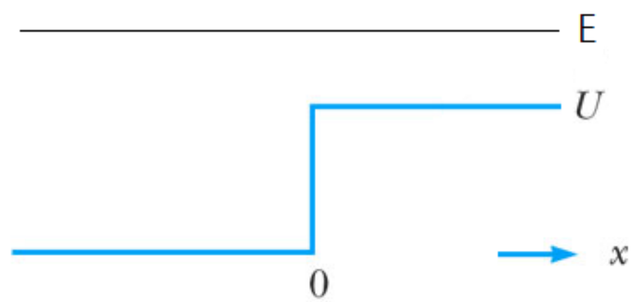
PHYS 2D
PROBLEM SESSION

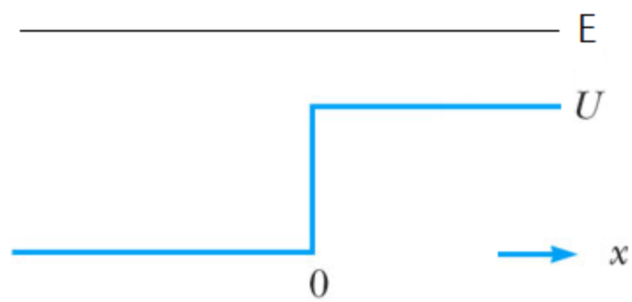
2012/5/24

7.2



- Step potential, E>U, find wave function and R & T
- Region I: x<0
  - $\psi(x)=A\exp(ikx)+B\exp(-ikx)$
  - $E=\hbar^2k^2/2m$
- Region II: x>0
  - $\psi(x)=C\exp(ik'x)+D\exp(-ik'x)$, $D=0$
  - $E-U=\hbar^2k'^2/2m$
- Boundary conditions:
  - ψ & $d\psi/dx$ continuous at $x=0$

7.2

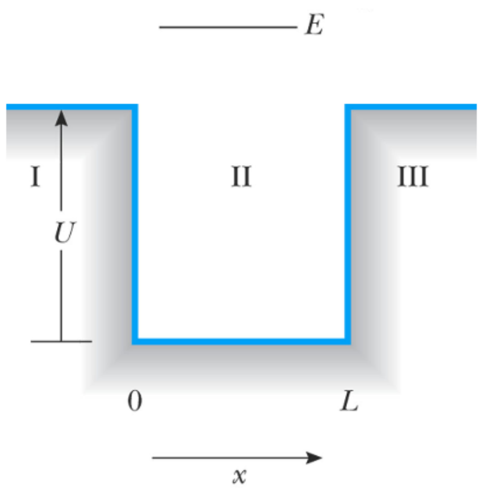


- Applying BC's
  - $A+B=C$
  - $k(A-B)=k'C$
  - $B=A(k-k')/(k+k')$
  - $R=|B/A|^2=[(k-k')/(k+k')]^2$
  - $T+R=1$, $T=1-R=4kk'/(k+k')^2$
    - R & T when $E \to U$ & $E \to \infty$
- $E \to U$, $k' \to 0$, $R \to 1$, $T \to 0$, total reflection
- $E \to \infty$, $k' \to k$, $R \to 0$, $T \to 1$, particle doesn't care

7.3

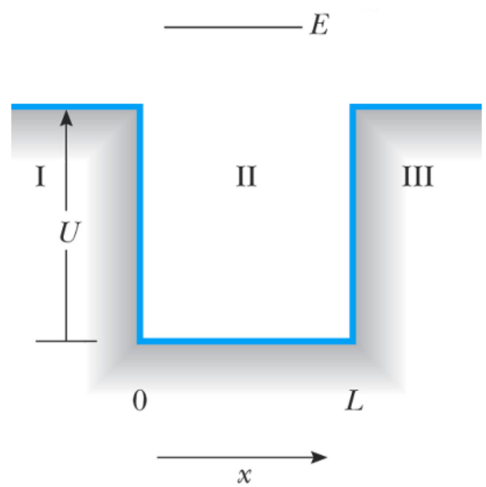
- Fraction of 25 MeV protons transmitted & reflected by 20 MeV step. How about electrons?
- $R=[(k-k')/(k+k')]^2$
- $E=\hbar^2k^2/2m=25$ MeV, $E-U=\hbar^2k'^2/2m=5$ MeV
- $k=(2mE)^{1/2}/\hbar$, $k'=[2m(E-U)]^{1/2}/\hbar$
- $R=[(E^{1/2}-(E-U)^{1/2})/(E^{1/2}+(E-U)^{1/2})]^2$
- $R=[(25^{1/2}-5^{1/2})/(25^{1/2}+5^{1/2})]^2=0.146$
- $T=1-R=0.754$
- Same for electron

