Physics 160 (Winter 2012)

Black Holes

The human mind has entertained countless extraordinary conceptions. But the most fantastic of these conceptions is that of a black hole.

- A hole in space with a well defined radius, into which anything will fall, and out of which nothing will emerge.
- A hole with a gravitational force so strong that even light is trapped and does not emerge
- A hole that curves space and warps time

This sounds like fantasy, but it isn't: well tested laws of physics predict firmly that black holes exist. Moreover, several black holes have already been detected in our Galaxy (The Milky Way).
In addition there is growing evidence that exceedingly massive BHs are at the centers of most galaxies (including the Milky Way), and that mass accretion into these massive BHs may account for the collosal energy output from distant quasars.

Overview : I will give a brief description of these 2 classes of BHs, and how to detect them

But, before I do this, I will place BHs in the context of fundamental Physics.

Physical Interactions:

There are 4 fundamental interactions in nature

Interaction strength.

(a) • Strong Interaction (strongest) : This is short-range attractive force that binds the nuclei of atoms. Recall, nucleus is composed of

- protons (+) positively charged minute particl
- neutrons (0) similar mass as proton but no ch

Repulsive electrostatic interaction between protons would disrupt nuclei were it not for the attractive strong interaction between protons, protons and neutrons, neutrons & neutrons

~~(b)~~ Also binds quarks that make up p, n.

(b) Electromagnetic Interaction:

Interaction between charges carried by matter.

- Attractive · between opposite chgs + & - ;
- Repulsive · between chgs. of same sign $^{++}_{--}$

Force $\propto 1/r^2$ $\oplus \xleftarrow{} r \xrightarrow{} \ominus$, so it is potentially long range. But on sufficiently large values of r, + and - charges cancel each other, so net chg. of universe is 0. No long-range significance

(c) Weak Interaction: This force that mediates decay one unstable nucleus into another nucleus: $n \rightarrow p + e^- + \bar{\nu}$

Important for $_2He^4$ production in Big Bang

(d) Gravitational Interaction (weakest):

Tests carried out over the past 96 years show that Einstein's theory of gravity, i.e., General Relativity, is the correct theory that replaces Newton's theory of gravity.

Newtonian theory: All of you have studied Newtonian Physics in which attractive force between 2 masses m_1 and M_2 separated by distance r_{12} is:

$F_{grav} = G m_1 M_2 / r_{12}^2$

- Newton's force acts instantaneously. But instantaneous interactions are prohibited not just by General Relativity, but also by Einstein's earlier theory, Special Relativity. In SR (and GR), no signal can propagate faster than the speed of light ($c = 3 \times 10^5$ km/s).
- Inconsistency between Newtonian gravitation and SR led to General Relativity (1915). As we shall see, GR did not replace Newtonian Gravity with a new force law. Rather GR revolutionized our understanding of spacetime

Importance of Gravity

- Universal Interaction between all masses (actually mass and energy).
- Attractive: Since there are no opposite sign "charges (masses)", there is no shielding of gravity as in with electrical charges. Thus, gravity is longest range "force": dominates other forces on large scales.

- **Significance:** Despite being the weakest force (e.g. force between e^- and p^+

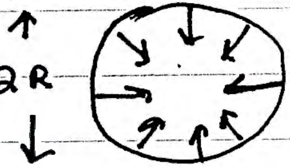
$$\left(\frac{F_{grav}}{F_{electric}} = \frac{Gm_pm_e/r^2}{e^2/r^2} = \frac{Gm_pm_e}{e^2} \sim 10^{-39}! \right)$$

it is gravity that determines the large-scale structure of the Universe.
  - Density structures
  - Expansion

Newtonian Domain

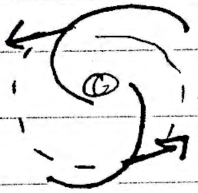
Still, for most problems, Newtonian physics work.

Examples → • Structure of the Sun - and most "normal stars" the inward pull of gravity balanced by outward push of gas pressure (gradients)



$R \sim 10^{11}$ cm

- Galactic Dynamics: All stars are in giant gravitationally bound configurations of stars, gas, dark-matter. Gravitational pull of dark matter balanced by ~~centrifugal~~ centripetal acceleration of rotating stars.



$R \simeq 3 \times 10^{22}$ cm

General Relativity Domain

But there are experimentally verified deviations from Newtonian gravity. Corrections are of order

$$\boxed{GM/Rc^2}$$

Sun: $M = 2 \times 10^{33}$ g, $R = 7 \times 10^{10}$ cm, $c = 3 \times 10^{10}$ cm/s

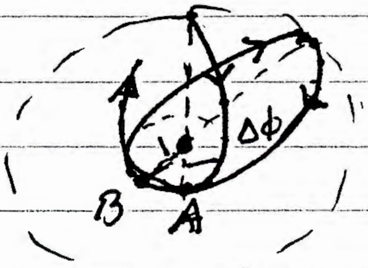
Since $G = 6.7 \times 10^{-8}$, $GM/Rc^2 \simeq 2 \times 10^{-6}$

Planetary orbits

Newton

Closed elliptical orbit about sun at focus

Einstein



Open precessing orbits in which position of closest approach to sun shifts by $\Delta\phi \sim \overset{43''}{h}$ per century ~~[illegible]~~

Thus shift of the perihelion of Mercury's orbit was predicted by Einstein and subsequently observed.

