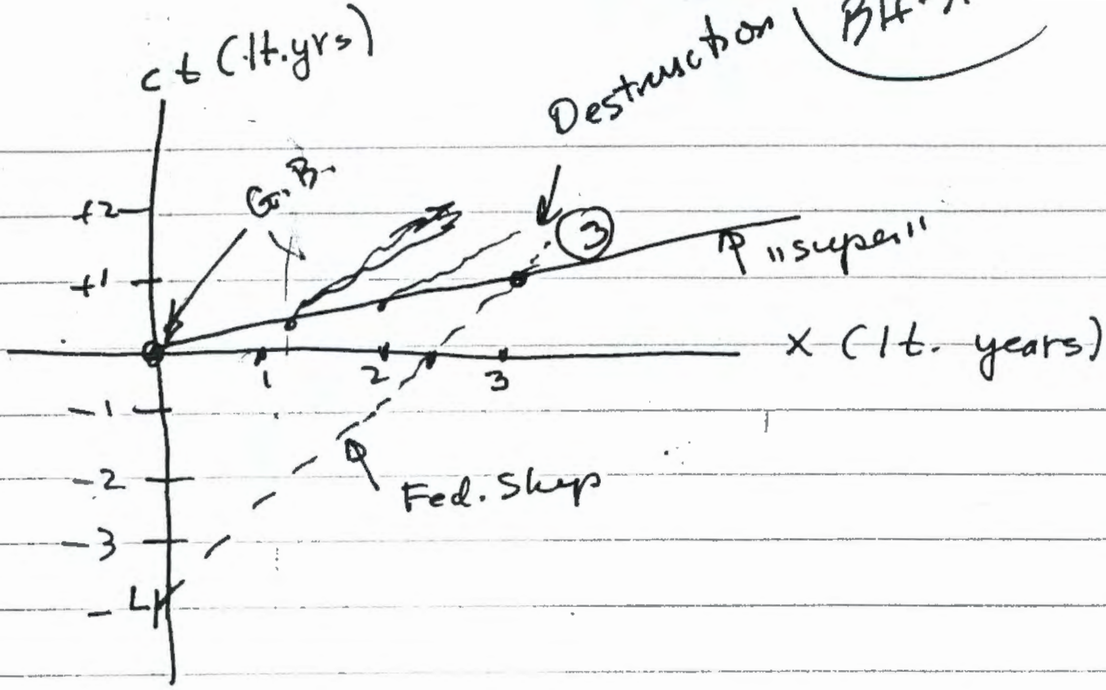


Klingon Rest Frame

Klingon Rest Frame:

- Peace treaty of Shalimar signed 4 years before great betrayel
- Peace treaty says: klingons to stop attacking federation troops in return to access to federation data base
- Federation negotiators leave shalimar after signing on ship moving at $v = 0.6c$
  $\Delta x / \Delta t = 0.6c \Rightarrow \Delta x / c\Delta t = 0.6$
- Within 4 years, klingons use database to construct a projectile, the "super" with $v > c$ velocity. At $ct = 0$, they launch super at $v = 3c$ toward fed ship $(\Delta x / c\Delta t) = 3$
- At event 3 Super destroys Fed. Ship

Finally at event ③, super overtakes the Fed ship and destroys it.

Federation Ship Coordinate Frame (ct', x')

Examine all 3 events in Fed Ship rest-frame. Use Lorentz transformation $(x,t) \to (x', t')$.

- stretch factor:

  Relative velocity between (x,t) and (x',t') is $v = 0.6c$

  Therefore $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = \dfrac{1}{\sqrt{1-(0.6)^2}} = \dfrac{1}{\sqrt{1-.36}}$

  $\gamma = \dfrac{1}{\sqrt{.64}} = \dfrac{1}{.8} = 1.25 = 5/4$

- Destruction Event: ③

  - Klingon Frame:

    $ct_3 = +1 \; ; \; x_3 = 3$

  - Fed. Ship frame

    $x_3' = \gamma(x_3 - vt_3) \; ; \; t_3' = \gamma\left(t_3 - \frac{v}{c^2}x_3\right)$

