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The Quantum Theory of Light

3-2 Assume that your skin can be considered a blackbody. One can then use Wien's displacement law, $\lambda_{\max} T = 0.2898 \times 10^{-2} \text{ m} \cdot \text{K}$ with $T = 35^\circ\text{C} = 308 \text{ K}$ to find

$$\lambda_{\max} = \frac{0.2898 \times 10^{-2} \text{ m} \cdot \text{K}}{308 \text{ K}} = 9.41 \times 10^{-6} \text{ m} = 9410 \text{ nm} .$$

3-4 (a) From Stefan's law, one has $\frac{P}{A} = \sigma T^4$. Therefore,

$$\frac{P}{A} = (5.7 \times 10^{-8} \text{ W/m}^2 \text{K}^4)(3000 \text{ K})^4 = 4.62 \times 10^6 \text{ W/m}^2 .$$

(b)
$$A = \frac{P}{4.62 \times 10^6 \text{ W/m}^2} = \frac{75 \text{ W}}{4.62 \times 10^6 \text{ W/m}^2} = 16.2 \text{ mm}^2 .$$

3-5 (a) Planck's radiation energy density law as a function of wavelength and temperature is given by $u(\lambda, T) = \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda_B T} - 1)}$. Using $\frac{\partial u}{\partial \lambda} = 0$ and setting $x = \frac{hc}{\lambda_{\max} k_B T}$, yields an extremum in $u(\lambda, T)$ with respect to λ . The result is

$$0 = -5 + \left(\frac{hc}{\lambda_{\max} k_B T} \right) \left(e^{hc/\lambda_{\max} k_B T} \right) \left(e^{hc/\lambda_{\max} k_B T} - 1 \right)^{-1} \text{ or } x = 5(1 - e^{-x}).$$

(b) Solving for x by successive approximations, gives $x \cong 4.965$ or

$$\lambda_{\max} T = \left(\frac{hc}{k_B} \right) (4.965) = 2.90 \times 10^{-3} \text{ m} \cdot \text{K} .$$

3-10 The energy per photon, $E = hf$ and the total energy E transmitted in a time t is Pt where power $P = 100 \text{ kW}$. Since $E = nhf$ where n is the total number of photons transmitted in the time t , and $f = 94 \text{ MHz}$, there results $nhf = (100 \text{ kW})t = (10^5 \text{ W})t$, or

$$\frac{n}{t} = \frac{10^5 \text{ W}}{hf} = \frac{10^5 \text{ J/s}}{6.63 \times 10^{-34} \text{ J s}} (94 \times 10^6 \text{ s}^{-1}) = 1.60 \times 10^{30} \text{ photons /s} .$$

3-14 (a)
$$K = hf - \phi = \frac{hc}{\lambda - \phi} = \frac{1240 \text{ eV nm}}{350 \text{ nm}} - 2.24 \text{ eV} = 1.30 \text{ eV}$$

(b) At $\lambda = \lambda_c$, $K = 0$ and $\lambda = \frac{hc}{\phi} = \frac{1240 \text{ eV nm}}{2.24 \text{ eV}} = 554 \text{ nm}$

3-16 (a) $\phi = \frac{hc}{\lambda} - K, \phi = \frac{1\,240\text{ eV nm}}{300\text{ nm}} - 2.23\text{ eV} = 1.90\text{ eV}$

(b) $V_s = \frac{1\,240\text{ eV nm}}{400\text{ nm e}} - 1.90\text{ eV/e} = 1.20\text{ V}$

3-17 The energy of one photon of light of wavelength $\lambda = 300\text{ nm}$ is

$$E = \frac{hc}{\lambda} = \frac{1\,240\text{ eV nm}}{300\text{ nm}} = 4.13\text{ eV} .$$

(a) As lithium and beryllium have work functions that are less than 4.13 eV, they will exhibit the photoelectric effect for incident light with this energy. However, mercury will not because its work function is greater than 4.13 eV.

(b) The maximum kinetic energy is given by $K = \frac{hc}{\lambda} - \phi$, so

$$K(\text{Li}) = \frac{1\,240\text{ eV nm}}{300\text{ nm}} - 2.3\text{ eV} = 1.83\text{ eV} , \text{ and } K(\text{Be}) = \frac{1\,240\text{ eV nm}}{300\text{ nm}} - 3.9\text{ eV} = 0.23\text{ eV} .$$

3-18 (a) $K_{\max} = eV_s = s(0.45\text{ V}) = 0.45\text{ eV}$

(b) $\phi = \frac{hc}{\lambda} - K = \frac{1\,240\text{ eV nm}}{500\text{ nm}} - 0.45\text{ eV} = 2.03\text{ eV}$