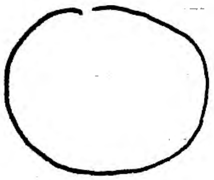
Difficult measurement since precession also caused by perturbations due to gravitational interactions between Mercury and other planets. When these effects are accurately measured one is left with residual, which cannot be explained by Newtonian Physics: ≃43'' just as predicted by G.R.

BH05-a

In fact GR important when $GM/Rc^2 \sim 1$
Do such objects exist?

Examples of Relativistic Objects: Found at the endpoints of stellar evolution. Answer depends on mass of progenitor

Stars:



- Stars supported against gravitational inward pull by outward push due to pressure gradients (dP/dr) generated by hot interior gas ($T_c \sim 1.5\times10^7$ K for sun)
- High T_c consequence of virial theorem on pressure required to hold-up weight of sun
- Lifetime of sun ($t_{age} \geq 4.6\times10^9$ yrs) requires energy input into core: otherwise sun would radiate all its thermal energy in $t_{KK} \sim 2\times10^7$ y
- Thermo-nuclear reactions in core supply energy:

$$4\times({}_1H^1) \longrightarrow {}_2He^4 + Q \quad (Q \simeq 26\,\text{MeV})$$

where $Q = -M_{He}c^2 + 4M_Hc^2$

- After ~ 10% of $M(H)$ consumed, star evolves into a different structure:

compact collapsed core + extended envelope
Let's focus on the core

- 

By products of collapse depend on the progenit
mass; i.e., nature of core depends on $M_{progenitor}$

BH054

$M_{progenitor}/M_\odot$	Byproduct	$M_{core}/M_\odot$ $M_{byproduct}/M_\odot$
$0.5 \rightarrow 10$	white dwarf	≤ 1.4
$10 \rightarrow 15$	Neutron star	≤ 3
> 15	Black Hole	> 3

Briefly: White dwarfs are compact objects $R \simeq 10^{-2} R_\odot$, "held-up" by degenerate e^- pressure: $GM/Rc^2 \simeq 10^{-4}$, $\langle M \rangle = 0.6 M_\odot$

Neutron stars: compact objects, $R \simeq 10^{-5} R_\odot$ $\langle M \rangle \simeq 1.4 M_\odot$, and on which $\frac{GM}{Rc^2} \simeq 0.1$ held up by degenerate neutron pressure

Black Holes:

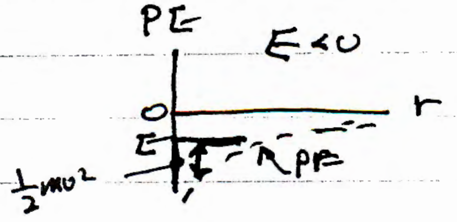
GR predicts: If mass compressed into a sufficiently small volume, the gravitational pull at the surface can be large enough to prevent anything from escaping, even light; i.e., test particle with mass, m, or massless photon with energy, $\epsilon = h\nu$, cannot escape.

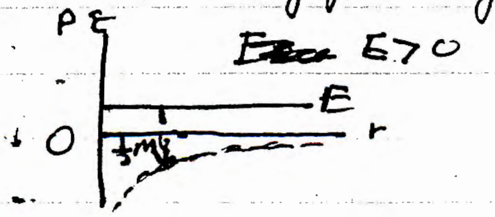
Crude argument gives correct result, even though it is physically incorrect

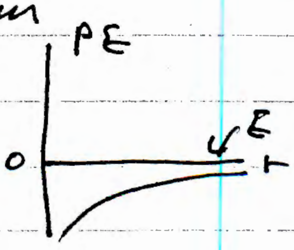
Suppose mass, M, collapsed within radius, r. Further suppose there is a test mass, m, at r, where $m \ll M$. Also let m have outward radial velocity, v

What is condition for m to escape gravitational field of M?

Newtonian argument: Energy diagram







$$\text{Energy } E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$E<0$: Particle will not escape. Rather go out to r_{max}

$E>0$: " " escape to $r \to \infty$

Minimal condition $E \geq 0$ or $\frac{1}{2}mv^2 \geq \frac{GMm}{r}$

Thus as $r \to \infty$ $\frac{1}{2}mv^2 \geq 0$

threshold condition is when $\frac{1}{2}mv^2 = 0$ at $r \to \infty$

$$\therefore \quad \frac{1}{2}mv^2 \geq \frac{GMm}{r} = \frac{1}{2}m v^2_{escape}$$

$$\Rightarrow \quad \boxed{v^2_{escape}(r) = \frac{2GM}{R}}$$

For fixed M, let's decrease starting radius, r. (M is more compact) to radius where $v^2_{escape} = c^2$. For such an object

