PHYS 2D
DISCUSSION SECTION

2012/5/08

- Quiz this Friday, Ch. 4 & 5
- Regrade request: until Friday
- Pick up quiz 2 today & tomorrow (problem session)
- Feedback/comments are welcome

Topics

- Overview of matter wave
- De Broglie's contribution
- Double-slit electron diffraction
- Davisson-Germer experiment
- Wave packet: idea & math
- Uncertainty principle

Matter Wave: Overview

- Matter has wave-like properties
- Mass particle described by a "wave function" $\Psi(x)$
- $|\text{Amplitude}|^2$ of Ψ is probability
- $|\Psi(x)|^2$=probability of finding the particle at x
- "Probability wave"

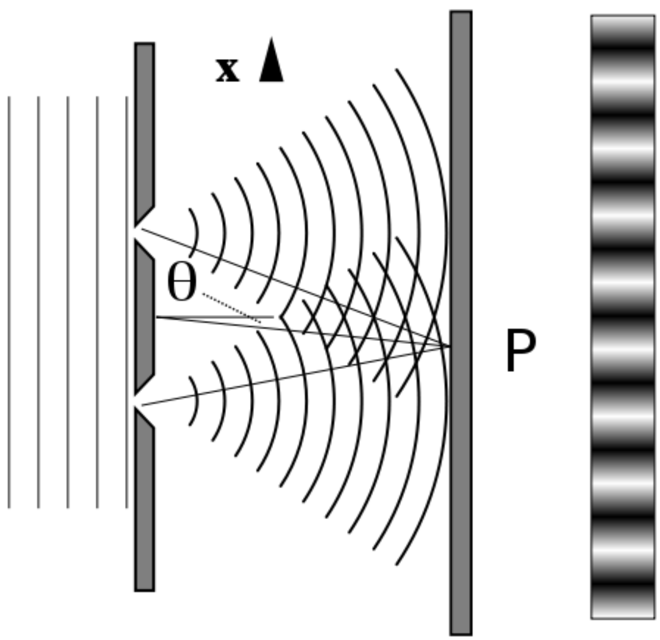
- Form of $\Psi(x)$ determined by Schrodinger's equation
- All sorts of weird things arise from this probability wave

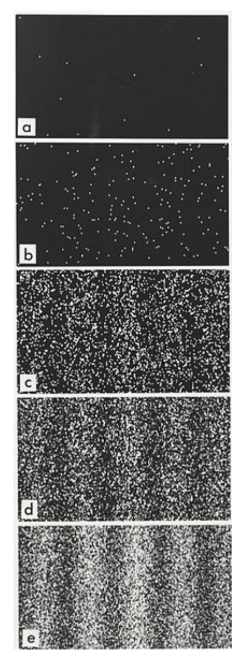
De Broglie's Matter Wave

- Wavelength of mass particles: λ =h/p (just like photons), p= γ mv
- Frequency of mass particles: f=E/h
- Explains angular momentum quantization in Bohr's atomic model
- Electron must form standing wave in orbit
  - $2\pi r = n\lambda = nh/p$
  - $L = rp = nh/2\pi = n\hbar$

Double-slit Electron Diffraction

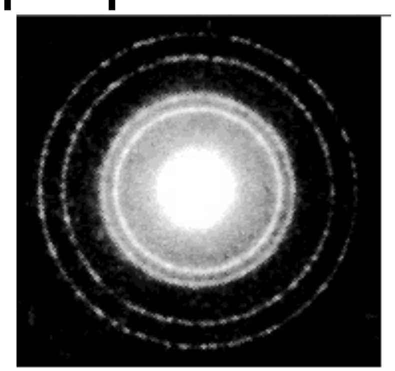
- Stripes happen for a single incoming electron
- Result will never look like this for “particles”





Davisson-Germer Experiment

- Scattering of electrons by nickel
- Nickel has crystal (ordered) structure
- The resulting distribution of scattered electrons fits that of a wave interference pattern
- Rings of 0 probability: destructive interference, only happens if e^-'s are waves
- Electrons have wave properties!