(c) $\lambda_c = \frac{hc}{\phi} = \frac{1\,240\text{ eV nm}}{2.03\text{ eV}} = 612\text{ nm}$

3-20 $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - K_{\max} ;$

First Source: $\phi = \frac{hc}{\lambda} - 1.00\text{ eV} .$

Second Source: $\phi = \frac{hc}{\frac{\lambda}{2}} - 4.00\text{ eV} = \frac{2hc}{\lambda} - 4.00\text{ eV} .$

As the work function is the same for both sources (a property of the metal),

$$\frac{hc}{\lambda} - 1.00\text{ eV} = \frac{2hc}{\lambda} - 4.00\text{ eV} \Rightarrow \frac{hc}{\lambda} = 3.00\text{ eV} \text{ and}$$

$$\phi = \frac{hc}{\lambda} - 1.00\text{ eV} = 3.00\text{ eV} - 1.00\text{ eV} = 2.00\text{ eV} .$$

3-25 $E = 300\text{ keV} , \theta = 30^\circ$

(a) $\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) = (0.002\,43\text{ nm}) [1 - \cos(30^\circ)] = 3.25 \times 10^{-13}\text{ m}$
 $= 3.25 \times 10^{-4}\text{ nm}$

$$(b) \quad E = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{E_0} = \frac{(4.14 \times 10^{-15} \text{ eVs})(3 \times 10^8 \text{ m/s})}{300 \times 10^3 \text{ eV}} = 4.14 \times 10^{-12} \text{ m}; \text{ thus,}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 4.14 \times 10^{-12} \text{ m} + 0.325 \times 10^{-12} \text{ m} = 4.465 \times 10^{-12} \text{ m}, \text{ and}$$

$$E' = \frac{hc}{\lambda'} \Rightarrow E' = \frac{(4.14 \times 10^{-15} \text{ eV s})(3 \times 10^8 \text{ m/s})}{4.465 \times 10^{-12} \text{ m}} = 2.78 \times 10^5 \text{ eV}.$$

$$(c) \quad \frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e, \text{ (conservation of energy)}$$

$$K_e = hc \left(\frac{1}{\lambda_0} - \frac{1}{\lambda'} \right) = \frac{(4.14 \times 10^{-15} \text{ eV s})(3 \times 10^8 \text{ m/s})}{\frac{1}{4.14 \times 10^{-12}} - \frac{1}{4.465 \times 10^{-12}}} = 22 \text{ keV}$$

3-27 Conservation of energy yields $hf = K_e + hf'$ (Equation A). Conservation of momentum yields

$$p_e^2 = p'^2 + p^2 - 2pp' \cos \theta. \text{ Using } p_{\text{photon}} = \frac{E}{c} = \frac{hf}{c} \text{ there results}$$

$$p_e^2 = \left(\frac{hf'}{c} \right)^2 + \left(\frac{hf}{c} \right)^2 - 2 \left(\frac{hf}{c} \right) \left(\frac{hf'}{c} \right) \cos \theta \text{ (Equation B). If the photon transfers all of its energy,}$$

$f' = 0$ and Equations A and B become $K_e = hf$ and $p_e^2 \left(\frac{hf}{c} \right)^2$ respectively. Note that in

general, $K_e = E_e - m_e c^2 = \left[p_e^2 c^2 + (m_e c^2)^2 \right]^{1/2} - m_e c^2$. Finally, substituting $K_e = hf$ and

$$p_e^2 = \left(\frac{hf}{c} \right)^2 \text{ into } K_e = \left[p_e^2 c^2 + (m_e c^2)^2 \right]^{1/2} - m_e c^2, \text{ yields } hf = \left[(hf)^2 + (m_e c^2)^2 \right]^{1/2} - m_e c^2$$

(Equation C). As Equation C is true only if h , or f , or m_e , or c is zero and all are non-zero this contradiction means that f' cannot equal zero and conserve both relativistic energy and momentum.

3-28 (a) From conservation of energy we have $E_0 = E' + K_e = 120 \text{ keV} + 40 \text{ keV} = 160 \text{ keV}$. The photon energy can be written as $E_0 = \frac{hc}{\lambda_0}$. This gives

$$\lambda_0 = \frac{hc}{E_0} = \frac{1240 \text{ nm eV}}{160 \times 10^3 \text{ eV}} = 7.75 \times 10^{-3} \text{ nm} = 0.00775 \text{ nm}.$$

(b) Using the Compton scattering relation $\lambda' - \lambda_0 = \lambda_c(1 - \cos \theta)$ where

$$\lambda_c = \frac{h}{m_e c} = 0.00243 \text{ nm} \text{ and } \lambda' = \frac{hc}{E'} = \frac{1240 \text{ nm eV}}{120 \times 10^3 \text{ eV}} = 10.3 \times 10^{-3} \text{ nm} = 0.0103 \text{ nm}.$$

Solving the Compton equation for $\cos \theta$, we find

$$-\lambda_c \cos \theta = \lambda' - \lambda_0 - \lambda_c$$

$$\cos \theta = 1 - \frac{\lambda' - \lambda_0}{\lambda_c} = 1 - \frac{0.0103 \text{ nm} - 0.00775 \text{ nm}}{0.00243 \text{ nm}} = 1 - 1.049 = -0.049$$

The principle angle is 87.2° or $\theta = 92.8^\circ$.