$$\frac{2GM}{r} = c^2$$ or

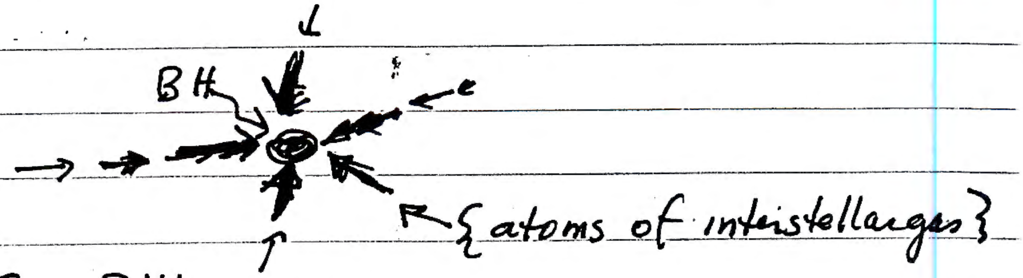
- $\frac{2GM}{rc^2} = 1$
- $r = \frac{2GM}{c^2}$

As we shall see, this is correct answer predicted by GR

Observational Consequences

① Stellar Black Holes

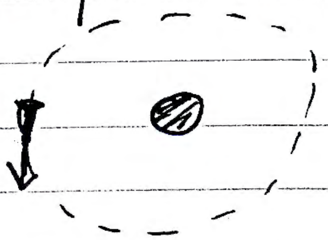
Black holes affect their surroundings. Gravitational field sucks in surrounding diffuse interstellar gas



Evidence for BH's presence

ⓐ Atoms of interstellar gas ($n \sim 1\,cm^{-3}$: compare to stars where $n > 10^{24}\,cm^{-3}$) are pulled in by gravity of BH. Gas accelerates inward, reaching velocity $v \sim c$ near BH ($r \to \frac{2GM}{c^2}$.

- ⓑ Rocket ship in circular orbit around BH



(i) at $r \gg 2GM/c^2$: gas far from BH is cool emits long wavelength radiation ($\lambda \simeq$ cm → M.) i.e., radio waves.

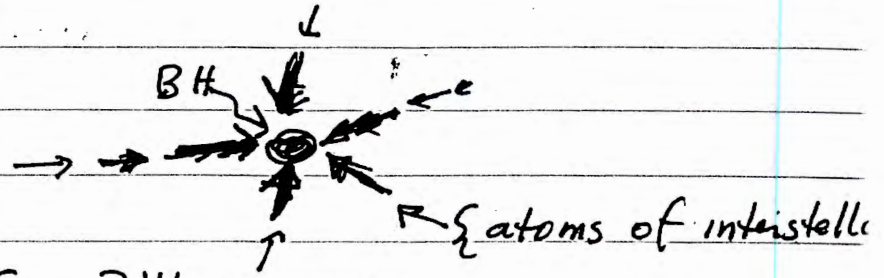
(ii) Nearer to BH: atoms move faster, collisions between them heats gas up. Radiation emitted at smaller λ's : UV ($\lambda \simeq 10^{-5}$ cm or ~ 1000 Å)

(iii) Even closer, gas is so hot it emits X-rays ($\lambda \simeq 10^{-8}$ cm, $\lambda \sim 1$ Å)

Observational Consequences

① Stellar Black Holes

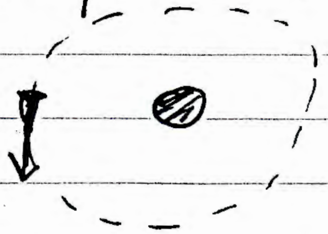
Black holes affect their surroundings. (
tational field sucks in surrounding diffe
interstellar gas



Evidence for BH's presence

ⓐ Atoms of interstellar gas ($n \sim 1\,\mathrm{cm}^{-3}$:
to stars where $n > 10^{24}\,\mathrm{cm}^{-3}$) are pulled
gravity of BH. Gas accelerates inward,
reaching velocity $v \sim c$ near BH (r

- ⓑ Rocket ship in circular orbit around



(i) at $r \gg 2GM/c^2$: gas far from BH
emits long wavelength radiation ($\lambda \simeq$ c
i.e., radio waves.

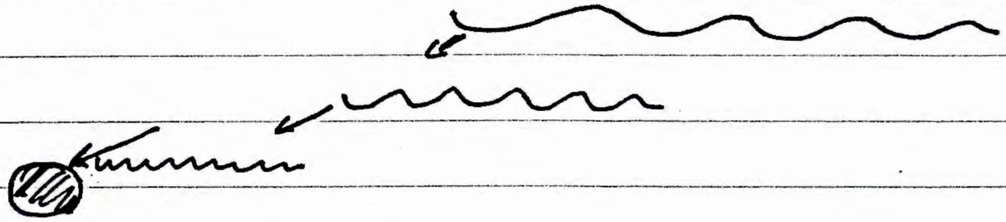
(ii) Nearer to BH: atoms move faster,
between them heats gas up. Radiatio em
smaller λ's : UV ($\lambda \simeq 10^{-5}$ cm or $\sim$10

(iii) Even closer, gas is so hot
emits X-rays ($\lambda \simeq 10^{-8}$ cm, $\lambda \sim 1$ Å

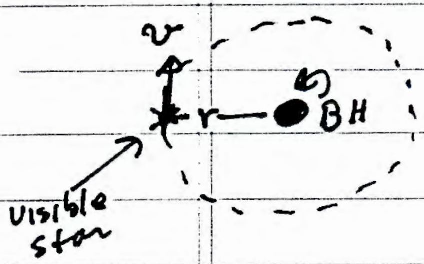
Detection of such X-rays led to the discovery of the first known BH, Cygnus X-1.