    $x_3' = \frac{5}{4}(3 - (0.6)(ct_3)) = \frac{5}{4}[3 - 0.6 \times 1]$

    $x_3' = \frac{5}{4} \times 2.4$

    $\boxed{x_3' = 3}$

    $ct_3' = \gamma\left(ct_3 - \frac{v}{c}x_3\right)$

    $= \frac{5}{4}(1 - 0.6 \times 3) = \frac{5}{4}(1 - 1.8)$

    $ct_3' = \frac{5}{4}(-0.8)$

Therefore $\boxed{ct_3' = -1}$

• Shalimar Event ②

$x_{sh} = 0$; $ct_{sh} = -4$

$x'_{sh} = \gamma(x_{sh} - vt_{sh})$

$= \frac{5}{4}(0 - 0.6(ct_{sh}))$

$x'_{sh} = \frac{5}{4}(0 - 0.6(-4)) = \frac{5}{4} \times 2.4$

$\boxed{x'_{sh} = +3}$ (Same as x'_3 since Rocket stand still in its own frame

$ct'_{sh} = \gamma(ct_{sh} - \frac{v}{c} x_{sh})$

$= \frac{5}{4}[-4 - 0.6 \times 0]$

$\boxed{ct'_{sh} = -5}$

• Great Betrayel event

$x_{GB} = 0$, $ct_{GB} = 0$

$x'_{GB} = \gamma(x_{GB} - vt_{GB}) = \gamma(x_{GB} - \frac{v}{c}[ct_{GB}])$

$\therefore x'_{GB} = \frac{5}{4}(0 - 0.6 \times 0) = 0$

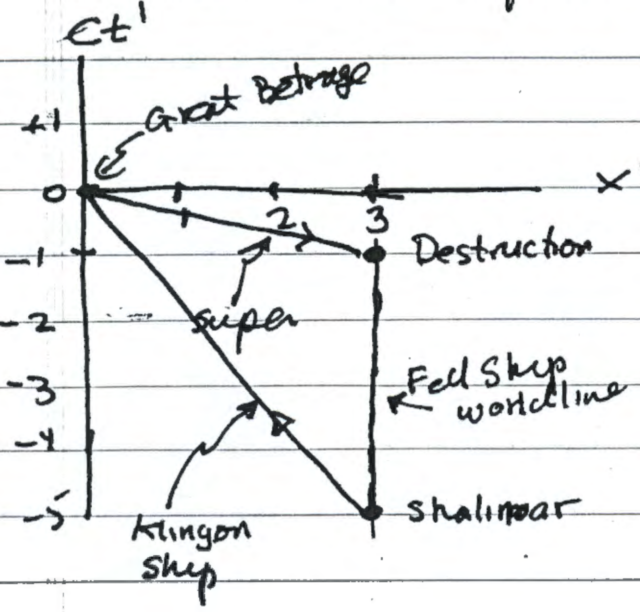
$\boxed{x'_{GB} = 0}$

$ct'_{GB} = \gamma(ct_{GB} - \frac{v}{c} x_{GB}) =$

$= \frac{5}{4}(0 - 0.6 \times 0)$

$\boxed{ct'_{GB} = 0}$

Fed Frame space-time Diagram



Event	x'	ct'
Destruction	3	-1
Shalimar	3	-5
Betrayal	0	0

Interpretation

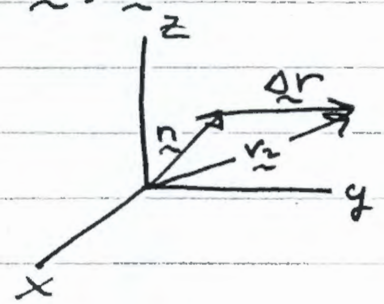
- world line of super tilts downward and to the right. Destruction of Fed Ship, therefore, occurs at $ct'_3 = -1$, 1 year before ~~Fed~~ super is fired at Great betrayal event ($ct'_{GB} = 0$)

- Result: If $v > c$, then we have break down of causality. we can influence the past ("Back to the future") No evidence for this at all

4 Vectors and Kinematics

(3D) Normally, we think of positional displacement vectors in 3D. Consider 2 nearby points with displacement vectors $\underset{\sim}{r_1}, \underset{\sim}{r_2}$.

Let $\Delta \underset{\sim}{r} = \underset{\sim}{r_2} - \underset{\sim}{r_1}$



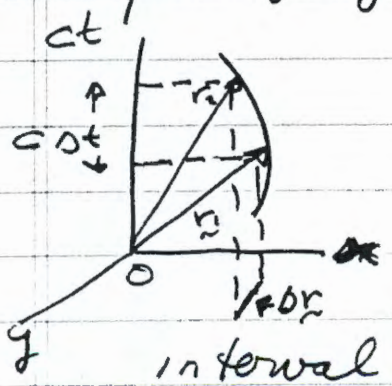
Components: $\Delta \underset{\sim}{r} = (\Delta x, \Delta y, \Delta z)$

Length: $|\Delta \underset{\sim}{r}|^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$

Note, $|\Delta \underset{\sim}{r}|$ is rigid length measured at both ends at same time. For rigid rotations, displacements, and Galilean transformations $|\Delta \underset{\sim}{r}|^2$ is invariant under coordinate transformations

(Note $|\Delta \underset{\sim}{r}|^2 = dS^2$) dS is 3-D interval

(4D) Now suppose 2 events are displaced in time as well as space; e.g. particle world-line.



4D "displacement vector"

$\Delta \underset{\sim}{R} = (c\Delta t, \Delta x, \Delta y, \Delta z)$.

But, in SR, spacetime is non-Euclidean. Recall that invariant interval $\Delta S^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$ is the same for all inertial observers connected to O by Lorentz transformation.

