PHYSICS 2D FINAL EXAM WINTER QUARTER 2012
PROF. HIRSCH MARCH 22nd, 2012

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v / c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v / c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c} \ / \ \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\ MeV/c^2$ Proton: $m_p = 938.26\ MeV/c^2$ Neutron: $m_n = 939.55\ MeV/c^2$

Atomic mass unit: $1\ u = 931.5\ MeV/c^2$; electron volt: $1\text{eV} = 1.6 \times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law : $\lambda_m T = \dfrac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_B T)}$

Planck's law : $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \overline{E}(\lambda, T) = \dfrac{8\pi}{\lambda^4} \times \dfrac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV\,A$; $k_B = (1/11{,}600) eV/K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function; Bragg equation: $n\lambda = 2d \sin\vartheta$

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243 A$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n = C \dfrac{Z^2}{K_\alpha^2} \dfrac{1}{\sin^4(\phi/2)}$; $ke^2 = 14.4\ eV\,A$; $\hbar c = 1973\ eV\,A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097 \times 10^7\ m^{-1} = \dfrac{1}{911.3 A}$

Bohr atom: $E_n = -\dfrac{ke^2 Z}{2r_n} = -E_0 \dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6 eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529 A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\ a(k)\ e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta\omega \Delta t \sim 1$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Schrodinger equation: $-\dfrac{\hbar^2}{2\text{m}} \dfrac{\partial^2 \Psi}{\partial x^2} + \text{U(x)}\Psi(\text{x,t}) = \text{i}\hbar \dfrac{\partial\, \Psi}{\partial\, t}$; $\Psi(\text{x,t}) = \psi(\text{x})\text{e}^{-\text{i}\frac{\text{E}}{\hbar}t}$

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Time – independent Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2}+\mathrm{U}(\mathrm{x})\psi(\mathrm{x})=\mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi=1$

∞ square well: $\psi_{\mathrm{n}}(x)=\sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n=\frac{\pi^2\hbar^2n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e}=3.81eVA^2$ (electron)

Harmonic oscillator: $\Psi_{\mathrm{n}}(x)=H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n=(n+\frac{1}{2})\hbar\omega$; $E=\frac{p^2}{2m}+\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\omega^2A^2$; $\Delta n=\pm1$

Expectation value of $[Q]$: $<Q>=\int\psi^*(x)[Q]\psi(x)\,dx$; Momentum operator: $p=\frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[\mathrm{Q}]\ \Psi=q\ \Psi$ (q is a constant) ; uncertainty: $\Delta Q=\sqrt{<Q^2>-<Q>^2}$

Step potential: reflection coef: $R=\frac{(k_1-k_2)^2}{(k_1+k_2)^2}$, $T=1-R$; $k=\sqrt{\frac{2m}{\hbar^2}(E-U)}$

Tunneling: $\psi(x)\sim \mathrm{e}^{-\alpha x}$; $T=e^{-2\alpha\Delta x}$; $T=e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x)=\sqrt{\frac{2m[U(x)-E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi+\mathrm{U}(\vec{\mathrm{r}})\Psi(\vec{\mathrm{r}},\mathrm{t})=\mathrm{i}\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\vec{\mathrm{r}},\mathrm{t})=\psi(\vec{\mathrm{r}})\mathrm{e}^{-\mathrm{i}\frac{\mathrm{E}}{\hbar}t}$

3D square well: $\Psi(\mathrm{x,y,z})=\Psi_1(x)\Psi_2(y)\Psi_3(z)$; $\mathrm{E}=\frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{\mathrm{n},\ell,\mathrm{m}_\ell}(r,\theta,\phi)=R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$; $Y_\ell^{m_\ell}(\theta,\phi)=P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

Angular momentum: $\vec{\mathrm{L}}=\vec{r}\times\vec{p}$; $[L_z]=\frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $[L^2]Y_\ell^{m_\ell}=\ell(\ell+1)\hbar^2Y_\ell^{m_\ell}$; $[\mathrm{L}_z]Y_\ell^{m_\ell}=m_\ell\hbar Y_\ell^{m_\ell}$