(iv) Closer: emitted λ decreases to γ rays ($\lambda \ll 1\,\text{Å}$). Then as $r \to 2GM/c^2$, radiation disappears

Summary

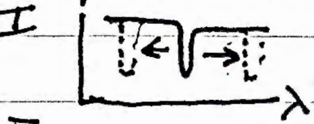


ⓒ Masses of Black Holes : Measure mass with the same technique used to measure the mass of the sun and stars in binary orbits.



Visible star orbiting blank sky: Measure two quantities: Velocity, v, and period, T

- v inferred from periodic shifting of spectral absorption lines in visible star: Doppler effect



- T from period

- Physics:

$$F = m_* a$$

$$\frac{GM_{BH} m_*}{r^2} \swarrow \qquad \searrow \frac{m_* v_c^2}{r} \quad (\text{circular orbit})$$

$$\Rightarrow \frac{GM_{BH} m_*}{r^2} = \frac{m_* v_c^2}{r} \Rightarrow \boxed{M_{BH} = r v_c^2 / G}$$

Example:

Cyg X-1 $v_c = 190$ km/s , $T = 10^6$ sec ,

since $2\pi r / T = v \Rightarrow r = vT/2\pi = 3 \times 10^{12}$ cm

$$M \simeq 8 M_\odot$$

Too massive for a neutron star, must be a BH

To understand such fascinating objects, we will

- use Newton's laws (1687) required to deduce M_{BH}. Ok when orbital radius $r \gg \frac{2GM}{c^2}$
- GR when considering region near BH; i.e., $r \sim 2GM/c^2$ (1915)
- Current understanding of BHs clarified by subsequent work due to Oppenheimer, Hawking, and Penrose in 20th century.

GR tells us

"No hair theorem" Despite complicated processes that leads to its formation, all the properties of a BH are determined by 2 numbers:

- Mass
- Spin Angular Momentum

(Contrast to normal stars where age, chemical abundance, B fields etc. are also important)

GR predicts: "Horizon" radius $r_H = 2GM/c^2$
$= 2.9 (M/M_\odot)$ km

So for BH with $M = 10 M_\odot$, $r_H = 29$ km

- Orbits: GR used to compute orbits near BHs for
  - test particles with mass
  - photons (with zero-mass)
  - Curvature of space

• Internal Structure

Density: $\langle\rho\rangle \simeq M \Big/ \frac{4\pi r_H^3}{3}$

for $M = 10 M_\odot \Rightarrow \langle\rho\rangle = \dfrac{10 \times 2\times10^{33}}{4(29\times10^5)^3} \simeq 2\times10^{14}\ \mathrm{g/cm^3}$

By comparison:

$\langle\rho\rangle_{H_2O} \sim 1\ \mathrm{g\,cm^{-3}}$

$\langle\rho\rangle_\odot \sim$ few $\mathrm{g\,cm^{-3}}$

Note: $\langle\rho\rangle \propto M \Big/ \left(\frac{2GM}{c^2}\right)^3 \propto \dfrac{1}{M^2}$

$\langle\rho\rangle$ decreases as M increases

But, mass of BH does not fill sphere with radius $r = r_H$ uniformly. Rather theory predicts all of the mass is concentrated into a miniscule region of space called a <u>singularity</u>. This is region which is so small, that classical GR is no longer operative. Rather laws of quantum mechanics apply.

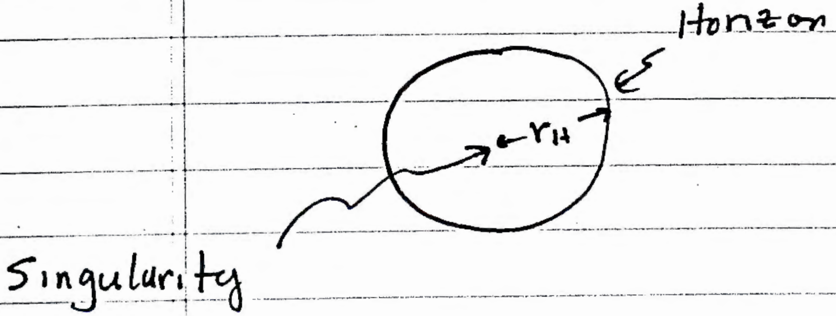
We shall see this occurs when $r = r_{sing} \simeq \left(\frac{G\hbar}{c^3}\right)^{1/2}$ where $\hbar = h/2\pi$ and $h = 6.626 \times 10^{-27}$ erg·s is Planck's constant.

