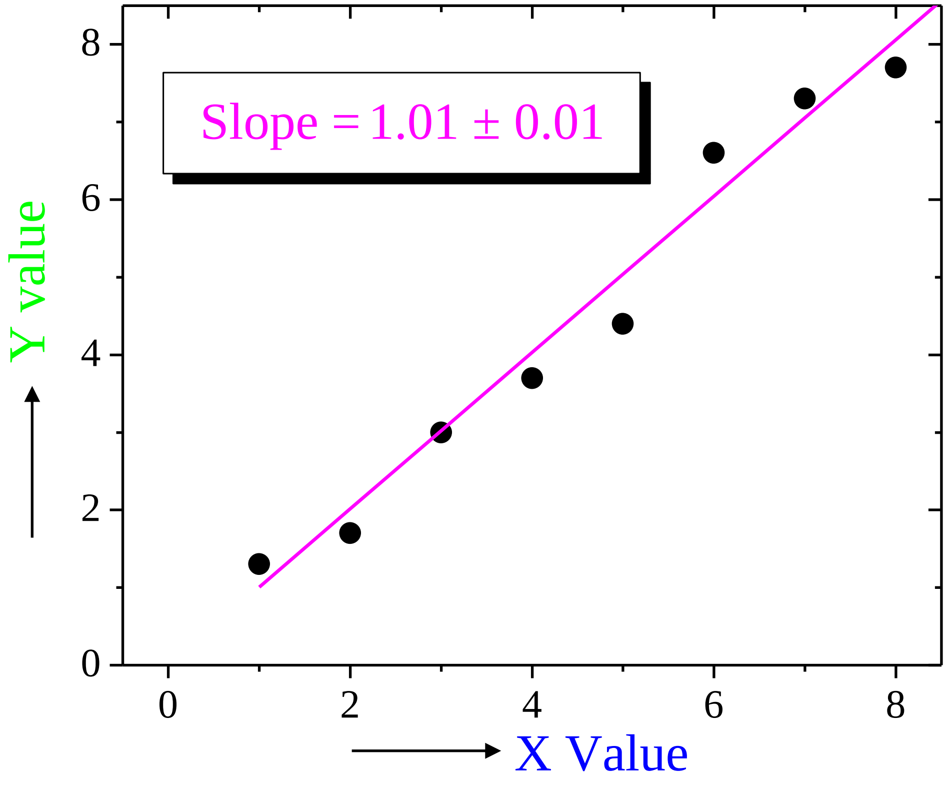
Linear Relationships: $y = A + Bx$ (Taylor, Chapter 8)

- Data would lie on a straight line, except for errors
- What is 'best' line through the points?
- What is uncertainty in constants?
- How well does the relationship describe the data?



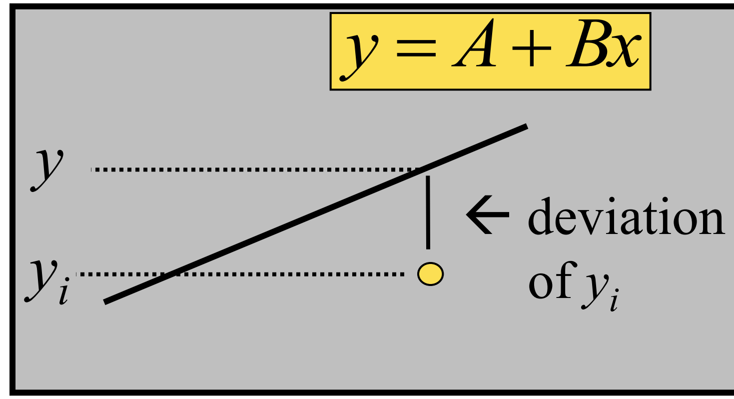
Analytical Fit

- Best means ‘minimize the square of the deviations between line and points’
- Can use error analysis to find constants, error



The Details of How to Do This (Chapter 8)

