Problem 1

$$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2 m_e L^2}(n_1^2+n_2^2+n_3^2) \equiv E_0(n_1^2+n_2^2+n_3^2)$$

ground state: $n_1=n_2=n_3=1$; $E_{111} = 3E_0 = 1\,eV \Rightarrow \frac{\hbar^2\pi^2}{2m_e L^2} = \frac{1}{3}\,eV \Rightarrow$

$$L^2 = \frac{3\hbar^2\pi^2}{2m_e \cdot eV} = \frac{3}{2}\cdot\frac{7.62\,eV\,\mathring{A}^2\pi^2}{eV} = 112.8\,\mathring{A}^2 \Rightarrow \boxed{L = 10.6\,\mathring{A}}$$

(b) 5 lowest energy levels: $E_0 = \frac{1}{3}eV$

$n_1\ n_2\ n_3$	$n_1^2+n_2^2+n_3^2$	E	degeneracy
1 1 1	3	$3E_0 = 1eV$	1
2 1 1	6	$6E_0 = 2eV$	3
2 2 1	9	$9E_0 = 3eV$	3
3 1 1	11	$11E_0 = 3.67eV$	3
2 2 2	12	$12E_0 = 4eV$	1

(c) Can fit 2 electrons per state. So in these 5 energy levels,

Total # of states = 1+3+3+3+1 = 11 states, 2 electrons/state

$\Rightarrow$ **can accommodate 22 electrons**

Problem 2

$$\psi(r,\theta,\phi) = C r^2 e^{-2r/a_0} \sin^2\theta \, e^{-2i\phi}$$

$\Rightarrow$ $\boxed{m_l = -2}$ $\Rightarrow l \geq 2$. From the θ dependence $\Rightarrow$ $\boxed{l = 2}$

So $n \geq 3$. From the r^2 term $\Rightarrow$ $\boxed{n = 3}$

From the general form $e^{-Zr/na_0} = e^{-2r/a_0} \Rightarrow \frac{Z}{n} = 2 \Rightarrow$ $\boxed{Z = 6}$

(b) $P(r) = r^2 R^2(r) = r^6 e^{-4r/a_0}$

$$P'(r) = 0 = 6r^5 - 4\frac{r^6}{a_0} \Rightarrow \boxed{r = \frac{3}{2} a_0}$$

In Bohr atom: $r_{n=3} = \frac{a_0}{Z} n^2 = a_0 \cdot \frac{9}{6} = \frac{3}{2} a_0$ agrees.

(c) $$\langle r \rangle = \frac{\int_0^\infty dr \, r \, P(r)}{\int_0^\infty dr \, P(r)} = \frac{\int_0^\infty dr \, r^7 e^{-4r/a_0}}{\int_0^\infty dr \, r^6 e^{-4r/a_0}}$$

Using $\int_0^\infty dr \cdot r^p e^{-\lambda r} = \frac{p!}{\lambda^{p+1}}$ $\Rightarrow$

$$\langle r \rangle = \frac{7!}{\left(\frac{4}{a_0}\right)^8} \frac{\left(\frac{4}{a_0}\right)^7}{6!} = \frac{7}{\frac{4}{a_0}} = \frac{7}{4} a_0$$

$\Rightarrow$ $\boxed{\langle r \rangle = \frac{7}{4} a_0 = 1.75 a_0}$

It is larger than the most probable r.

Problem 3

(a) $r_n = \frac{a_0}{Z} n^2$. So for the same n, $\boxed{r(He^+) = \frac{r(H)}{2}}$

Speed: $L = n\hbar = m_e v r \Rightarrow v = \frac{L}{m_e r}$; so for the same n,

$$\Rightarrow \frac{v(He^+)}{v(H)} = \frac{r(H)}{r(He^+)} = 2$$

(b) Period: $T = \frac{2\pi r}{v} \Rightarrow \frac{T(He^+)}{T(H)} = \frac{r(He^+)}{r(H)} \frac{v(H)}{v(He^+)} = \frac{1}{2} \cdot \frac{1}{2} \boxed{= \frac{1}{4}}$

(c) $i = \frac{dq}{dt} = \frac{q}{T}$. For H, $q = e$; for He^+, $q = 2$, so

$$\frac{i(He^+)}{i(H)} = \frac{q(He^+)}{q(H)} \cdot \frac{T(H)}{T(He^+)} = 2 \cdot 4 \boxed{= 8}$$

(d) $B = \frac{\mu_0 i}{2r} \Rightarrow \frac{B(He^+)}{B(H)} = \frac{i(He^+)}{i(H)} \cdot \frac{r(H)}{r(He^+)} = 8 \cdot 2 \boxed{= 16}$

Spin-orbit splitting: $\Delta E = -\vec{\mu} \cdot \vec{B}$ so it is proportional to B, since $\vec{\mu}$ is the magnetic moment due to electron spin, same for both

$$\Rightarrow \boxed{\frac{\Delta E(He^+)}{\Delta E(H)} = \frac{B(He^+)}{B(H)} = 16}$$

(e) $\boxed{\Delta E(He^+) = 16 \cdot \Delta E(H) = 16 \cdot 4.5 \times 10^{-5} eV = 7.2 \times 10^{-4} eV}$