$\boxed{\text{Velocity in SR}}$

Define 4 velocity: $U^t = c\dfrac{dt}{d\tau}$; $U^x = \dfrac{dx}{d\tau}$, $U^y = \dfrac{dy}{d\tau}$, $U^z = \dfrac{dz}{d\tau}$

τ is proper time ($= \frac{ds}{c}$)

Scalar Product $U \cdot U =$

$= -(U^t)^2 + (U_x)^2 + (U_y)^2 + (U_z)^2$

Minkowski Metric (flat space but non-Euclidean spacetime)

Scalar Product: $\underset{\sim}{v} \cdot \underset{\sim}{v} = \eta_{ab} v^a v^b = \sum_a \sum_b \eta_{ab} v^a v^b$

sum $a = 0 \rightarrow 3$
$b = 0 \rightarrow 3$

$$\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$0 \rightarrow t$
$1 \rightarrow x$
$2 \rightarrow y$
$3 \rightarrow z$

Since η_{ab} has finite elements only on diagonal

$$v \cdot v = \eta_{00}(v^0)^2 + \eta_{11}(v^1)^2 + \eta_{22}(v^2)^2 + (\eta_{33})(v^3)^2$$

$$= (-1)(v^t)^2 + (1)(v^x)^2 + (1)(v^y)^2 + (1)(v^z)^2$$

$$v \cdot v = -(v^t)^2 + (v^x)^2 + (v^y)^2 + (v^z)^2$$

Interval: By analogy

$$ds^2 = \eta_{ab}\, dx^a dx^b$$
$$= \eta_{00}(c\,dt)^2 + \eta_{11}(dx^2) + \eta_{22}(dy^2) + \eta_{33}(dz^2)$$
$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Back to Definitions

$$v^t = c\frac{dt}{d\tau}\ ; \quad v^x = \frac{dx}{d\tau}\ ; \quad v^y = \frac{dy}{d\tau}\ ; \quad v^z = \frac{dz}{d\tau}$$

- Recall: $\frac{dt}{d\tau} = \gamma$ where $\gamma = 1/\sqrt{1-(u/c)^2}$

  therefore $\boxed{v^t = c\gamma}$

- $\frac{dx}{d\tau} = \frac{dt}{d\tau}\frac{dx}{dt} = \gamma u_x$ (u_x is regular 3-D ~~speed~~ velocity component along x axis)

Therefore: $\boxed{v^a = (\gamma c, \gamma u_x, \gamma u_y, \gamma u_z)}$

Scalar Product: $v \cdot v = -\gamma^2 c^2 + \gamma^2 \underbrace{(u_x^2 + u_y^2 + u_z^2)}_{u^2}$

$$v \cdot v = -\gamma^2 c^2 + \gamma^2 u^2 = \gamma^2 (u^2 - c^2)$$

$$v \cdot v = \frac{u^2 - c^2}{1 - u^2/c^2} = -c^2$$

$$\boxed{v \cdot v = -c^2}$$

Lorentz Invariant? Go to $\mathcal{O}'$ frame where $u_x' = u_y' = u_z' = 0$ (frame moving with test particle). In that case $u = 0 \Rightarrow \gamma = 1$

4 Vel: $v'^a = (c, 0, 0, 0) \Rightarrow v' \cdot v' = (-1)c^2 + 0 = -c^2$

As a result: $v \cdot v = v' \cdot v'$

So 4-scalar product the same in both frames.

Alternative Proof: Recall definition of proper time

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

Dividing by $d\tau^2$

$$-c^2 = -c^2\left(\frac{dt}{d\tau}\right)^2 + \left(\frac{dx}{d\tau}\right)^2 + \left(\frac{dy}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2$$

$$\boxed{-c^2 = -(v^t)^2 + (v^x)^2 + (v^y)^2 + (v^z)^2}$$

Momentum

4 Momentum $\underset{\sim}{P} = (P_t, P_x, P_y, P_z)$

$$\underset{\sim}{P} = (mv_t, mv_x, mv_y, mv_z)$$

Therefore

$$P^{o} = (m\gamma c, m\gamma u_x, m\gamma u_y, m\gamma u_z)$$

Scalar Product $P \cdot P = -m^2c^2$ (since $\underset{\sim}{P} \cdot \underset{\sim}{P} = m^2 \underset{\sim}{v} \cdot \underset{\sim}{v}$)

Also $P \cdot P = -P_t^2 + \underbrace{P_x^2 + P_y^2 + P_z^2}_{p^2} = -m^2c^2$ $\quad p = \gamma m \underset{\sim}{u}$ 3-D relativistic momentum

Define quantity E by $P_t \equiv E/c$

therefore $-(E/c)^2 + p^2 = -m^2c^2$

Therefore $\boxed{E^2 = c^2p^2 + m^2c^4 \quad (1)}$

Since $E/c = P_t = \gamma mc$, $\boxed{E = \gamma mc^2 \quad (2)}$

Physical Significance of E

(1) Non relativistic limit

$$\lim_{u \to 0} E = \gamma m c^2 = \frac{mc^2}{\sqrt{1-(u/c)^2}} = mc^2\left[1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 + O\left(\frac{u}{c}\right.\right.$$

Thus in this limit: $E = mc^2 + \frac{1}{2} m u^2$

Clearly E is an energy term since $\frac{1}{2} m u^2$ is familiar non-relativistic kinetic energy

~~[illegible]~~

But notice: as $u \to 0$, $\lim_{u \to 0} E = mc^2$

Free particle at rest has energy, i.e., rest-mass energy.