- (c) Using the conservation of momentum Equations 3.30 and 3.31 one can solve for the recoil angle of the electron.

$$p = p' \cos \theta + p_e \cos \phi$$

$p_e \sin \phi = p' \sin \theta$; dividing these equations one can solve for the recoil angle of the electron

$$\begin{aligned} \tan \phi &= \frac{p' \sin \theta}{p - p' \cos \theta} = \left(\frac{h}{\lambda'} \right) \frac{\sin \theta}{\frac{h}{\lambda_0} - \frac{h}{\lambda' \cos \theta}} = \left(\frac{hc}{\lambda'} \right) \frac{\sin \theta}{\frac{hc}{\lambda_0} - \frac{hc}{\lambda' \cos \theta}} \\ &= \frac{120 \text{ keV} (0.9988)}{160 \text{ keV} - 120 \text{ keV} (-0.049)} = 0.7232 \end{aligned}$$

and $\phi = 35.9^\circ$.

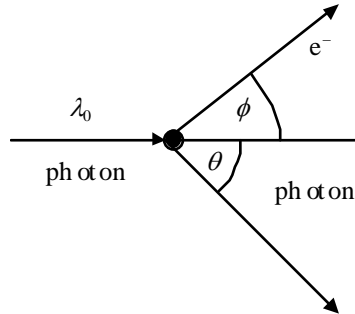
- 3-30 Maximum energy transfer occurs when the scattering angle is 180 degrees. Assuming the electron is initially at rest, conservation of momentum gives

$$hf + hf' = p_e c = \sqrt{(m_e c^2 + K)^2 - m_e^2 c^4} = \sqrt{(511 + 50)^2} = 178 \text{ keV}$$

while conservation of energy gives $hf - hf' = K = 30 \text{ keV}$. Solving the two equations gives $E = hf = 104 \text{ keV}$ and $hf' = 74 \text{ keV}$. (The wavelength of the incoming photon is

$$\lambda = \frac{hc}{E} = 0.0120 \text{ nm}.$$

- 3-31 (a) $E' = \frac{hc}{\lambda'}$, $\lambda' = \lambda_0 + \Delta\lambda$
- $$\lambda_0 = \frac{hc}{E_0} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{0.1 \text{ MeV}} = 1.243 \times 10^{-11} \text{ m}$$
- $$\Delta\lambda = \left(\frac{h}{m_e c} \right) (1 - \cos \theta) = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(1 - \cos 60^\circ)}{(9.11 \times 10^{-31} \text{ kg})(3 \times 10^8 \text{ m/s})} = 1.215 \times 10^{-12} \text{ m}$$
- $$\lambda' = \lambda_0 + \Delta\lambda = 1.364 \times 10^{-11} \text{ m}$$
- $$E' = \frac{hc}{\lambda'} = \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3 \times 10^8 \text{ m/s})}{1.364 \times 10^{-11} \text{ m}} = 9.11 \times 10^4 \text{ eV}$$
- (b) $\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$
 $K_e = 0.1 \text{ MeV} - 91.1 \text{ keV} = 8.90 \text{ keV}$



- (c) Conservation of momentum along x : $\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'}\right) \cos \theta + \gamma m_e v \cos \phi$. Conservation of momentum along y : $\left(\frac{h}{\lambda'}\right) \sin \theta = \gamma m_e v \sin \phi$. So that

$$\frac{\gamma m_e v \sin \phi}{\gamma m_e v \cos \phi} = \left(\frac{h}{\lambda'}\right) \sin \theta \left[\left(\frac{h}{\lambda_0}\right) - \left(\frac{h}{\lambda'}\right) \cos \theta \right]$$

$$\tan \phi = \frac{\lambda_0 \sin \theta}{(\lambda' - \lambda_0) \cos \theta}$$

Here, $\theta = 60^\circ$, $\lambda_0 = 1.243 \times 10^{-11} \text{ m}$, and $\lambda' = 1.364 \times 10^{-11} \text{ m}$. Consequently,

$$\tan \phi = \frac{(1.24 \times 10^{-11} \text{ m})(\sin 60^\circ)}{(1.36 - 1.24 \cos 60^\circ) \times 10^{-11} \text{ m}} = 1.451$$

$$\phi = 55.4^\circ$$