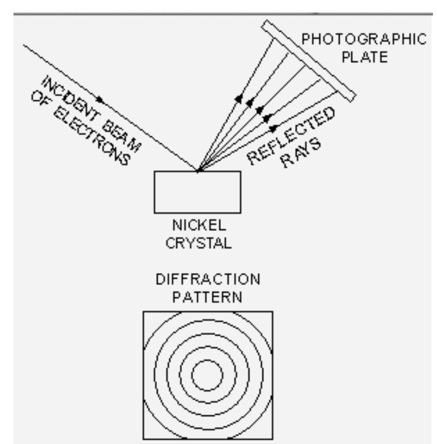
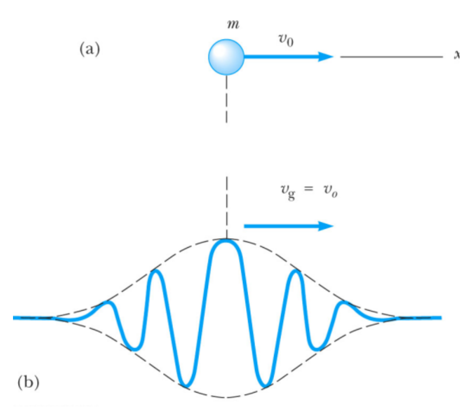


Figure 1. Diffraction of electron beam by nickel crystal (Davisson and Germer's experiment)

Wave Packet

- Particles are described by probability waves
- Classical particles are always found in a limited region in space
- To reflect this, our wave function must have high amplitudes in a certain region, very low amplitudes in all other places, so it's almost always found in that region
- The speed the "wave packet" moves must reflect the classical particle speed

- Need to know basics about waves

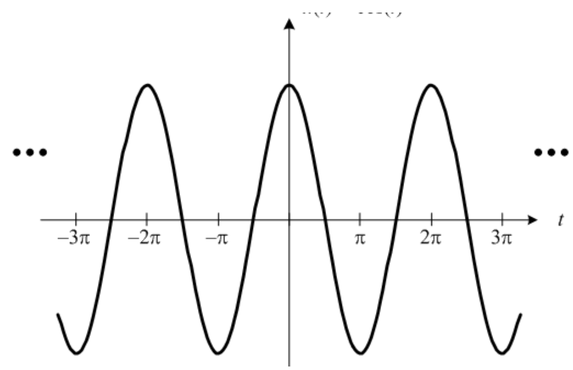


Phase Velocity

- Phase velocity: $v_p=\omega/k$
- Wave propagating in +x with time:

$$y(x,t) = A\cos(kx-\omega t)$$

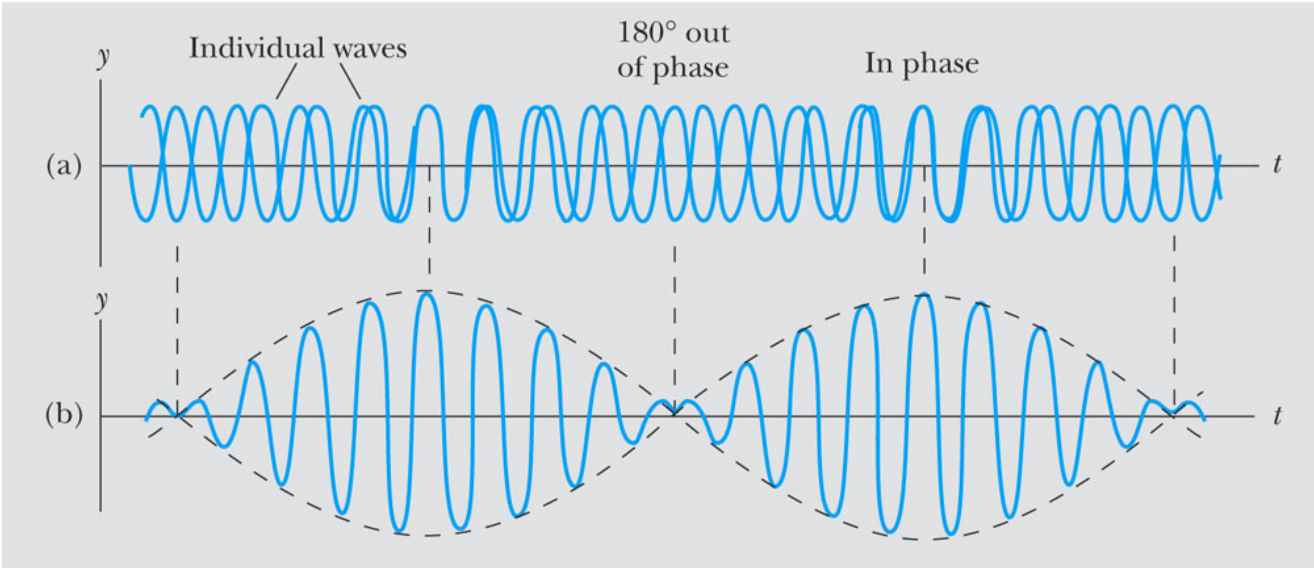
- If we follow the same point in the wave as it propagates, then $k\Delta x-\omega\Delta t=0$, or $\Delta x=(\omega/k)\Delta t=v_p t$
- v_p=speed that point moves=speed of wave