(2) Mass-Energy:

This is famous mass-energy equation of Einstein. Since 4 momentum is conserved, mass can be converted into energy and visa-versa

(3) Kinetic Energy

$$K = E - mc^2$$

energy in excess of rest-mass energy

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i.e., mass can be converted into other forms of energy such as radiation, kinetic energy, etc.

Before giving example, let's look at what happens when ~~[illegible]~~ we increase momentum arbitrarily!

$$p = \gamma m u = \frac{mu}{\sqrt{1-(u/c)^2}}$$

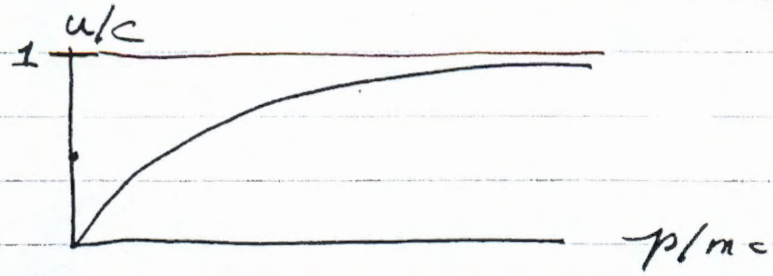
implies: $u = u(p)$ ∴ $p^2 = \dfrac{m^2u^2}{1-(u/c)^2}$

Solve for u/c: $[1-(u/c)^2]p^2 = m^2u^2$

or $u^2[m^2 + (p/c)^2] = p^2$

$$u^2 = \frac{p^2}{m^2 + (p/c)^2} = \frac{p^2c^2}{m^2c^2 + p^2} = \frac{c^2\left(\frac{p}{mc}\right)^2}{1+(p/mc)^2}$$

therefore $\boxed{(u/c)^2 = \dfrac{(p/mc)^2}{1+(p/mc)^2}}$ (5)



As $p \gg mc$ $u \to c$ asymptotically

~~Recall $p = (\gamma m)u = Mu$~~

~~So as p increase, effective inertial mass increases~~

Let's see why increasing p arbitrarily does not result in arbitrary increase of u above c.

$\therefore$ Recall $\gamma^2 = \dfrac{1}{1-(u/c)^2}$

But from eq. (5) we have

$$1-(u/c)^2 = 1 - \frac{(p/mc)^2}{1+(p/mc)^2} = \frac{1+(p/mc)^2-(p/mc)^2}{1+(p/mc)^2}$$

$$\text{or } 1-(u/c)^2 = \frac{1}{1+(p/mc)^2}$$

Therefore $\gamma^2 = 1+(p/mc)^2$ or $\boxed{\gamma = \sqrt{1+(p/mc)^2}}$ (6)

$\therefore$ Since $u = \dfrac{p}{\gamma m} = \dfrac{p}{\sqrt{1+(p/mc)^2}\cdot m}$

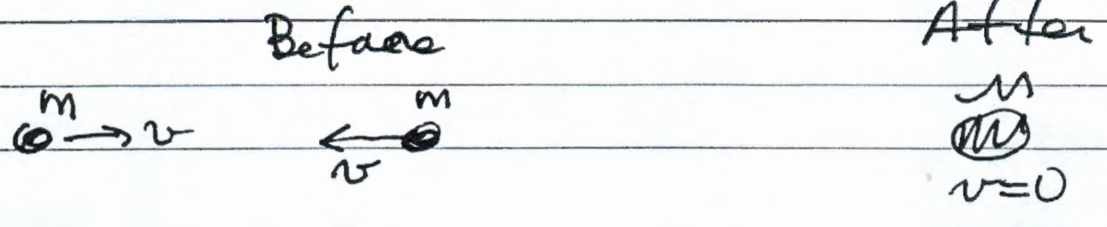
So as $p \to \infty$

$$\lim_{p\to\infty} u = \frac{p}{\frac{p}{mc}\times m} = c \ !$$

u prevented from exceeding c by increase in "effective inertial mass" γm.

Example of Mass-Energy Equivalence

Inelastic Collision



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In SR: Energy is conserved:

$$E_{before} = \gamma mc^2 + \gamma mc^2 = 2\gamma mc^2 \; ; \; \gamma = \frac{1}{\sqrt{1-(u/c)^2}}$$

$$E_{After} = Mc^2 \quad (\gamma = 1 \text{ since } u = 0)$$

Mass- Energy Conservation:

$$E_{before} = E_{after}$$

$$2\gamma mc^2 = Mc^2$$

$$M = \frac{2m}{\sqrt{1-(u/c)^2}}$$

Since $\frac{1}{\sqrt{1-(\frac{u}{c})^2}} > 1$

the implication is that $M > 2m$. Composite mass exceeds sum of its constituents. Where does the excess mass come from?

Answer: Loss of kinetic energy.