Radial probability density: $P(r)=r^2\,|R_{n\ell}(r)|^2$; Energy: $\mathrm{E_n}=-\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$

Ground state of hydrogen and hydrogen - like ions: $\Psi_{1,0,0}=\frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$

Orbital magnetic moment: $\vec{\mu}=\frac{-e}{2m_e}\vec{L}$; $\mu_z=-\mu_Bm_l$; $\mu_\mathrm{B}=\frac{e\hbar}{2m_e}=5.79\times10^{-5}eV/T$

Spin 1/2: $s=\frac{1}{2}$, $|S|=\sqrt{s(s+1)}\hbar$; $S_z=m_s\hbar$; $m_s=\pm1/2$; $\vec{\mu}_s=\frac{-e}{2m_e}g\vec{S}$

Orbital + spin mag moment: $\vec{\mu}=\frac{-e}{2m_e}(\vec{L}+g\vec{S})$; Energy in mag. field: $U=-\vec{\mu}\cdot\vec{B}$

Two particles : $\Psi(\vec{r}_1,\vec{r}_2)=+/-\Psi(\vec{r}_2,\vec{r}_1)$; symmetric/antisymmetric

Screening in multielectron atoms: $\mathrm{Z}\to\mathrm{Z_{eff}}$, $1<\mathrm{Z_{eff}}<Z$

Orbital ordering:

1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 6d ~ 5f

Justify all your answers to all 8 problems. Write clearly.

Problem 1 (10 points)

Masses m_1 and m_2 are moving towards each other at the same speed u. When they collide they stick together to form a mass M moving at speed u' along the same line as the original velocities.

(a) Assuming Newtonian mechanics (no relativity), find expressions for u' and M in terms of m_1, m_2 and u.

(b) Using relativistic mechanics, find u' in terms of m_1, m_2 and u. Is it larger, smaller or equal than the value found in (a)?

(c) Using relativistic mechanics find an expression for M. Is it larger, smaller or equal than the value found in (a)? If it is not equal, explain why it is different.

Problem 2 (10 points)

A black body is at temperature 5000 °K.

(a) At what wavelength λ_o does it emit maximum power? Give your answer in A.

If its temperature is now increased to 10000 °K:

(b) By what factor does the power emitted at the wavelength λ_o found in (a) change? Does it increase or decrease?

(c) By what factor does the total power emitted at all wavelengths change? Increase or decrease?

(d) By what factor does the power emitted at wavelength 100,000,000,000λ_o change? Increase or decrease?

Problem 3 (10 points)

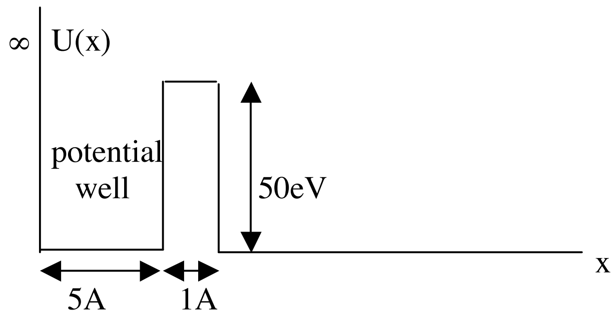
Radiation with wavelengths in the range 100A to 150A incident on a gas of hydrogen-like ions of atomic number Z at room temperature ionizes some of those ions.

(a) What is Z for those ions?

(b) What is the maximum kinetic energy of an electron produced by this radiation?

(c) If the radiation is in the range 110A to 150A, how many absorption lines are seen, and what are their wavelengths, in A?

