PHYS 2D
DISCUSSION SECTION

2012/4/11

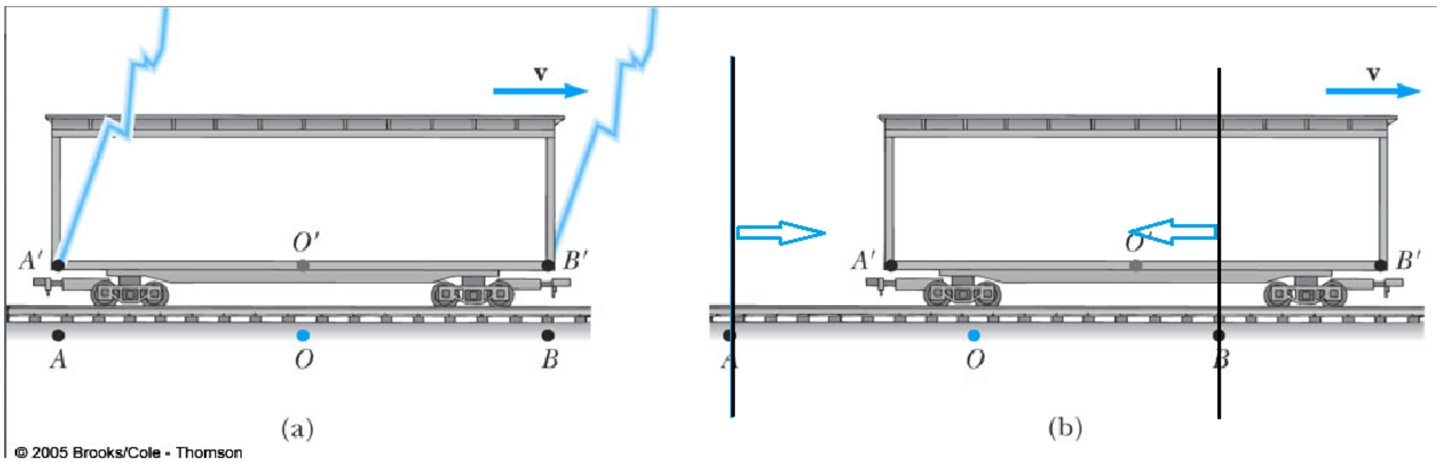
- Email me topics/questions you'd like to discuss
- Problem section tomorrow 8pm Pepper Canyon 109
- Specific problems for problem section?
- Quiz on Friday

Topics for Today

- Simultaneity
- Time dilation & Length contraction
- Proper time/length
- Lorentz Transformation
- Twin Paradox
- Relativistic Energy

Simultaneity

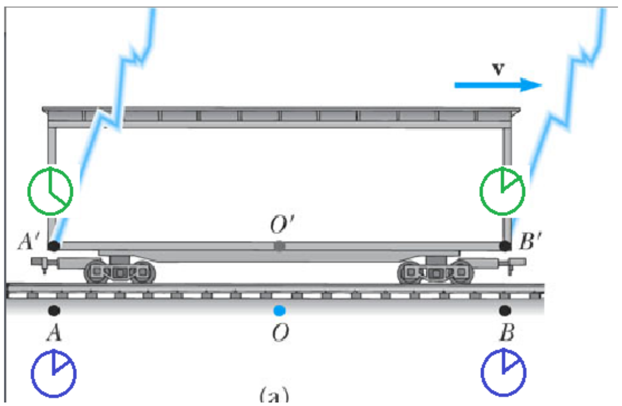
- From rest frame PoV, light signal from B hits O' first



- From moving frame PoV, light signals from A' & B' both have velocity c & traverse 1/2 the compartment
- Lightning must have hit B' first