- Want to find *A, B* that minimize difference between data and line
- Since line above some data, below other, minimize sum of squares of deviations
- Find *A, B* that minimize this sum



$$y_i - y = y_i - A - Bx_i$$

$$\sum_{i=1}^{N} (y_i - A - Bx_i)^2$$

$$\frac{\partial}{\partial A} = \sum y_i - AN - B\sum x_i = 0$$

$$\frac{\partial}{\partial B} = \sum x_i y_i - A\sum x_i + B\sum x_i^{\ 2} = 0$$

Finding *A* and *B*

- After minimization, solve equations for *A* and *B*
- Looks nasty, not so bad…
- See Taylor, example 8.1

$$\frac{\partial}{\partial A} = \sum y_i - AN - B\sum x_i = 0$$

$$\frac{\partial}{\partial B} = \sum x_i y_i - A\sum x_i + B\sum x_i^{\,2} = 0$$

$$A = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{\Delta}$$

$$B = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\Delta}$$

$$\Delta = N\sum x_i^{\,2} - \left(\sum x_i\right)^2$$

Uncertainty in Measurements of *y*

- Before, measure several times and take standard deviation as error in *y*
- Can't now, since y_i's are different quantities
- Instead, find standard deviation of deviations

$$\sigma_x = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

$$\sigma_y = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N}(y_i - A - Bx_i)^2}$$

Uncertainty in A and B

- A, B are calculated from x_i, y_i
- Know error in x_i, y_i ; use error propagation to find error in A, B
- A distant extrapolation will be subject to large uncertainty

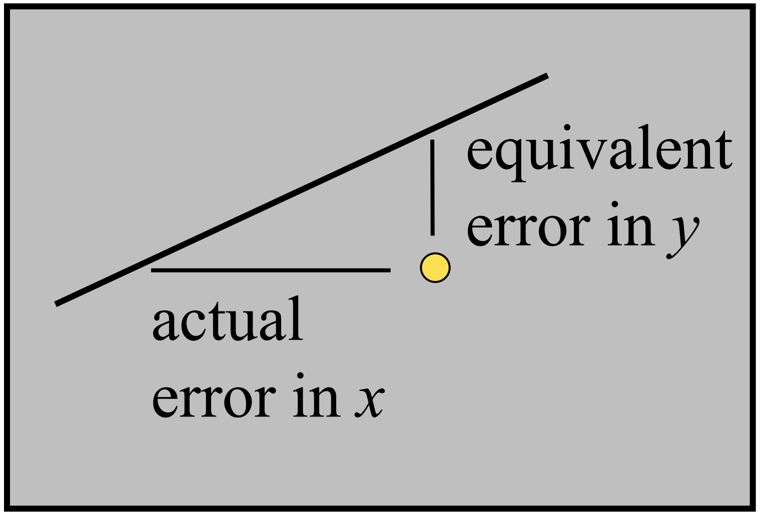
$$\sigma_A = \sigma_y \sqrt{\frac{\sum x_i^2}{\Delta}}$$

$$\sigma_B = \sigma_y \sqrt{\frac{N}{\Delta}}$$

$$\Delta = N \sum x_i^2 - \left(\sum x_i\right)^2$$

Uncertainty in x

- So far, assumed negligible uncertainty in x
- If uncertainty in x, not y, just switch them
- If uncertainty in both, convert error in x to error in y, then add errors



$$\Delta y = B \Delta x$$

$$\sigma_y(equiv) = B\sigma_x$$

$$\sigma_y(equiv) = \sqrt{\sigma_y{}^2 + (B\sigma_x)^2}$$

Other Functions

- Convert to linear
- Can now use least squares fitting to get ln *A* and *B*

$$y = Ae^{Bx}$$

$$\ln y = \ln A + Bx$$

χ^2 Testing
(Taylor Chapter 12)

- You take N measurements of some parameter x which you believe should be distributed in a certain way (e.g., based on some hypothesis).
- You divide them into n bins (k=1,2,...,n) and count the number of observations that fall into each bin (O_k).
- You also calculate the expected number of measurements (E_k), in the same bins, based on some hypothesis.
- Calculate:

$$\chi^2 = \sum_{i=1}^{n} \frac{\left(O_k - E_k\right)^2}{E_k}$$

- If $\chi 2 < n$, then the agreement between the observed and expected distributions is acceptable.
- If $\chi 2 >> n$, there is significant disagreement.