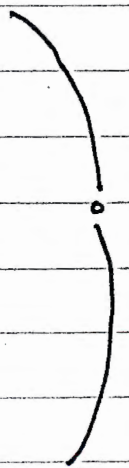
$$r_{sing} \simeq 10^{-33} \text{ cm}!$$

<u>Contrast</u>:

$r_{Bohr} \simeq 10^{-8}$ cm

$r_{proton} \simeq 10^{-13}$ cm





Singularity and any matter at $r < r_H$ are hidden from view from the outside. When mass collapses within $r = r_H$, all that is left behind is grav. pull.

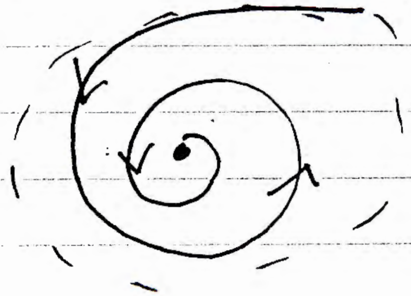
Other types of BHs

- **Galactic Center**

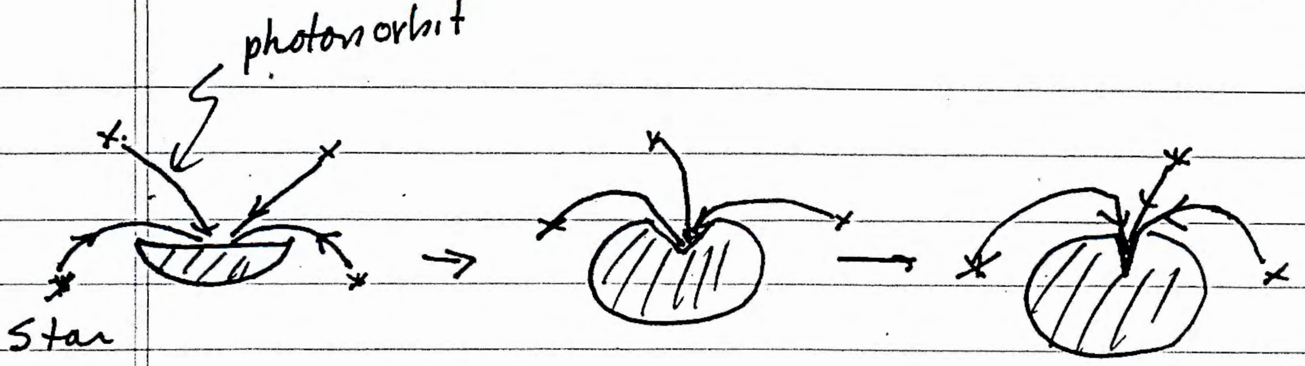
at the center of our Galaxy there exists a BH with mass $M_{BH} = 4.1 \times 10^6 M_\odot$.

- $r_H = 2GM/c^2 = 12.2 \times 10^6$ km
- Orbits:

If space craft orbits at $r < 3r_H$; i.e., at $r < 3 \times 12.2 \times 10^6$ km $= 3.7 \times 10^7$ km, something strange happens. Even if spacecraft starts out in circular orbit, it will now spiral into the BH. This happens even though orbital angular momentum L & energy are conserved. While this is impossible in Newtonian theory, it can happen in GR.



As you descend towards BH, sky around you becomes increasingly black until every star seems to be located at same point.



This is subject of this class
- We will discuss Physics of BHs.
- Origin of BHs
- Recent modifications: Hawking radiation

Preliminaries : But before we launch into BH physics, we need to study GR. And before we discuss GR, it is necessary to review special relativity, SR. And before SR I will next time review concepts of space & time incorporated into Newtonian theory

Special Relativity: (Hartle Chp 4)

① Space-Time Concepts

In 1905 Einstein submitted an article to Annalen der Physick (Annals of Physics), which came to grips with a paradox about light that had bothered him for the previous 10 years. Upon turning the final page of the article, the editor of the journal, Max Planck, realized that the accepted 19th century scientific order had been overthrown. that is, Einstein had completely overturned traditional (intuitive) concepts of space and time and replaced them with a new concept, Special Relativity, containing properties that fly in the face of familiar experience

Ⓐ Paradox

In the mid 1800's Maxwell succeeded in unifying electricty & magnetism with the concept of the <u>electromagnetic field</u>.