Simultaneity

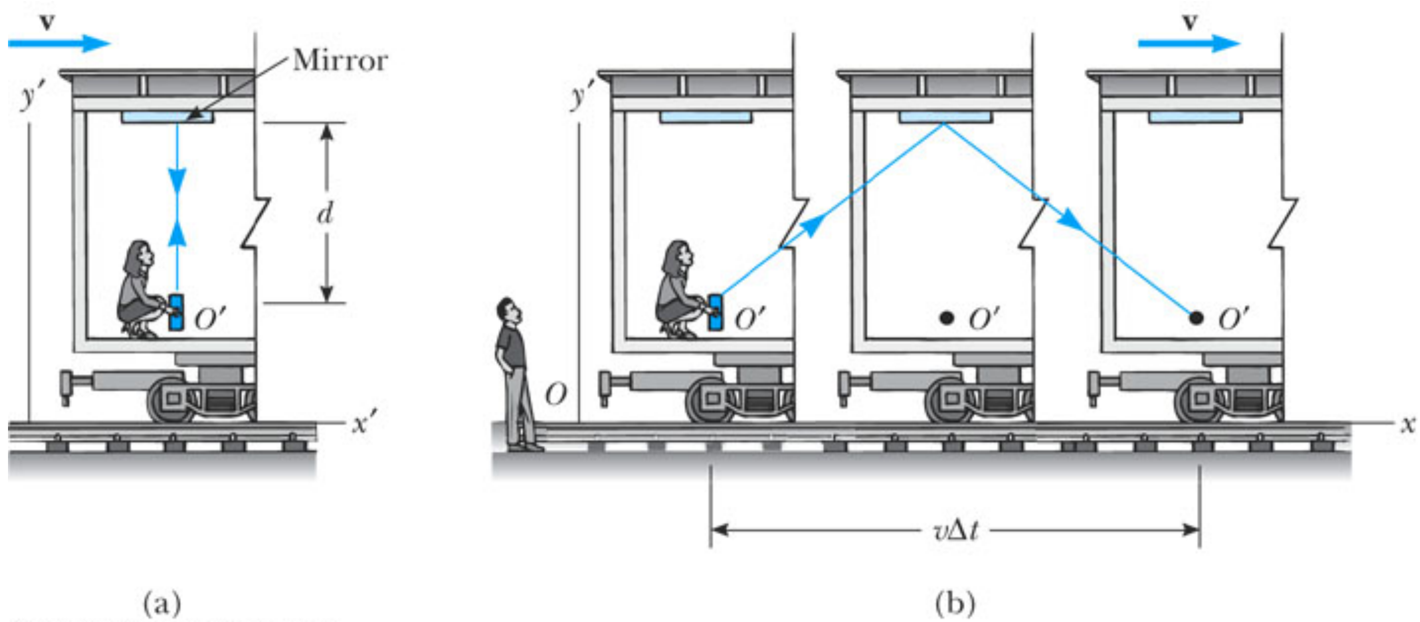
- Discrepancy arises from constancy of c
- Clocks at A' & B' are not synchronized



$t' = \gamma (t - v x / c^2)$

- Define $t_B' = t_B = 0$, $x_B' = x_B = 0$, then $t_A = t_{B=0}$
- $t_A' = \gamma(t_A - v x_A / c^2) = -\gamma v x_A / c^2 > 0$ $(x_A < 0)$
- $t_A' > 0$, $t_B' = 0$, A happened later
- Simultaneity is meaningless for different frames

Time Dilation



- $\Delta t = \gamma \Delta t'$, $\Delta t'$ is the proper time (2 measurement events happen **at the same place** in K' frame)
- $\Delta t > \Delta t'$ always, where Δt is measured in a moving frame K (2 events happen at different places in K)

Length Contraction

- Derivation: slide 7, lecture 4
- Proper length $\Delta L' = \gamma \Delta L$, where K' is the object's rest frame
- Proper time is measured in the object's rest frame
- Proper length is also measured in the object's rest frame
- $\Delta L' > \Delta L$, object appears shorter in moving frame K

Lorentz Transformation

Lorentz Transformation	Inverse Lorentz Transformation
$x' = \gamma(x - vt)$	$x = \gamma(x' + vt')$
$y' = y$	$y = y'$
$z' = z$	$z = z'$
$t' = \gamma(t - vx/c^2)$	$t = \gamma(t' + vx'/c^2)$

- Complete description of spacetime coordinate transformation
- Can rederive time dilation & length contraction

Lorentz Transformation

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$t' = \gamma(t - vx/c^2)$	$t = \gamma(t' + vx'/c^2)$

- Suppose K' is the rest frame

Time dilation:

- To get the proper time, events are measured **at the same place,** which happens in the rest frame K'
- So we have $\Delta x'=0$
- Plug into inverse LT: $\Delta t= \gamma\Delta t'$ (Δt is larger)