Group Velocity

- Simple example for group velocity: Addition of 2 waves with slightly different wavelength & frequency

$$y = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$



Group Velocity

$$y = A\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$$

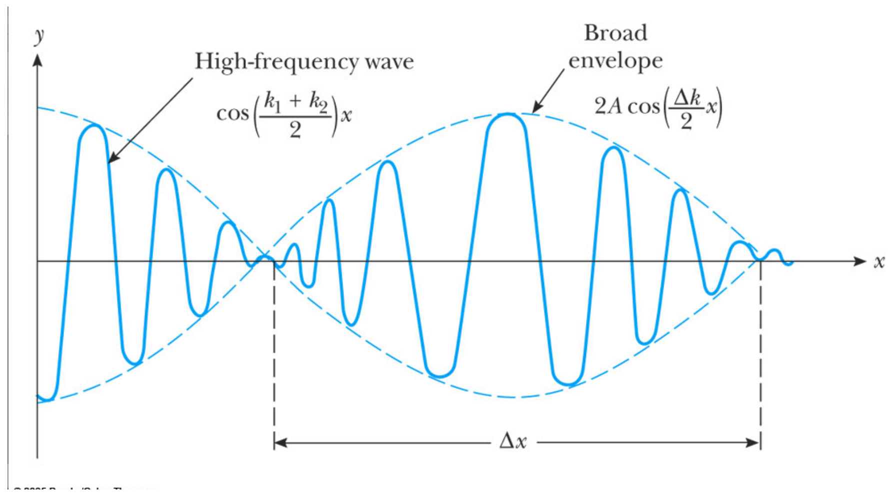
$$\therefore y = 2A\left[\left(\cos\left(\frac{k_2 - k_1}{2}x - \frac{\omega_2 - \omega_1}{2}t\right)\right)\left(\cos\left(\frac{k_2 + k_1}{2}x - \frac{\omega_2 + \omega_1}{2}t\right)\right)\right]$$

$$\boxed{y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)\right)\cos(kx - \omega t)\right]}$$

- Red: Very close to the original wave
- Blue: Large wavelength, modifies amplitude of red wave

Group Velocity

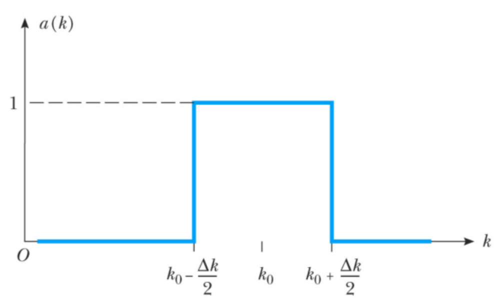
$$y = 2A\left[\left(\cos\left(\frac{\Delta k}{2}x - \frac{\Delta \omega}{2}t\right)\right)\cos(kx - \omega t)\right]$$