Recall: $K = E - mc^2$

$$K_{before} = 2\gamma mc^2 - 2mc^2 = \cancel{2(\gamma-1)mc^2}$$

$$K_{after} = 0$$

$$\Delta K = K_{after} - K_{before} = 0 - \left[\frac{2m}{\sqrt{1-(u/c)^2}} - 2m\right]c^2$$

Loss of k.E. $\frac{\Delta K}{c^2} = -(M - 2m)$

Thus loss of kinetic energy = gain in mass

Mass $\Longleftrightarrow$ Energy

General Relativity and Curved Spacetime

Newtonian Gravity

Successes: Newtonian Gravitation was highly successful for over 200 years.
- Explained motions of planets
- " " of bodies on earth surface

The idea of universal gravitation, that same force that caused bodies to drop to earth's surface, was also responsible for planetary orbits, was one of the major revolutionary ideas in science

Failure: Incompatible with Special Relativity

$$m_1 \bullet \longleftarrow r \longrightarrow \bullet M_2$$

Force of gravity between m_1 and m_2 is attractive and has magnitude $F_{12} = \frac{G m_1 m_2}{r^2}$

Instantaneous. But this force is communicated instantaneously; "action at a distance" idea is an hypothesis that Newton never explained. Thus if M_2 were suddenly to decrease to $M_2/2$, m_1 would know about it instantly. But according to Einstein, m_1 would not "feel" change until a time lag r/c.
- Einstein realized SR and Newtonian gravity were in conflict. Newton was wrong. Rather Maxwell's theory of E&M was

Recap

Introduced many new concepts last time:

(1) Four vectors

4 velocity - • Velocity $v^a = (v^t, v^x, v^y, v^z)$

where $v^a = \frac{dx^a}{d\tau}$; $x^a = (ct, x, y, z)$

• Length and metric

Scalar product. $v \cdot v = \eta_{ab} v^a v^b$

Minkowski metric

$$\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\therefore v \cdot v = -(v^t)^2 + (v^x)^2 + (v^y)^2 + (v^z)^2$$

• Using fact that $dt/d\tau = \gamma$, we found

$$v^a = (\gamma c, \gamma u_x, \gamma u_y, \gamma u_z)$$

(2) Energy and Momentum

4-momentum

$$p^a = (\underbrace{m\gamma c}_{p^t}, \underbrace{m\gamma u_x}_{p^x}, \underbrace{m\gamma u_y}_{p^y}, \underbrace{m\gamma u_z}_{p^z})$$

- Energy $\frac{E}{c} = p^t \Rightarrow$

$$E^2 = c^2p^2 + m^2c^4$$

$$E = \gamma mc^2$$

- when $u=0$, $E = mc^2$. so object at rest contains rest-mass energy

- "Effective inertial mass" γm increases as momentum increases, which prevents, u ~~[illegible]~~ from exceeding c since $u = p/\gamma m$ and $\lim_{p \to \infty} \gamma \to \frac{p}{mc}$

Started discussing General Relativity

- Conflict between SR and Newtonian Laws
  - Newton: action at a distance - instantaneous propagation of information
  - Einstein: nothing propagates faster than c.
  - Newton's eqs. are not Lorentz invariant but Maxwell's equations are. Einstein concluded that ~~[illegible]~~ wrong. Maxwell was right and Newton wrong.

This was an incredible act of self confidence during the early 20th century, given all the evidence favoring Newtonian gravity. But Einstein persisted. In fact precession of perihelion of Mercury's orbit was evidence, however slight, of cracks in the Newtonian edifice.

Principle of Equivalence: Local Inertial Frames

Today, we know Einstein was correct. Whenever Einstein's theory of gravitation, General Relativity (GR), and Newtonian theory were in conflict, GR turned out to be correct!

Main Postulates of GR

(I) Gravity is not a force superposed on an infinite flat Euclidean Space. Rather gravity is a manifestation of the curvature of spacetime

Complex set of arguments led to this realization. First step

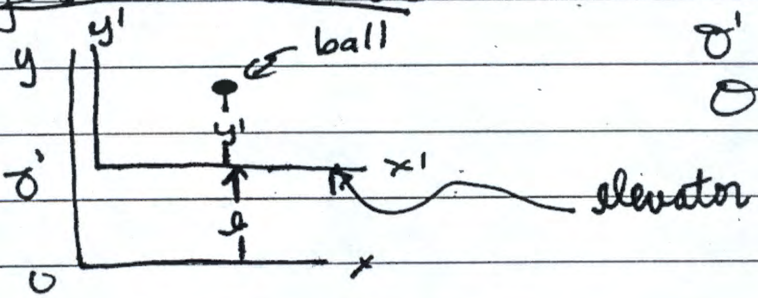
(II) Principle of Equivalence: All the Laws of Physics in a small freely falling reference frame must be the same as in an inertial frame for a gravity-free Universe; we can eliminate gravity in free-fa

The observer in the elevator cannot by <u>local</u> experiments distinguish an elevator in free-fall acceleration g from an inertial frame in which $g = 0$

<u>Reason</u>: Gravity is unique among 4 fundamental interactions: it is the only force in which a particle's response to a force is independent of its inertial mass: all objects accelerate at same rate independent of their inertial masses

$$\left.\begin{aligned} F &= ma \\ \frac{GM\cancel{m}}{r^2} &= \cancel{m}a \end{aligned}\right\} \quad a = \frac{GM}{r^2} = g$$