Lorentz Transformation

Lorentz Transformation

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma(t - vx/c^2)$

Inverse Lorentz Transformation

$x = \gamma(x' + vt')$

$y = y'$

$z = z'$

$t = \gamma(t' + vx'/c^2)$

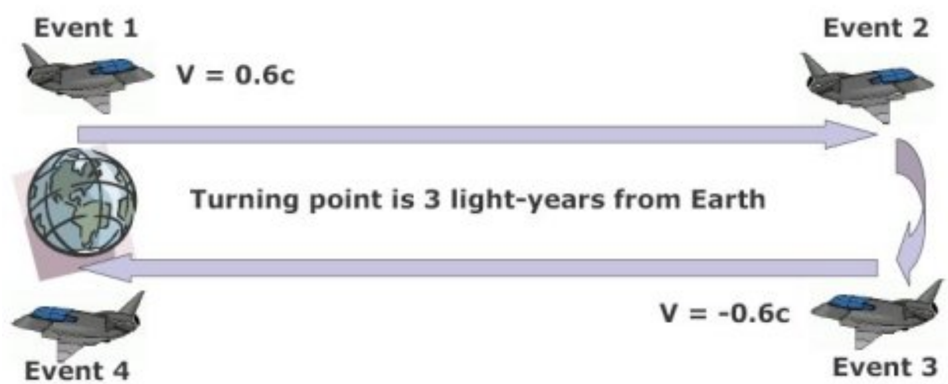
- Suppose K' is the rest frame

Length Contraction:

- Length measurement: must measure both ends of the object **at the same time**
- Want to find length in moving frame K
- So we must have $\Delta t=0$ (same time in K), plug into LT
- $\Delta x'=\gamma\Delta x$ (Δx is smaller)

Twin Paradox

- Twins Pam & Jim, Jim is on Earth, Pam is on rocket with v=0.6c, traveling 3 light-years (Earth PoV)



- Paradox: Jim sees Pam moving while he's at rest, Pam sees Jim moving while she's at rest
- Jim thinks Pam's clock ticks slower, Pam thinks Jim's clock tick slower

Twin Paradox

- Solution: Pam switched between 2 inertial frames, so the problem is not symmetrical
- http://en.wikipedia.org/wiki/Twin_paradox
- At the turning point, “The traveling twin reckons that there has been a jump discontinuity in the age of the resting twin.”, due to the change of frames

- Find your own answer that convinces you

Energy & Momentum

- Define $\vec{\mathbf{p}} \equiv \dfrac{m\vec{\mathbf{u}}}{\sqrt{1-\dfrac{u^2}{c^2}}} = \gamma m\vec{\mathbf{u}}$

- Newton's 2nd law $\vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$

$$F = \frac{dp}{dt} = \frac{d}{dt}\left\{\frac{mv}{\left[1-(v/c)^2\right]^{1/2}}\right\}$$

$$F = \frac{dp}{dt} = \frac{m}{\left[1-(v^2/c^2)\right]^{3/2}}\left(\frac{dv}{dt}\right)$$

- $$K = \int_{x_1}^{x_2} \frac{m\,\frac{du}{dt}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\,dx \qquad dx = u\,dt$$

$$= \int_{t_1}^{t_2} \frac{m\,\frac{du}{dt}}{\left(1-\frac{u^2}{c^2}\right)^{3/2}}\,u\,dt \qquad du = \frac{du}{dt}\,dt$$

$$= \int_0^u \frac{m\,u\,du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}} \qquad y = 1-\frac{u^2}{c^2},\ dy = -2\,\frac{u}{c^2}\,du$$

$$= \int_1^{1-\frac{u^2}{c^2}} \frac{m\left(\frac{c^2}{-2}\right)dy}{y^{3/2}} = \left[y^{-\frac{1}{2}}\,mc^2\right]_1^{1-\frac{u^2}{c^2}}$$

$$= \gamma mc^2 - mc^2$$

Energy

- $K = \gamma mc^2 - mc^2$
- Define total energy $E \equiv K + mc^2$, kinetic energy+rest mass
- Mass & energy can be interexchanged
- $E = \gamma mc^2$
- $p = \gamma mu$, $(pc)^2 + (mc^2)^2 = \dfrac{m^2 u^2}{1 - \frac{u^2}{c^2}} c^2 + m^2 c^4$

$$= \frac{m^2 u^2}{1 - \frac{u^2}{c^2}} c^2 + \frac{m^2 c^2 - m^2 u^2}{1 - \frac{u^2}{c^2}} c^2 = \frac{m^2 c^4}{1 - \frac{u^2}{c^2}} = (\gamma mc^2) = E^2$$

- $E^2 = (pc)^2 + (mc^2)^2$

Energy Conservation

- Total relativistic energy E is conserved
- $\sum_i E_i^{before} = \sum_i E_i^{after}$ & $E^2 = (pc)^2 + (mc^2)^2$
- Can be used in particle physics problems

- Questions?