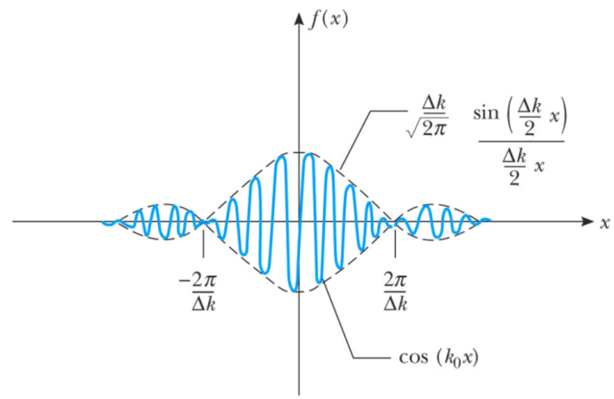
- Group velocity: speed of the blue envelope
- Blue: $v_g = \Delta\omega/\Delta k$, just like Red: $v_p = \omega/k$

Wave Packet

- A wave packet concentrated in some region of space can be construct with the superposition of waves of different wavelengths/frequencies
- Example:
- Superposition of waves with $k=k_0-\Delta k \sim k_0+\Delta k$



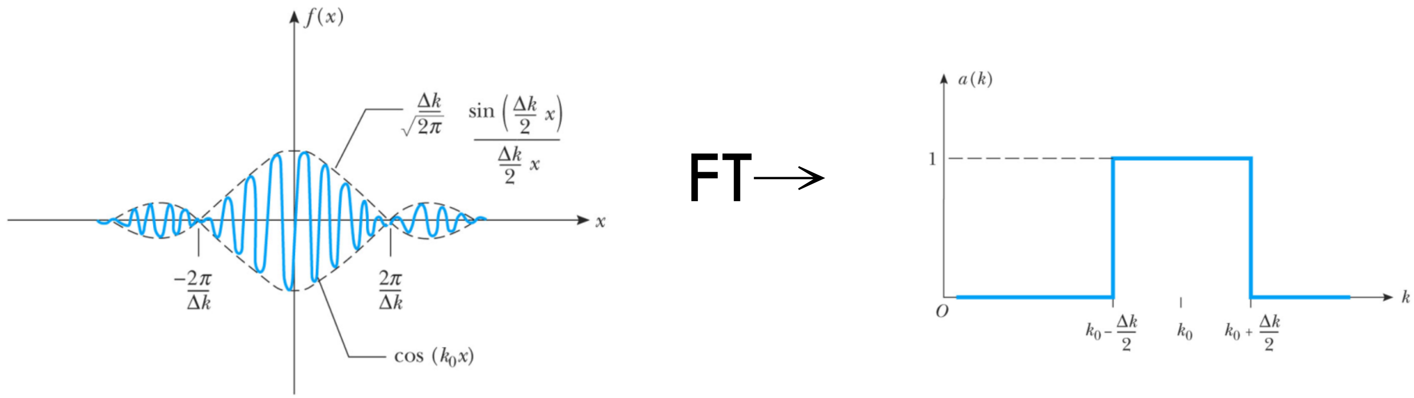
yields



(looking at some fixed t, i.e. t=0)

Fourier Integral

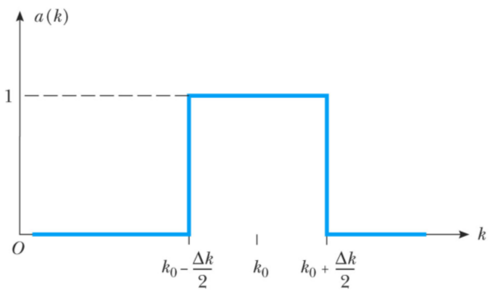
- The frequency ingredients are related to the actual wave by Fourier transform

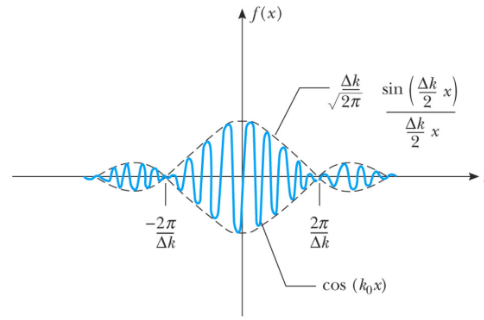


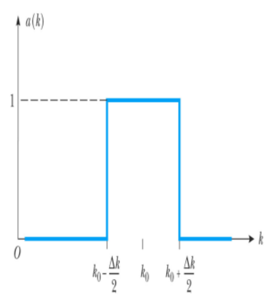
$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} a(k) e^{ikx} dk \qquad a(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