Every object in an elevator accelerates with $a = g$ <u>including the elevator</u>



O' is elevator
O is lab frame

Motion of ball: $y = l(t) + y'$

$$\frac{d^2y}{dt^2} = \frac{d^2l}{dt^2} + \frac{d^2y'}{dt^2}$$

$$-g = -g + \frac{d^2y'}{dt'^2}$$

$$\Rightarrow \boxed{\frac{d^2y'}{dt^2} = 0}$$

Since elevator and observer accelerate at same rate

Ball does <u>not</u> accelerate relative to elevator. Thus elevator is a local inertial frame

Aside on gravitational and inertial mass

Inertial Mass : Newton's 2nd Law

$\underset{m_I}{\bullet} \longrightarrow \underset{\sim}{F}$: $F = m_I a$

$a = F/m_I$

Inertial mass is resistance to force

Gravitational Mass :

Newton's Law of Universal Gravitation

$\overset{m}{\bullet} \longleftarrow r \longrightarrow \bullet M$

$F = G m_G M_G / r^2$: m_G's measure strength of grav. force

Combine 2 Laws

$$\frac{G m_G M_G}{r^2} = m_I a$$

Since $g \equiv \dfrac{G M_G}{r^2}$,

$$g m_G = m_I a$$

Measurements : $\dfrac{m_I - m_G}{m_I} < \text{few} \times 10^{-13}$

That is

$$1 - \frac{m_g}{m_I} = 1 - \frac{a}{g} = \frac{g-a}{a}$$

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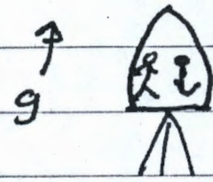
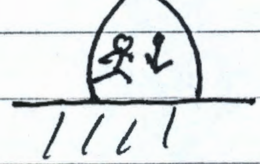
Back to Equivalence Principle

Equality of m_I and m_G not only means gravity can be eliminated in a locally freely falling frame, but that gravity can be generated by uniform acceleration —

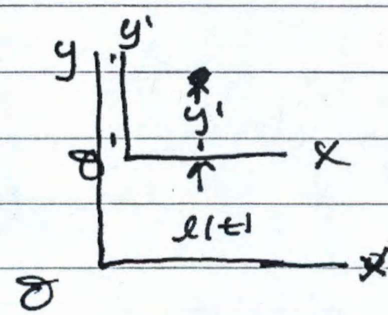
- Acceleration of rocket in gravity-free space: ~~Rocket~~ Rocket accelerates with $+g$.



- Observer lets go of a ball. It falls to floor with acceleration $-g$ according to observer in rocket



- Observer does same experiment in stationary rocket on surface of the earth: Same Result!



O' rocket
O earth

Eqs. for accelerating Rocket in space:

Again $y = y' + l(t)$

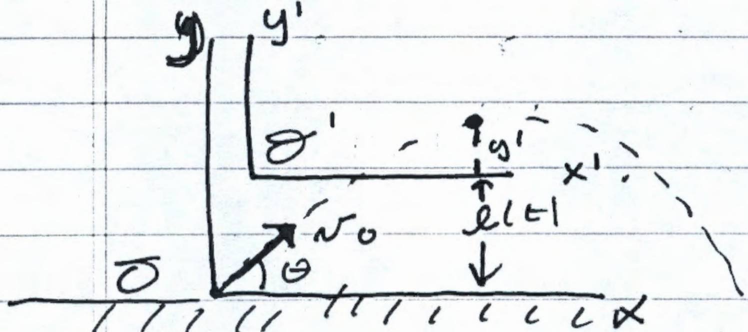
$$\frac{d^2y}{dt^2} = \frac{d^2y'}{dt^2} + \frac{d^2l}{dt^2}$$

In this case: $\frac{d^2y}{dt^2}=0$, $\frac{d^2l}{dt^2} = +g$

$$\therefore \quad 0 = \frac{d^2y'}{dt^2} + g \Rightarrow \boxed{\frac{d^2y'}{dt^2} = -g}$$

- But Einstein went one step further by postulating the **all** the laws of physics, not just those involving gravity, will be the same in the two reference frames. This leads to bending of light, etc.

First, some exotic Examples



(i) Cannon fixed to ground in O fires ball at v_0 and angle θ wrst ground

(ii) O' is freely falling observer who sees ball trajectory

(iii) $l(t)$ is vertical separation of O and O' at t.

(iv) x, x' coincide

$y = l(t) + y'$ is vertical position of ball at t.

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Therefore: $\dfrac{d^2y}{dt^2} = \dfrac{d^2y'}{dt^2} + \dfrac{d^2\ell}{dt^2}$

Cannon ball in free fall $\Rightarrow \dfrac{d^2y}{dt^2} = -g$

Observers $\mathcal{O}'$ in free-fall $\Rightarrow \dfrac{d^2\ell}{dt^2} = -g$

As a result $\boxed{\dfrac{d^2y'}{dt^2} = 0}$

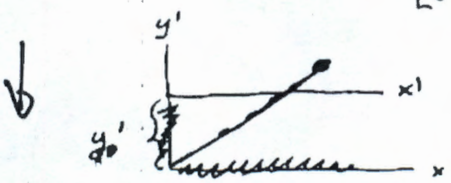
Last equation means cannonball does <u>not</u> accelerate according to freely falling observer.

