

Problem 1 (15 pts)

Photons of wavelength 0.2\AA are incident on a metal that has workfunction 4eV . Detectors placed around the metal detect both photons and electrons coming from the metal.

- Some of the photons detected have wavelength 0.4328\AA . Assuming that this corresponds to the K_α line, deduce the value of the atomic number of this metal.
- Some of the photons detected result from Compton scattering of the incoming photons by the electrons in the metal. What are the largest and the smallest wavelength of those photons?
- For the electrons that originate in Compton scattering that are ejected in the forward direction (i.e. the direction of the incoming photons), what is their kinetic energy, in eV? Take into account the work function.
- Some of the electrons ejected from the metal have much higher kinetic energy than those in (c). What is the highest possible value of the kinetic energy of those electrons, in eV?
- For each one of the photons detected with wavelength 0.4328\AA there is an electron ejected from the metal. What is the kinetic energy of that electron?

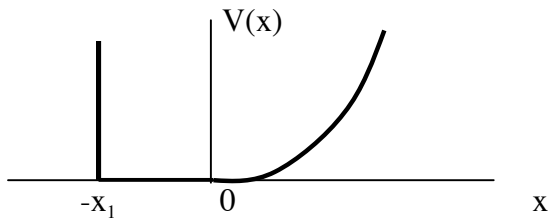
Problem 2 (15 pts)

A particle of mass m is in a stationary state of the one-dimensional potential $V(x)$ given by

$$V(x) = \infty \quad \text{for } x < -x_1$$

$$V(x) = 0 \quad \text{for } -x_1 < x < 0$$

$$V(x) = \frac{\hbar^2}{2mx_0^4} x^2 \quad \text{for } x > 0$$



The value of x_0 is given, x_1 is unknown. The wavefunction for this particle in the region $x > 0$ is known to be

$$\Psi(x) = Ae^{-\lambda x^2}$$

Since the particle is in a stationary state, the wavefunction satisfies the time-independent Schrodinger equation for all x . Using the Schrodinger equation,

- Find λ in terms of x_0 . Justify your answer.
- Find the energy of this particle in terms of x_0 and the mass m . Justify your answer.
- Find the wavefunction $\Psi(x)$ for $-x_1 < x < 0$. Your answer should depend only on A and x_0 , not on x_1 nor anything else. Use continuity of Ψ and Ψ' . Note that the energy of the particle is the same everywhere.
- Find two possible values of x_1 , expressed in terms of x_0 .
- Plot schematically the wavefunction in the interval $-x_1 < x < x_1$ for the two values of x_1 found in (d).

Problem 3 (15 pts + 3 pts extra credit)

An electron is in a stationary state in one dimension described by the wavefunction

$$\Psi(x) = Bxe^{-x/(2x_0)} \text{ for } x > 0$$

$$\Psi(x) = 0 \text{ for } x \leq 0$$

with $x_0 = 2A$.

(a) Find B.

(b) Find the average position, $\langle x \rangle$, in A. Hint: it's easier if you write it as the ratio of two integrals to eliminate B.

(c) Find the uncertainty in the position, Δx , in terms of x_0 .

(d) Find the average momentum, i.e. the expectation value of the momentum operator.

(e) Find the uncertainty in the momentum, Δp , and check whether or not the uncertainty principle is satisfied.

(f) Find the expectation value of the kinetic energy of this electron, in eV.

Use that: $\int_0^{\infty} dr r^p e^{-\lambda r} = \frac{p!}{\lambda^{p+1}}$.

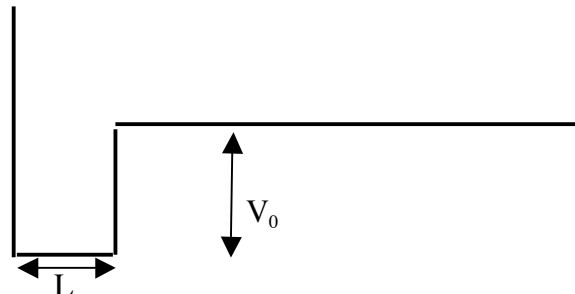
Problem 4 (15 pts)

An electron is in the lowest energy state of the potential shown in the figure. Its energy is $E = 2\text{eV}$. $V_0 = 6\text{eV}$.

$$V(x) = \infty \text{ for } x < 0.$$

$$V(x) = 0 \text{ for } 0 < x < L$$

$$V(x) = V_0 \text{ for } x > L$$



(a) Give the form of the wavefunction in the region $0 < x < L$. Give the numerical value (with units) of all the constants in your answer except for one of them.

(b) Same for the range $x > L$.

(c) Find the value of L in A. Your answer should be $L = 3.487A$. Show all steps in your derivation even if you find a different answer.

(d) What would be the ground state energy for an electron in an infinite well of length $L = 3.487A$?

(e) What would be the length of an infinite well where the ground state energy of an electron is $E = 2\text{eV}$? Give the answer, in A, and call it L_{eff} .

(f) Explain how the value L_{eff} found in (e) can be approximately understood quantitatively in terms of the length of this well $L = 3.487A$ and the wavefunction parameters given in (b).

Use $\hbar^2 / m_e = 7.62\text{eVA}^2$

Problem 5 (15 pts)

A gas of hydrogen-like ions has all the electrons in the same energy state. An electron in one of these ions absorbs a photon of wavelength 80\AA and is ejected with kinetic energy 32.6eV .

- What are the possible values of Z for these ions?
- In a magnetic field of 2000 T , these ions would show three closely spaced emission lines, with energy separation less than 0.2 eV . Assuming the Z of these ions is the smallest of the possible values found in (a), give the quantum numbers and the possible wavefunctions for these electrons. You don't have to give the value of the normalization constant in the wavefunction.
- What are the wavelengths of the three spectral lines mentioned in (b), in \AA ?
- In the absence of applied magnetic field, two closely spaced spectral emission lines are seen. Give an approximate value for the splitting of these lines, in eV (you may look in the book for any information you may need to answer this). Justify your answer.

Use $\mu_B = 5.79 \times 10^{-5} \text{ eV/T}$

Problem 6 (15 pts)

The vibrations of the atoms in a solid can be described by $3N$ harmonic oscillators, with N the number of atoms. Assume for a given solid the frequency of oscillation is ω , and $\hbar\omega = 0.15\text{eV}$.

- What is the Einstein temperature T_E for this solid?
- What is the average energy of an oscillator at temperature 10^6 K ?
- What is the average energy of an oscillator at zero temperature?
- What is the average energy of an oscillator at the Einstein temperature?
- At temperature T_E , if there are $10,000$ oscillators with energy 1.575 eV , how many oscillators are there with energy 1.875 eV ?

Problem 7 (10 pts + 5 pts extra credit)

- The lowest energy state for a proton confined to a length L_1 has the same energy as the lowest energy state for an electron confined to a length L_2 . What is L_2/L_1 ?
- 1000 electrons of kinetic energy 1eV are incident on a rectangular barrier of height 10eV and width 1\AA . How many tunnel through?
- Three electrons are in a cubic box of length L . The system is in a state of lowest possible energy. Assume electrons don't interact with each other but have spin $1/2$. How many different states are there with that energy? Describe all the states in words or with pictures or both.