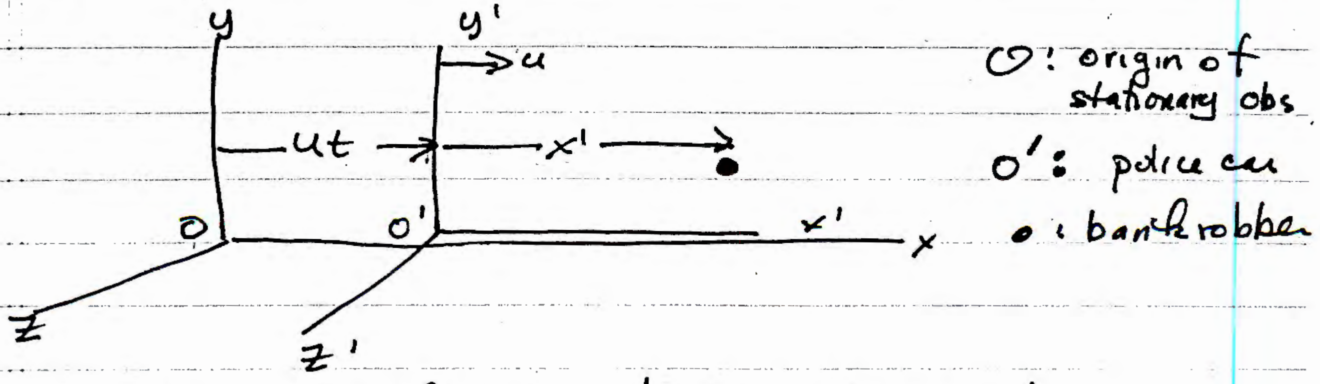
Examples (i) Effets of lightening on power generators
(ii) Sprinkle iron filings near magnet & look at pattern (lines of force)
(iii) Electrostatic shocks

Maxwell's theory is described by 4 first order linear partial DEs; i.e., Maxwell's equations. Solutions for the propagation of EM disturbances show that in vacuum they travel at a fixed, unchanging speed, the speed of light, c. Maxwell showed that light is a wave that never slows down.

Einstein Question:

What would happen if I chased a beam of light at light speed? What would the velocity of a quantum of light be with respect to me?

Newtonian Answer: Use Galilean coordinate transformation. Suppose police car chases a bank robber on the freeway. Wrst "stationary" observer O, police car O' moves with velocity u and bank robber with v



Galilean transformation between O and O'

$$\left.\begin{array}{l} x = x' + ut \\ y = y' \\ z = z' \end{array}\right\}$$

coordinates of bank robber
at t=0, O coincides with O'

Take Time Derivatives: $\frac{d}{dt} = \frac{d}{dt'}$

$$\frac{dx}{dt} = \frac{dx'}{dt} + u$$

$$v = v' + u \quad \text{or} \quad v' = v - u$$

Bank robber's speed recorded by police car is $v' = v - u$ if police car moves at $u = v$, same speed, then $v' = 0$; i.e., bank robber appears stationary to police car, O'.

In Newtonian theory the speed of light has no special significance. Thus if $v = c$ and $u = c$, then $v' = 0$. Thus Einstein would catch up to light wave (photon) and see light wave standing still.

But Einstein reasoned that there is no such thing as stationary light (in vacuum). In fact this is a prediction of Maxwell's equations.

Implication (1): Something wrong with Galilean transformation. While they work when $v \ll c$, they break down when $v \to c$.

Implication (2): By abandoning Galilean transformation, we change our conception of space & time.

- Simultaneity between events needs to be revised

BH-16

Examples of Change

Common Intuition: Different perceptions of 2 observers

① Trees along highway move wrst driver, but stationary wrst hitchhiker sitting on fence

② Dashboard in car is stationary wrst driver, but is moving wrst hitchhiker

Special Relativity

Einstein's theory shows that differences between perceptions of two observers are more subtle and profound.

SR: Observers in relative motion have different perceptions of time as well as space

ⓐ Same watch ticks at different rates according to 2 observers

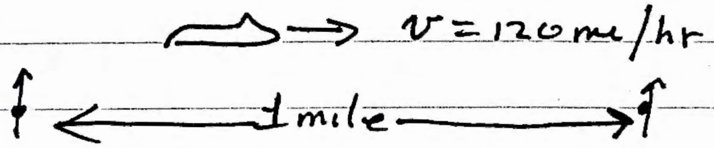
ⓑ Lengths of given object will be different for 2 observers.

Disagreement not due to inaccuracies of measuring devices or experimental errors. Most accurate devices confirm that space & time - measured by distances and durations - are not experienced identically in reference frames in relative motion.

<u>Special Relativity</u> resolves the conflict between our intuition of motion and the speed of light. But one pays a price; i.e., individuals in relative motion will not agree on their observations of space and time.

Examples

(1)



Driver streaks down mile long strip at 120mi/hr. Stationary observer watches racing car. Both have identical stopwatches synchronized at the starting time.

● Stationary Observer :

ⓐ Time to go between posts

$\rightarrow \Delta t = \Delta x / v = 1\,\text{mi} / 120\,\text{mi/hr} = \frac{1}{120}\,\text{hr}$

$\Delta t = 0.5$ minutes $= 30$ seconds

● Driver

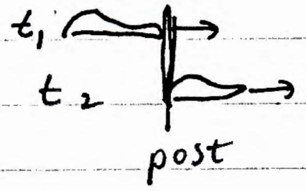
$\Delta t' = 29.999\,999\,999\,999\,52$ seconds

$$\boxed{\Delta t > \Delta t'}$$

ⓑ Car Length

● Measuring Length of Car

Stationary observer:



$\Delta l = v \Delta t = v(t_2 - t_1)$

Driver : measured length of car is showroom:

$\Delta l' = 16\,\text{ft}$

But stationary observer finds $\Delta l = 15.999\,999999\,99974\,\text{ft}$

$$\boxed{\Delta l < \Delta l'}$$

So we have "time dilation" and "length contraction". These effects get amplified as $v \to c$.

