Physics 4E Quiz 2 Solutions Spring 2012

Problem 1

The radius of the orbit is $r = a_0 \frac{n^2}{Z} = 4a_0 \Rightarrow n^2 = 4Z$

there are many possibilities, e.g. $Z=1, n=2$; $Z=4, n=4$, etc.

Now the wavelength of photon emitted when $n \to 1$ is

$$\frac{hc}{\lambda} = E_0 Z^2\left(1 - \frac{1}{n^2}\right) \Rightarrow Z^2 = \frac{hc}{\lambda E_0 (1-\frac{1}{n^2})} \Rightarrow$$

$$\Rightarrow Z = \left(\frac{hc}{\lambda E_0}\right)^{1/2} \frac{1}{(1-1/n^2)^{1/2}}$$; with $\lambda = 10\,\text{Å}$, $\frac{hc}{\lambda E_0} = \frac{12,400}{10 \cdot 13.6} =$

$$= 91.2 = 9.5^2 \Rightarrow \boxed{Z=9}$$, so from $n^2 = 4Z \Rightarrow \boxed{n=6}$

(b) From $L = n\hbar = m_e v_n r_n$, $r_n = \frac{a_0 n^2}{Z}$ one finds

$$\frac{v_n}{c} = \alpha \frac{Z}{n}$$, with $\alpha = \frac{ke^2}{\hbar c}$ $\Rightarrow \boxed{\frac{v_n}{c} = \frac{3}{2}\alpha}$

(c) Longest wavelength photon is $n=6 \to n=7$; $\frac{hc}{\lambda} = E_0 Z^2\left(\frac{1}{6^2} - \frac{1}{7^2}\right) \Rightarrow$

$$\lambda = \frac{hc}{E_0 Z^2\left(\frac{1}{6^2} - \frac{1}{7^2}\right)} \Rightarrow \boxed{\lambda = 1527.4\,\text{Å}}$$

Problem 2

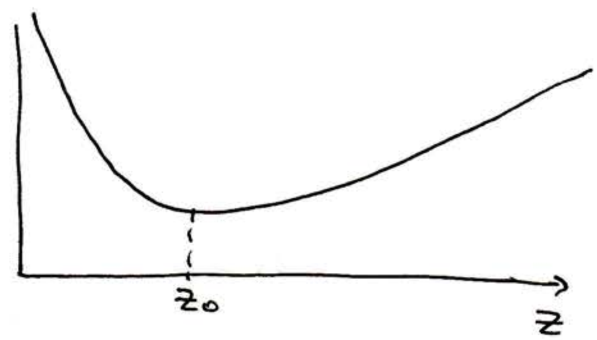
$$\Delta z \, \Delta p \sim \hbar \Rightarrow \Delta p \sim \frac{\hbar}{\Delta z} \quad , \text{ and } \Delta z = z_0$$

$$\bar{E} = \frac{\overline{p^2}}{2m_e} = \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e z_0^2} = \frac{3.81}{10^2}\,\text{eV} \Rightarrow \boxed{\bar{E} = 0.0381\,\text{eV}}$$

In J, using $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J} \Rightarrow \boxed{\bar{E} = 6.096 \times 10^{-21}\,\text{J}}$

(b)

$$E = \frac{\hbar^2}{2m_e z^2} + Mgz$$



For small z, first term dominates.
For large z, second term dominates

Equilibrium corresponds to minimum energy, at $z = z_0$.

(c) Equilibrium is determined by $\left.\frac{dE}{dz}\right)_{z=z_0} = 0 \Rightarrow$

$$-\frac{\hbar^2}{m_e z_0^3} + Mg = 0 \Rightarrow \quad M = \frac{\hbar^2}{m_e z_0^3 g} \Rightarrow$$

$$\Rightarrow M = 7.62\,\text{eV}\,\text{Å}^2 \cdot \frac{1}{10^3\,\text{Å}^3 \cdot 9.81\,\text{m}}\,\text{s}^2 = \frac{7.62 \times 1.6 \times 10^{-19}\,\text{J} \cdot \text{s}^2}{10^3\,\text{Å} \cdot 9.81\,\text{m}} =$$

$$= \frac{7.62 \times 1.6 \times 10^{-19} \cancel{10^3}\,\text{gr} \cdot \cancel{\text{m}^2}}{\cancel{10^3} \cdot 10^{-10} \cancel{\text{m}} \cdot 9.81 \cancel{\text{m}}} = \frac{7.62 \times 1.6}{9.81} \times 10^{-9}\,\text{gr}$$

$$\Rightarrow \boxed{M = 1.24 \times 10^{-9}\ \text{grams}}$$

Problem 3

(a) The n-th orbit fits n de Broglie wavelengths.

Given $r_n = a_0 n^2 \Rightarrow \quad 2\pi r_n = 2\pi a_0 n^2 = n\lambda \Rightarrow$

$\lambda = 2\pi a_0 n \Rightarrow$ for $n = 3$, $\lambda = 6\pi a_0 \Rightarrow \boxed{\lambda = 9.97\,\text{Å}}$

(b)

$$y = y_0 \cos(k_1 x - \omega_1 t) + y_0 \cos(k_2 x - \omega_2 t)$$

$k_1 = 11\,\text{Å}$, $k_2 = 13\,\text{Å}$, $\omega_1 = 30\,s^{-1}$, $\omega_2 = 42\,s^{-1}$

$\bar{k} = \dfrac{k_1 + k_2}{2} = 12\,\text{Å}^{-1}$, $\bar{\omega} = \dfrac{\omega_1 + \omega_2}{2} = 36\,s^{-1}$, $\Delta k = k_2 - k_1 = 2\,\text{Å}^{-1}$, $\Delta\omega = \omega_2 - \omega_1 = 12\,s^{-1}$

phase velocity: $v_p = \dfrac{\bar{\omega}}{\bar{k}} = \dfrac{36\,s^{-1}}{12\,\text{Å}^{-1}} \Rightarrow \boxed{v_p = 3\,\dfrac{\text{Å}}{s}}$

group velocity: $v_g = \dfrac{\Delta\omega}{\Delta k} = \dfrac{12\,s^{-1}}{2\,\text{Å}^{-1}} \Rightarrow \boxed{v_g = 6\,\dfrac{\text{Å}}{s}}$

(c) $y(x, t=0) = 2y_0 \cos\left(\dfrac{\Delta k}{2} x\right) \cos(\bar{k} x)$

Largest negative value for the envelope is for $\dfrac{\Delta k}{2} x = \pi \Rightarrow \cos\left(\dfrac{\Delta k}{2} x\right) = \cos\pi = -1$

$\Rightarrow \quad x = \dfrac{2\pi}{\Delta k} = \dfrac{2\pi}{2}\,\text{Å} = \pi\,\text{Å} = \boxed{3.14\,\text{Å}}$