

# Solutions to Quiz 1, Physics 1b (Levine)

Written by Dan Klein  
dsklein@physics.ucsd.edu

January 2012

All three versions of this quiz have the same questions, just rearranged. These solutions will follow the order of version A, but I've included the numberings for versions B and C so you can match them to the problems on your quiz.

## Answers

**Version A:** 1-a 2-b 3-d 4-a 5-b 6-a 7-d 8-b

**Version B:** 1-a 2-b 3-a 4-a 5-b 6-b 7-d 8-d

**Version C:** 1-a 2-a 3-b 4-a 5-b 6-d 7-d 8-b

## Problem A1/B4/C4

This problem requires you to convert potential energy into kinetic energy. When our electron starts out, it has some potential energy because the electric field from the fixed point charge wants to push it away, but it has no kinetic energy because it starts from rest. After the electron is allowed to move far away from the fixed charge ( $r \rightarrow \infty$ ), it will have some kinetic energy because it is moving, but no potential energy, because the electric potential drops to zero at large distances (1).

The electrical potential energy due to two charged particles is given by:

$$U_e = \frac{k_e q_1 q_2}{r} \quad (1)$$

Recall from mechanics that the kinetic energy of a moving particle is given by:

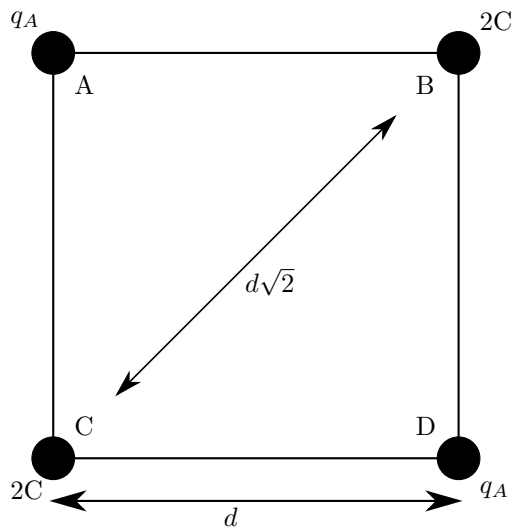
$$K = \frac{1}{2} m v^2 \quad (2)$$

Since energy is conserved, the two quantities must be equal, so:

$$\frac{k_e q \cdot q_e}{r_i} = \frac{1}{2} m_e v_f^2$$

Here,  $q$  is the charge of the fixed point,  $q_e$  is the charge of the electron,  $r_i$  is the initial distance between the two charges,  $m_e$  is the mass of the electron, and  $v_f$  is the final velocity we're solving for. Plugging in the values, as well as the electric constant  $k_e$ , we get  $v_f = 2.5 \times 10^7$  m/s (*choice a.*). (**Fun Fact:** this is almost 10% of the speed of light!)

### Problem A2/B2/C8



I laid out my square as in this diagram. Yours may look slightly different, but the math will be the same.

We know that the force on a charged particle is equal to the charge of the particle times the electric field at the particle's location:

$$\vec{F} = q \cdot \vec{E}(\vec{r}) \quad (3)$$

Thus, in order for the force on charge B to be zero, the electric field must be zero at point B's location. The principle of superposition tells us that the total electric field at point B's location is the sum of the electric fields from particles A, C and D:

$$\vec{E}(\vec{r}_B) = \vec{E}_A + \vec{E}_C + \vec{E}_D \quad (4)$$

Looking at the geometry of the setup (and being careful with the direction of our vectors), we can see that the three electric fields are given by:

$$\vec{E}_A = \frac{k_e q_A}{d^2} \hat{i}, \quad \vec{E}_C = \frac{k_e (2)}{(d\sqrt{2})^2} \left( \frac{\hat{i} + \hat{j}}{\sqrt{2}} \right), \quad \vec{E}_D = \frac{k_e q_A}{d^2} \hat{j}$$

(Note that  $\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$  is a unit vector (length 1) that points at a  $45^\circ$  angle. It is the same as writing  $\hat{i} \cos 45^\circ + \hat{j} \sin 45^\circ$ ). If we add these all up, we get that the total electric field at point B is:

$$\vec{E}(\vec{r}_B) = k_e \left( \frac{q_A}{d^2} + \frac{2}{2\sqrt{2}d^2} \right) \hat{i} + k_e \left( \frac{q_A}{d^2} + \frac{2}{2\sqrt{2}d^2} \right) \hat{j} = 0$$

We can greatly simplify this equation by cancelling out the  $k_e$ , the factor of  $\frac{1}{d^2}$ , and the 2's to get  $q_A + \frac{1}{\sqrt{2}} = 0$ , or  $q_A = \frac{-1}{\sqrt{2}}C = -0.71C$  (*choice b*).

### Problem A3/B8/C7

This problem involves a simple application of Gauss's Law, which states that the total flux of electric field through a closed surface is proportional to the total charge enclosed within that surface:

$$\Phi_e = \vec{E} \cdot A = \frac{Q_{enc}}{\epsilon_0} \quad (5)$$

The obvious choice for our Gaussian surface is the sphere of radius 0.6m given in the problem. The total flux of electric field through this spherical surface will equal (the surface area of the sphere)  $\times$  (the electric field strength at the surface):

$$4\pi r^2 \cdot E(r) = \frac{Q_{enc}}{\epsilon_0}$$

From here it's easy to see that  $Q_{enc} = 4\pi r^2 \cdot E(r) \cdot \epsilon_0 = -4 \times 10^{-9}C$  (*choice d*).