Recall $c = 3\times10^{10}\,\text{cm/s} = 186{,}000\,\text{mi/s} = 670\times10^{6}\,\frac{\text{mi}}{\text{hr}}$

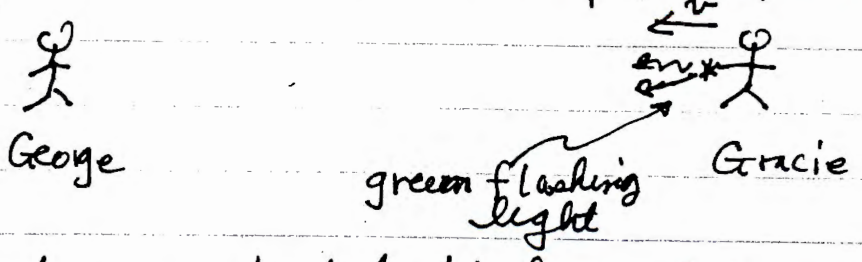
Key Ingredients in Special Relativity

① Principle of Relativity

All the Laws of physics are the same in all inertial reference frames (i.e., frames with constant relative velocity)

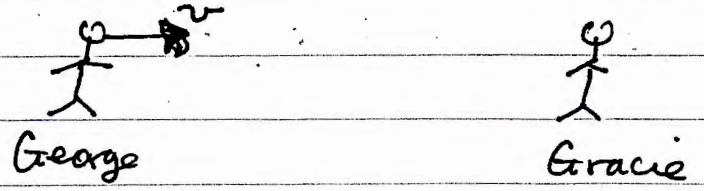
Galileo/newton assumed only laws of motion obeyed this principle.

ⓐ Equivalence of inertial space explorers



George : floating in absolute blackness feels stationary.
Gracies moves with speed v toward George

(b) Gracie



But according to Gracie, it is George who is moving.

Same situation from 2 points of view (reference frames). Each observer feels stationary and perceives the other as moving. Symmetry between equally valid descriptions.

Implication: Motion is relative. There is no meaning to concept of absolute motion, but only of relative motion.

Caveat: Motions must be force-free. Neither Gracie nor George is being pushed nor pulled. If either were accelerated due to rocket motors, etc. they would know they were not stationary (Galileo).

Uniform Motion

Einstein: Suppose you are in uniformly moving train (constant v) with shades drawn over windows. Without viewing external world, you cannot determine whether you are moving or sitting still wrst train station

$\boxed{\text{No experiment you do can determine motion of train}}$

All you can do is determine relative motion by looking outside.

<u>All Laws of Physics</u>: If George & Gracie conduct identical experiments in their uniformly moving space stations, the results will be the same. This is because laws of physics are identical. Constant velocity motion has no effect on the experiments

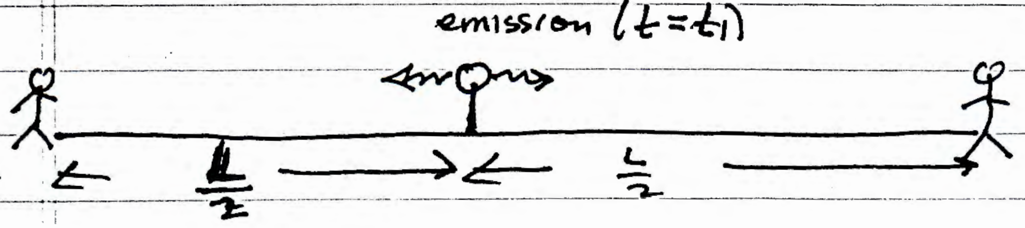
② <u>2nd Key Ingredient to SR: Speed of Light</u>

Light is special. Contrary to assertion that only relative motions ~~a~~ have meaning, all observers agree that speed of light $c = 3 \times 10^{10}$ cm/s, no matter how fast they move.

Consequences for Spacetime

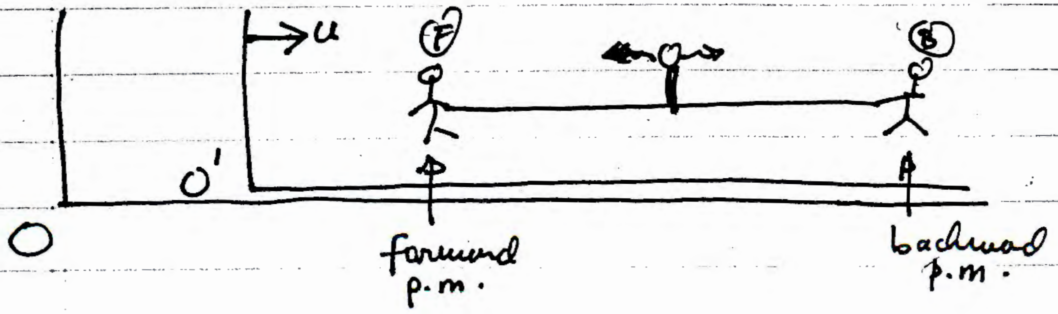
Example Prime ministers of India & Pakistan sit at ends of long negotiating table, having just concluding a ceasefire agreement. But <u>neither of them will sign the accord befoe the other</u>

Banki-Moon (U.N. Secy. Genl.) comes up with a brilliant idea. Light bulb 1/2 way between them is turned on. The emitted light will reach both prime ministers at the same time since they are equidistant from the bulb



Each leader agrees to sign their copies of the agreement when each sees light at same instant. Everything works.