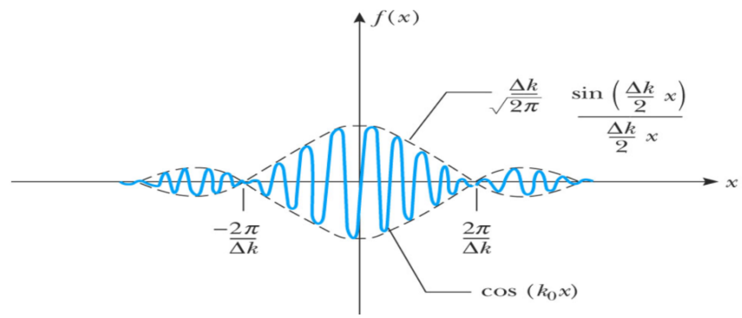
Fourier Integral

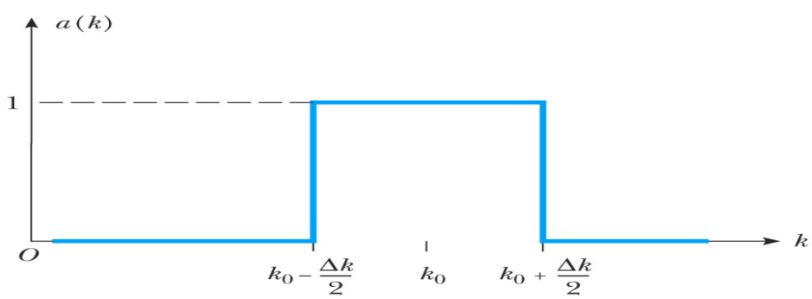
- Property of the Fourier integral: ΔkΔx=constant

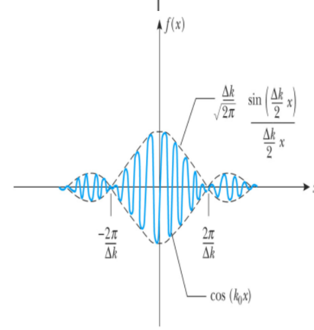




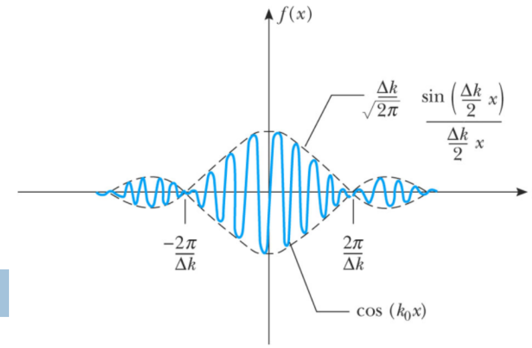








Uncertainty Principle



- Wide range of k is needed for narrow wave packet, narrow range of k is needed for wide wave packet
- When measured, the position of the particle can be anywhere inside the wave packet, k of the particle can be anywhere inside the range for k
- Width of packet: $\Delta x \geqq 0.5/\Delta k$ ($=0.5/\Delta k$ for Gaussian wave packets)
- $p=h/\lambda$, $k=2\pi/\lambda$, $\Delta p=\Delta k*h/2\pi=\hbar\Delta k$
- $\Delta x \Delta p \geqq \hbar/2$

Uncertainty Principle

- $\Delta x \Delta p \geqq \hbar/2$
- When measurement of position is very precise (wave packet narrow), the measurement of momentum gives a wide range of values, vice versa
- Uncertainty principle comes from particle being a wave

Uncertainty Principle

- If we look at a fixed x instead of fixed t, then $kx \rightarrow \omega t$, $\Delta\omega\Delta t \geqq 0.5$, $E = \hbar\omega$, $\Delta E \Delta t \geqq \hbar/2$
- A narrow wave packet in time comes from waves of a wide range of frequencies, vice versa
- Precise time measurement corresponds to large uncertainty of energy

- Questions?