Degrees of Freedom

- Number of degrees of freedom, d = number of observations, O_k, minus the number of parameters computed from the data and used in the calculation.
- d=n-c,
  - Where c is the number of parameters that were calculated in order to compute the expected numbers, E_k.
  - It can be shown that the expected average value of $\chi 2$ is d.
- Therefore, we define "reduced chi-squared":

$$\tilde{\chi}^2 = \frac{\chi^2}{d}$$

- If the reduced chi-squared is <1, there is no reason to doubt the expected distribution.

Fitting Summary

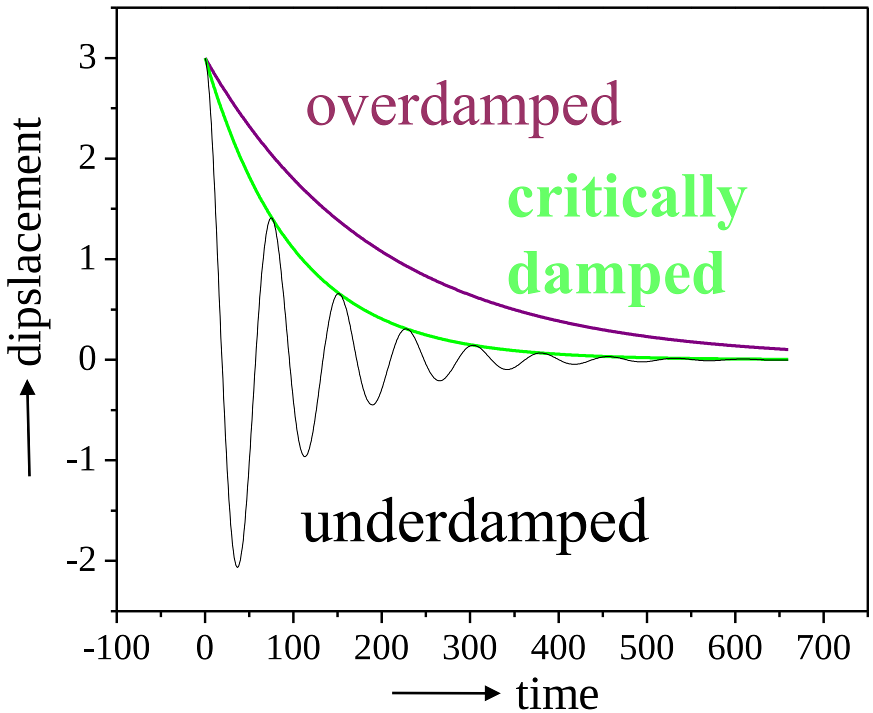
- You have a set of measurements and a hypothesis that relates them.
- The hypothesis has some unknown parameters that you want to determine.
- You “fit” for the parameters by maximizing the odds of all measurements being consistent with your hypothesis.

- Evaluate your fit based on the <u>goodness of fit</u>.

Experiment 3

- Goals: Test model for damping
- Model of a shock absorber in car
- Procedure: develop and demonstrate critically damped system
- check out setup, take data, do data make sense?
- Write up results - Does model work under all conditions, some conditions? Need modification?

Comparison of the various types of damping



Plotting Graphs

Give each graph a title

Determine independent and dependent variables

Determine boundaries

Include error bars

Demonstrate critical damping: show convincing evidence that critical damping was achieved

- Demonstrate that damping is critical
  - No oscillations (overshoot)
  - Shortest time to return to equilibrium position

Error propagation

(1) $k_{spring} = 4\pi^2 m/T^2$

$$\sigma_{kspring} = \varepsilon_{kspring} * k_{spring}$$

$$\varepsilon_{kspring} = \sqrt{\varepsilon_m^2 + (2\varepsilon_T)^2}$$

(2) $k_{by\text{-}eye} = m(g\Delta t^*/2\Delta x)^2$

$$\sigma_{by\text{-}eye} = \varepsilon_{by\text{-}eye} * k_{by\text{-}eye}$$

$$\varepsilon_{by\text{-}eye} = \sqrt{(2\varepsilon_{\Delta t^*})^2 + (2\varepsilon_{\Delta x})^2 + \varepsilon_m^2}$$

Remember

- Finish Experiment # 3
- Taylor chapter 12
- Taylor problem 12.3
- Review goals and questions from current and previous labs