Integrating last equation we have:

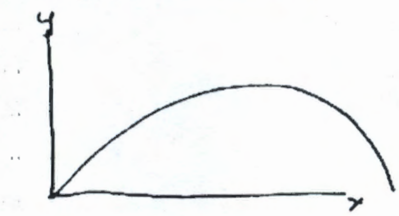
$\dfrac{dy'}{dt} = \text{const} = (V_0')_y$; integrate again $y' = (V_0')_y t + y_0'$

Since there are no x forces: $x'(t) = (V_0')_x t$

Therefore: $y' = \left[\dfrac{(V_0')_y}{(V_0')_x}\right] x' + y_0'$



Cannon-ball has st. line trajectory in $\mathcal{O}'$. $v = v_0$ for all time



Cannon-ball has well known parabolic trajectory in $\mathcal{O}$.

Einstein Explanation: In <u>small</u>, local reference frames,

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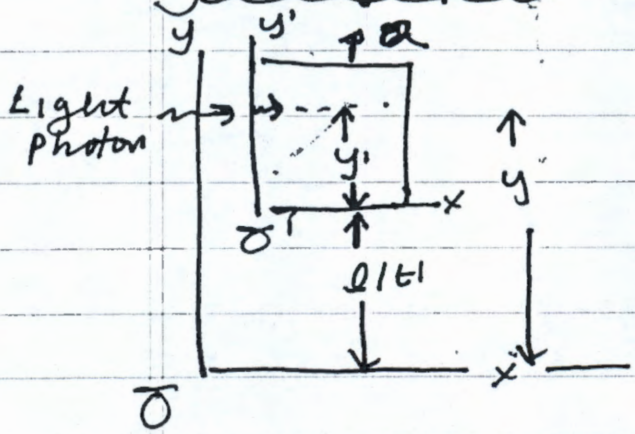
freely falling particles move in straight lines. In geometric terms, space is locally flat, and particles move along natural paths in flat space; i.e., straight lines.

Locally, we can always transform away gravity

Creating Gravity: As we saw before we can also create gravity by accelerating rocket in gravity-free empty space -

Light Bending: photon classical particle



y is photon vertical position in O

y' is photon vertical position in O'

O: No gravity $\Rightarrow \frac{d^2y}{dt^2} = 0, x = ct$

Photon moves horizontally with c

O' Accelerated observer $\quad y = l + y'$

$$\frac{d^2y}{dt^2} = \frac{d^2l}{dt^2} + \frac{d^2y'}{dt^2}$$

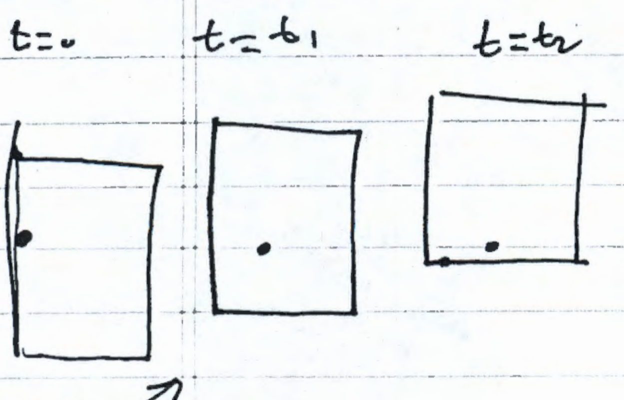
$$0 = a + \frac{d^2y'}{dt^2}$$

$$\Rightarrow \frac{d^2y'}{dt^2} = -a$$

$$\Rightarrow y' = y_0' - \frac{1}{2}at^2$$

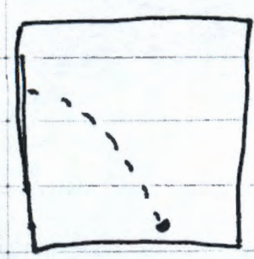
$$x' = ct \Rightarrow$$

$$y' = y_0' - \frac{a}{2}\frac{(x')^2}{c^2}$$



O: sees

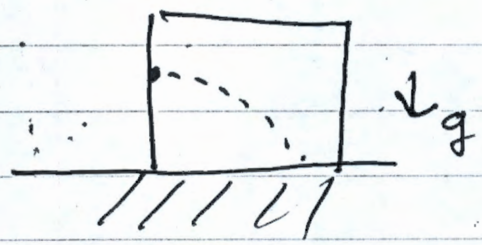
O' sees



Light bent along parabolic path

Equivalence Principle

Because accelerated rocket and earth frame are indistinguishable if $a=g$, light bends in parabolic path $y' = y_0' - \frac{g}{2}\frac{x^2}{c^2}$ whether the elevator accelerates in empty space or is on surface of the earth subject to gravitational pull g. Light will be bent in both cases.