(**Note:** we know the enclosed charge is *negative* because the electric field points inward. If the electric field pointed *outward*, that would mean that the enclosed charge was positive).

### Problem A4/B3/C1

There are two ways to do this problem: an easy way, and a way that requires a bit more work.

The more-work way is to use Coulomb's Law

$$\vec{E}(\vec{r}) = \frac{k_e q}{r^2} \hat{r} \quad (6)$$

as well as the Principle of Superposition to calculate the total electric field resulting from each of the two point charges. This requires a bit of punching into your calculator that we can avoid fairly easily...

The easy way is to use symmetry to our advantage. Our two point sources have equal charge, and are also equidistant from the midpoint where we want to know the electric field. That means that the electric field from one charge is equal in magnitude to the electric field from the other charge, as measured

at the midpoint. However, the fields also point in opposite directions, so they exactly cancel each other out. Thus, the electric field measured at the midpoint between the two charges has magnitude = 0 (*choice a*).

### Problem A5/B6/C3

We are trying to figure out the electric field between the two plates of a capacitor. First, we can find the electric potential between the plates, using the relation between electric potential, charge and capacitance:

$$Q = VC \quad (7)$$

We can rearrange this equation to get  $V = \frac{Q}{C}$ . We also know that

$$V = \vec{E} \cdot d \quad (8)$$

We can combine these equations to get  $E = \frac{Q}{Cd}$ ; plugging in the values given in the problem, we find  $E = 225 \text{ N/C}$  (*choice b*).

### Problem A6/B1/C2

This problem requires no math, just a conceptual understanding of the behavior of electric charges and fields. First, note that we have a conducting sphere of net charge -5 mC. This net charge is permanent—unless the sphere explicitly has charge added or removed, it will always have a charge of -5 mC. Next, a charge of +10 mC is inserted into the middle of the sphere. The electric field due to this +10 mC charge will attract the negatively charged electrons in the metal, until a charge of -10 mC accumulates on the interior surface of the sphere, balancing out the charge inserted in the center. This movement of charge will also cause some positive charges to accumulate on the outer surface of the conductor. Through all this, the net charge of the sphere must remain -5 mC. Thus, if -10 mC of charge accumulates on the inside surface of the sphere, then +5 mC of charge must accumulate on the outer surface (*choice a*).

### Problem A7/B7/C6

We have a  $3 \mu\text{C}$  charge located at coordinates (0,3) and we want to know the electric field at coordinates (4,9). We know the electric field due to a point charge is given by:

$$\vec{E}(\vec{r}) = k_e \frac{q}{r^2} \hat{r} \quad (9)$$

There are two parts of the electric field to find here: its magnitude and its direction.

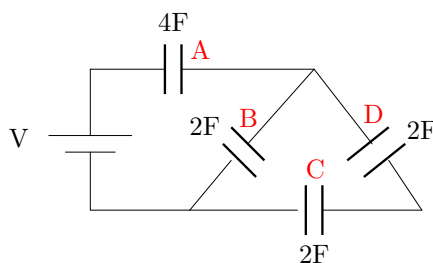
The magnitude of the electric field is fairly straightforward. We know  $q = 3\mu\text{C}$ , and we know (or can look up) the value of  $k_e$ . Our vector  $\vec{r}$  that points

from (0,3) to (4,9) is equal to  $(4\hat{i} + 6\hat{j})$ . Thus  $r^2 = 4^2 + 6^2 = 52$ . So, the magnitude of our electric field is  $|E| = k_e \frac{3 \times 10^{-6}}{52} = 519 \text{V/m}$ .

There are a couple ways to find the direction  $\hat{r}$  of the electric field. We could use trig, but that gets a bit tricky. The simplest way is to note that  $\hat{r}$  is just  $\vec{r}$  rescaled to have a length (i.e. magnitude) equal to 1. Since  $\vec{r} = 4\hat{i} + 6\hat{j}$  and has a magnitude  $|\vec{r}| = \sqrt{52}$ , we know  $\hat{r} = \frac{\vec{r}}{\sqrt{52}} = \frac{4\hat{i} + 6\hat{j}}{\sqrt{52}} = 0.554\hat{i} + 0.832\hat{j}$ .

Thus our final answer is  $\vec{E}(\vec{r}) = 519 \text{ V/m } (0.554\hat{i} + 0.832\hat{j})$  (*choice d*).

## Problem A8/B5/C5



This problem requires you to know how to combine capacitors in series and in parallel to find equivalent capacitances. To start off, let us label the capacitors A, B, C, D as shown above. Capacitors C and D are on the same wire, which means they are in series. Recall that the rule for addition of capacitors in series is:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} \quad (10)$$

We know that  $\frac{1}{2} + \frac{1}{2} = 1$ , so we can replace capacitors C and D in series with a single capacitor (call it E) of capacitance 1F. Next, let us combine our equivalent capacitor E with capacitor B. These two are on separate branches, which means they are in parallel. The rule for addition of capacitors in parallel is:

$$C_{total} = C_1 + C_2 + \dots + C_n \quad (11)$$

Thus we can replace capacitors E and B with a single capacitor (call it G) on a single wire, of capacitance 3F. Finally, let us combine capacitors A and G. They are in series, and the formula (10) tells us

$$\frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

so we can replace A and G with a single capacitor of capacitance  $\frac{12}{7} \text{ F} = 1.71 \text{F}$  (*choice b*).