Ban Ki-Moon now tries the same experiment with another set of warring nations that have also reached a peace agreement. But in this case the negotiating table is on a train moving with constant speed, u.



Again the experiment works on the train. Both Ⓕ and Ⓑ prime ministers receive the light signal simultaneously and sign the agreement.

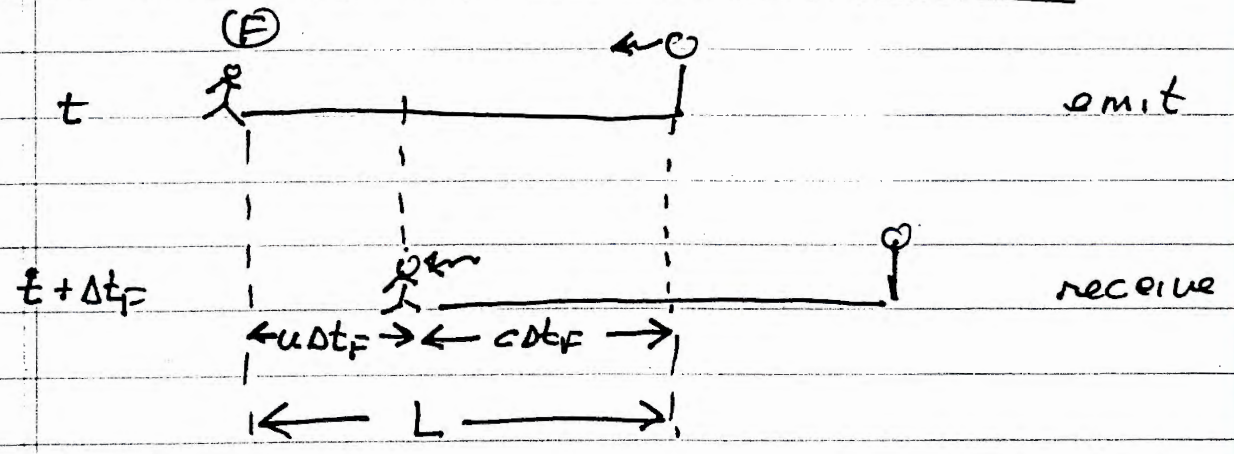
BH-~~#~~22

But. Banhi is shocked when she hears the fighting has broken out between people of each country, who had been watching the ceremony from the train platform.

why? People from the (F) country claims that their PM signed the agreement before the PM from the (B) country. But everyone on the train agrees they signed it simultaneously. Who is right? Both.

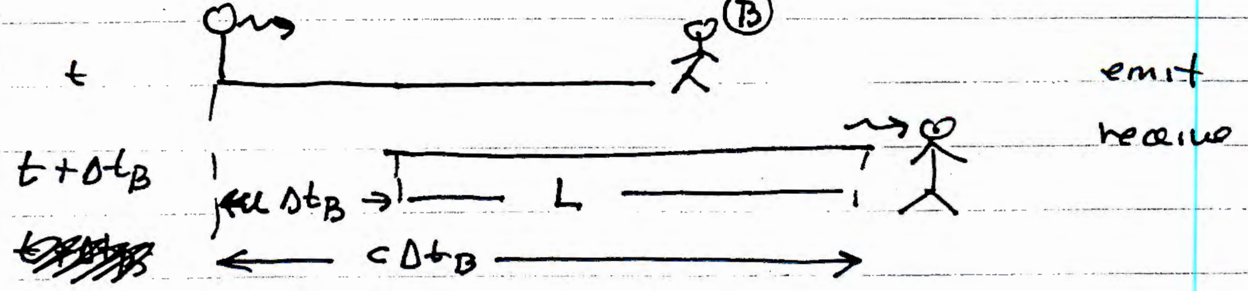
What Stationary Observer on Platform See

Forward.



$$u\Delta t_F + c\Delta t_F = L \Rightarrow \Delta t_F = \frac{L}{c+u}$$

Backward



$$u\Delta t_B + L = c\Delta t_B \Rightarrow \Delta t_B = \frac{L}{c-u}$$

In the second case light travels (with the same speed) for a longer distance than in the first case. So it takes longer for the light to reach the Ⓑ prime minister than the Ⓔ PM according to the stationary observers on the platform.

Who's right? Banki on train or people on platform? Both is the answer

End of Simultaneity