Book gives more examples of consequences of equivalence principle (EP). But keep in mind that EP was stepping stone in development of GR, much like cubism was intermediate step on the path to the development of modern art.

So let's to turn to non-local aspects of gravity which violate EP, and which are essence of GR.

Tidal Forces and Curvature

EP only works because $g=$ const. throughout elevator. But reality is different. In fact gravitational

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Lecture #5:

acceleration will not be precisely constant. Rather, it depends on distance and it will vary across test body in the manner of tidal forces.

Property of gravity ① Directed toward center of earth ② Falls off with distance from center of earth

Big astronaut in free-fall

Earth frame O

Astronaut frame O'

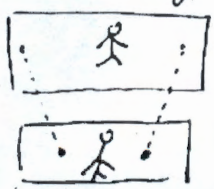
$$\underset{\sim}{a}'(r) = \underset{\sim}{a}(r) - \underset{\sim}{\bar{a}}(r_{cm})$$

Astronaut gets:
stretched in vertical direction
squeezed in horizontal direction

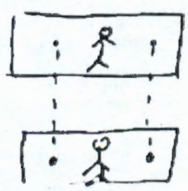
Bottom Line: Sufficiently large astronaut in big rocket ship can distinguish free fall in grav. field from truly inertial frame

Free Fall

Lg. free fall Rocket



Gravity free Inertial Frame



Particles move toward each other!

NO relative acceleration

∴ you can tell the difference! between inertial frame and free-fall frame in big rocket ship or big elevator.

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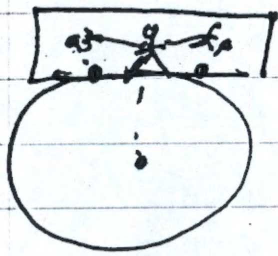
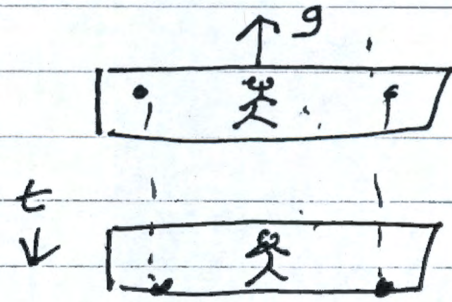
Likewise astronaught in large rocket can distinguish ~~[illegible]~~ rocket that accelerates in empty space with $a = g$ from rocket sitting on surface of earth

Earth's Surface



Balls land on floor at smaller horizontal separation than at start

Acceleration in Empty Space



Balls land on floor with same horizontal separation

Connection Between Curvature and Gravity

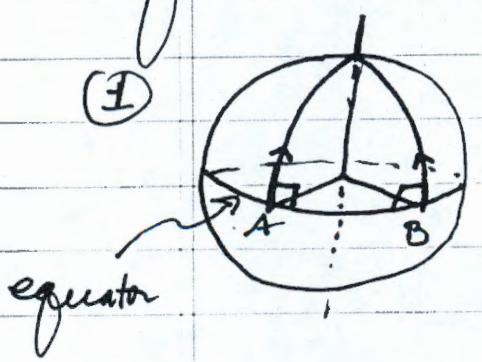
Idea: Unique trajectories dictated by curvature of spacetime

Bodies move along natural longitudes and latitudes of curved spacetime.

Surface-of-Earth Analogy: why not spacetime?
The surface of the earth is an excellent example of curved space that is locally flat, i.e., 2D sphere that is idealized model for the surface of the earth.

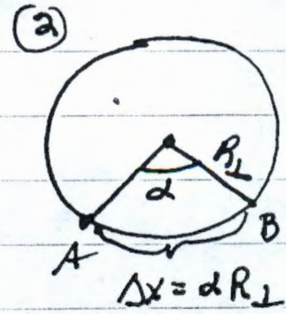
Example: Two surveyors start together at the equator. ~~They travel~~ One is 10 km to the east of the other. Both go 200 km straight north: defined as rt. angle (90°) from equator. Each travels locally along straight line.

(1)



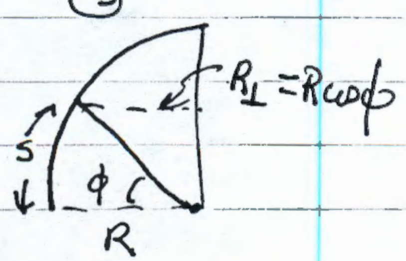
3D

(2)

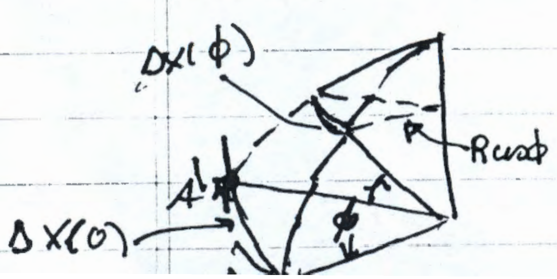


Top view

(3)



Side View



$$\left.\begin{array}{l}\Delta X(\phi) = (R\cos\phi)\alpha \\ \Delta X(0) = R\phi\end{array}\right\} \boxed{\Delta X(\phi) = \Delta X(0)\cos\phi}$$