Problem 4 (10 points + 2 points extra credit)



An electron is in the ground state of the1-dimensional potential well of width 5A shown in the figure. To the left the potential is infinite, to the right there is a potential barrier of width 1A and height 50eV.
(a) Find the ground state energy of this electron in eV assuming it is an infinite well.
(b) Estimate a better value for the ground state energy by taking into account that the electron wavefunction penetrates the potential barrier.
For the following parts use the value of the energy found in (a).
(c) Estimate the transmission probability of this electron through this barrier.
(d) Estimate the speed of this electron in the potential well. Find your answer first as v/c, then find v in A/s using that $c=3x10^{18}$ A/s.
(e) From the result of (d), estimate how many times per second will the electron hit the right wall.
(f) In a time interval of 1s, what is the probability that the electron will tunnel out of the well? Use the results of (c) and (e) to answer this.
Use $\frac{\hbar^2}{2m_e} = 3.81eVA^2$.

Problem 5 (10 points)

Consider a two-dimensional box of side lengths 5A in the x direction and 3A in the y direction (with the left lower corner located at (x,y)=(0,0)).
(a) Find the 3 lowest energy levels for an electron in this box. Giver their quantum numbers and energies, in eV.
(b) What is the lowest energy state for which the probability of finding the electron at distance 1.5A from the boundary of the box in the y direction (i.e. for y=1.5A) is zero for all x? Give its energy in eV, its quantum numbers and its wavefunction $\Psi(x,y)$.
(c) Assume there are 5 electrons in this box, that don't interact with each other. Taking into account that the electron has spin ½ and the Pauli exclusion principle, find the ground state energy of this system, in eV, by adding the energies of all the electrons.
(d) What is the wavelength (in A) of the lowest energy photon that this 5-electron system can absorb? Assume there are no selection rule but of course the Pauli principle is obeyed.

Problem 6 (10 points)
The electron in an excited state of hydrogen is described by the wavefunction
$\Psi(r,\theta,\phi) = Cr^2 e^{-r/3a_0} \sin^2\theta \, e^{i\phi}$
with C a constant.
(a) Give the values of the quantum numbers n, ℓ, m_ℓ. Justify.
(b) Find the most probable r for this electron. Compare with the value in the Bohr atom.
(c) Find the value of <r> in the following way: first find an expression for C^2 in terms of an integral, but don't do the integral. Then, find an expression for <r> that has C^2 and another integral, and replace C^2 in terms of the integral found earlier. Only now do both integrals and note that a lot of factors simplify.
(d) Find $< \frac{1}{r} >$ using the same procedure as in (c). Compare with the value in the Bohr atom.
Use $\int_0^\infty dr\, r^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$

Problem 7 (10 points)
Electrons in a gas of hydrogen atoms at room temperature are in the energy state n=2, $\ell = 1$, in the presence of a magnetic field of magnitude 15T. Assume they have all allowed values of m_ℓ.
(a) Find all possible values for the energies of the photons that will be emitted when these electrons make transitions to the n=1 state, ignoring the electron spin.
(b) Taking into account the electron spin: how many different energy levels does the n=2, $\ell = 1$ level split into, and what are their energy values relative to the energy of this level in the absence of the magnetic field? Give also the quantum numbers for each.
Assume that because of the large magnitude of the magnetic field the spin-orbit interaction can be ignored.
(c) Ignore the electron spin for this question. If this system is cooled to low temperatures while the electrons are in the n=2, $\ell = 1$ state with magnetic field 15T, most electrons will occupy the state with one particular m_ℓ value if the temperature is sufficiently low. Which m_ℓ value is that, and can you roughly estimate for how low a temperature this will happen?

Problem 8 (10 points)
An electron in a one-dimensional system is described by the wavefunction
$\Psi(x) = C\cos(kx)$
in the region -a<x<a, and $\Psi(x) = 0$ outside. k is a real number. Using that the momentum operator is given by $[p] = \frac{\hbar}{i}\frac{\partial}{\partial x}$, answer the following:
(a) Is the momentum p a sharp observable? Show mathematically why or why not.
(b) Is p^2 a sharp observable? Show mathematically why or why not.
(c) Find the uncertainties in p and p^2, i.e. Δp and $\Delta(p^2)$. Your answers should be consistent with your answers in (a) and (b). Assume Ψ is normalized.

Justify all your answers to all problems. Write clearly.