7.11



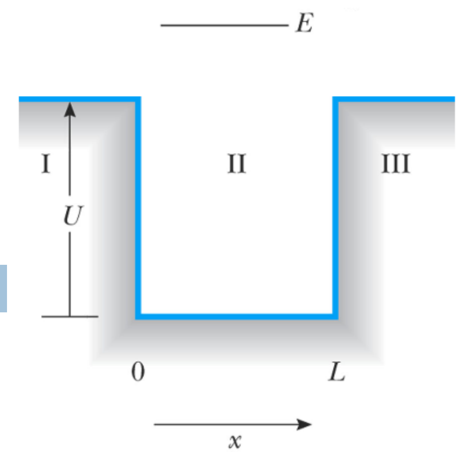
- Show that if $2L=\lambda_{II}$, $R=0$
- Region I: $x<0$
  - $\psi_I(x)=A\exp(ik_I x)+B\exp(-ik_I x)$
  - $E-U=\hbar^2 k_I^2/2m$
- Region II: $0<x<L$
  - $\psi_{II}(x)=C\exp(ik_{II} x)+D\exp(-ik_{II} x)$
  - $E=\hbar^2 k_{II}^2/2m$
- Region II: $x>L$
  - $\psi_{III}(x)=F\exp(ik_{III} x)$
  - $k_{III}=k_I$

7.11



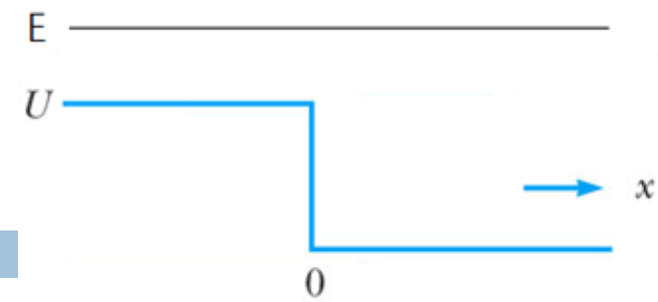
- Boundary conditions:
  - ψ & $d\psi/dx$ continuous at $x=0$ & $x=L$
- $x=0$:
  - $A+B=C+D$, $k_I(A-B)=k_{II}(C-D)$
- $x=L$: $(2L=\lambda_{II})$
  - $k_{II}L=k_{II}\lambda_{II}/2=\pi$, $\exp(\pm ik_{II}L)=-1$
  - $-C-D=F\exp(ik_IL)$, $k_{II}(-C+D)=k_IF\exp(ik_IL)$
- $C+D=A+B=-F\exp(ik_IL)$, $k_{II}(C-D)=k_I(A-B)=-k_IF\exp(ik_IL)$
- $-F\exp(ik_IL)=A+B=A-B$, $B=0$, $R=0$

7.11

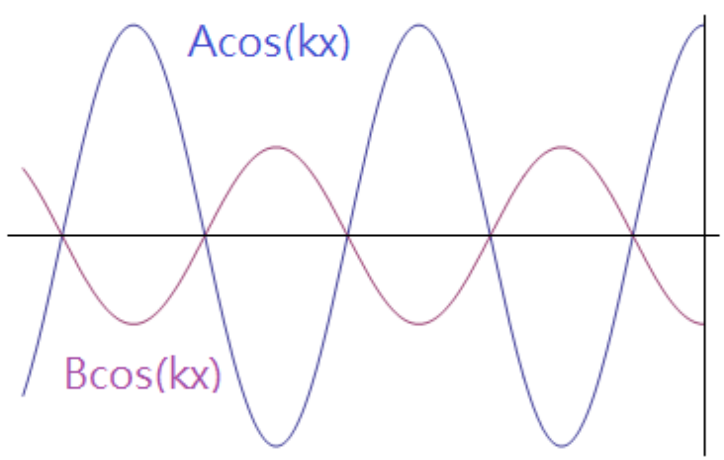


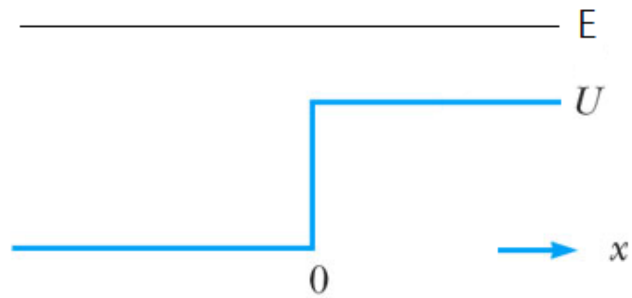
- When $2L=\lambda_{II}$, R=0
- Reflected waves from x=0 & x=L cancel
- Reflected wave is phase-shifted by π when going from high wave speed medium into low (I to II)
- $v=f\lambda$, $f=E/h$ is constant, v proportional to λ
- Reflected wave from x=0 is shifted by π
- Reflected wave from x=L has phase difference of $2\pi*2L/\lambda_{II}=2\pi$=no difference
- Total effect: Phases of 2 reflected waves differ by π

Phase Change of π



- $x<0$:
  - Low k, high λ, high wave speed
  - $\psi_I(x)=A\exp(ik_Ix)+B\exp(-ik_Ix)$
- $x>L$:
  - High k, low λ, low wave speed
  - $\psi_{II}(x)=C\exp(ik_{II}x)$
- $B/A=(k_I-k_{II})/(k_I+k_{II})<0$
- Look at real part at $t=0$ (assume A is real)





- $B/A=(k_I-k_{II})/(k_I+k_{II})>0$
- No phase change